Formation of protons in a new cumulative region of central rapidities and large transverse momenta due to coherent coalescence of quarks

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"Relativistic Nuclear Physics and Quantum Chromodynamics"

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Study of inclusive production cross-sections
of pions and protons in a new cumulative region:
central rapidities, large transverse momenta

$$d+d \rightarrow \pi+X$$

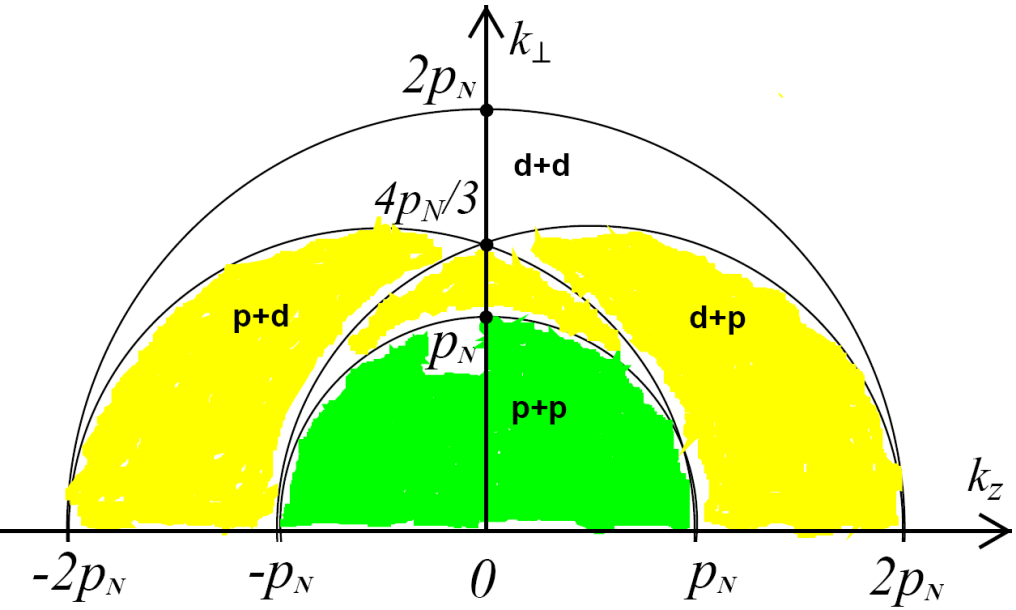
$$d+d \rightarrow p+X$$

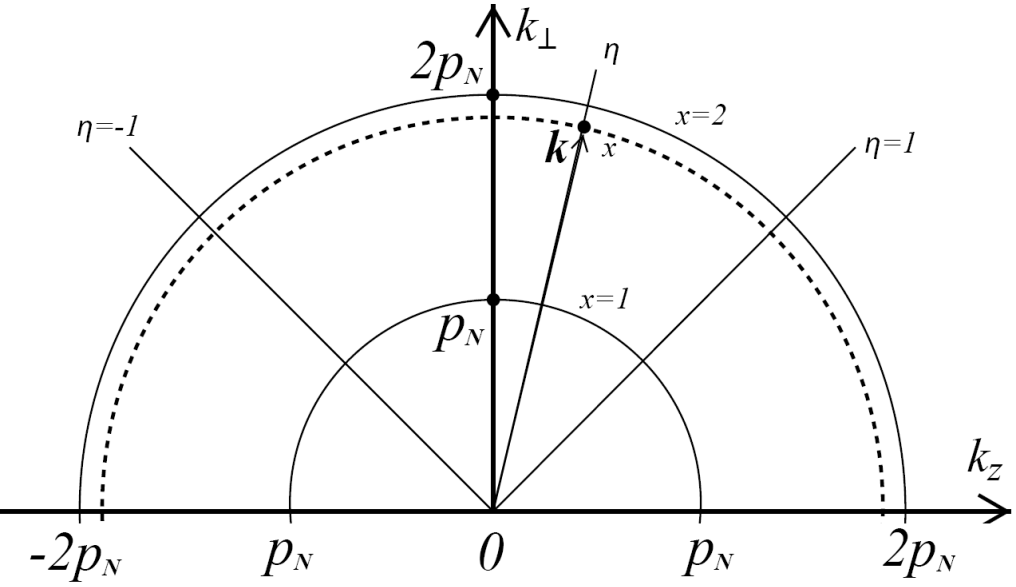
outside p+p kinematics:

$$p+p \rightarrow \pi+X$$
$$p+p \rightarrow p+X$$

The mechanisms of pion and proton production are different!

Cumulative regions in dd and pd collisions





$p_N >> m_N$ $\quad p_N \approx \sqrt{s}/2$

We expect
for the inclusive cross-section
of the production of particles with large transverse momenta at midrapidities:

$$\frac{k_0\, d^3\sigma}{d^3\mathbf{k}} = f(x,\eta) = C\, s^{-q_1}\, (f-x)^{q_2}\, F(\eta)$$

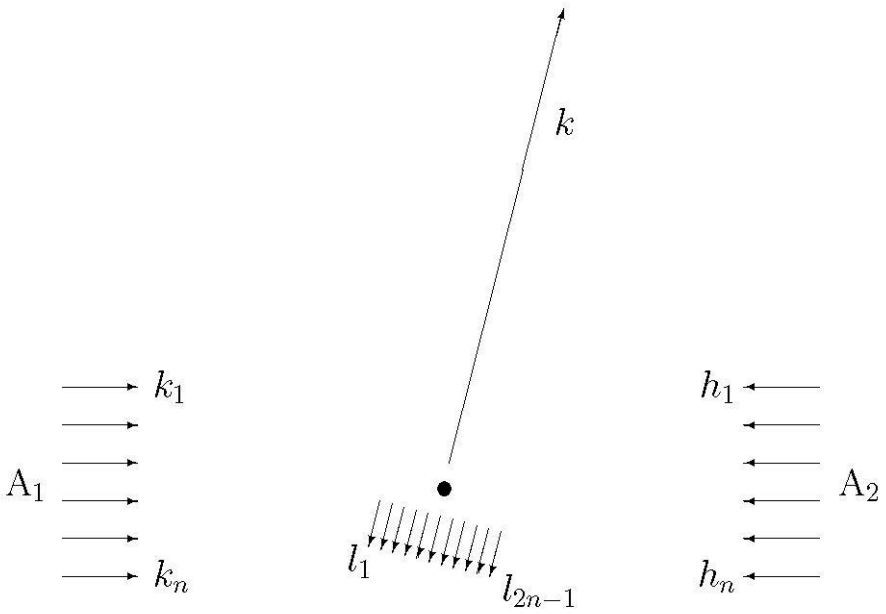
Quark Counting Rules with two asymptotic parameters:
$s >> m^2$ and $(f-x)$

q_1, q_2 are different for production of pions and protons!

x - cumulative number
$\eta = -\ln \mathrm{tg}\,\frac{\theta^*}{2}$ - pseudorapidity

The mechanism of pion production
in the new cumulative region of central rapidities and large transverse momenta due to flucton-flucton interaction.

Kinematics



d+d → π+X at quark level
($A_1 = A_2 = A = 2$, $n = 6$)

$p_N = P_A/A \qquad p_N \gg m_N$

Initial state:

$k_i \sim P_A/n = p_N/3$
$h_i \sim -P_A/n = -p_N/3$
$n = 3A$

Final state:

$k \sim P_A = A\,p_N = n\,p_N/3$

$l_i \sim -P_A/(2n-1) = -\frac{n}{3(2n-1)}p_N$

First small parameter: $\frac{m_N}{p_N} = \frac{2m_N}{\sqrt{s_{NN}}} \ll 1$

Second small parameter: $1 - \frac{k}{k_{max}} \ll 1$

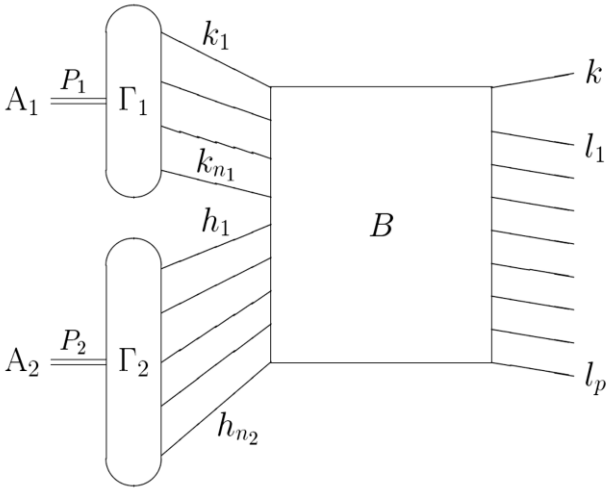
$k \to k_{max} \Rightarrow M_X{}^2 = (\sum_{i=1}^{2n-1} l_i)^2 \to M_{Xmin}{}^2$

$l_1 = l_2 = \ldots = l_{2n-1} = -\frac{k_{max}}{2n-1}$

V. Vechernin, S. Yurchenko
Int.J.Mod.Phys. E, 2441022,
DOI: 10.1142/S0218301324410222

$s = s_{NN}$

Quark counting rules for the inclusive cross sections of pion production in new cumulative region of large transverse momenta at mid-rapidities (the decrease with both the initial energy s and the cumulative number *x*)



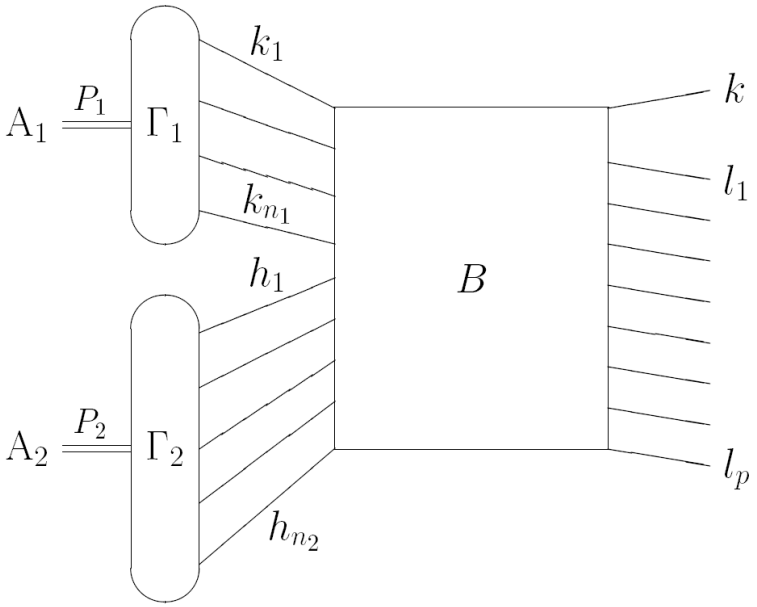
$$p = n_1 + n_2 - 1$$

To evaluate this behavior and find asymptotes at s>>m and (2-x)<<1 we need to generalize the quark counting rules, known now only for a) the inclusive cross sections in the fragmentation region (|t|<<s) and b) the elastic and quasielastic cross sections in the high pT region (|t|~s), to the case of inclusive cross sections in the high pT region (|t|~s).

$$I(x) \equiv (2\pi)^3 \, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p-\frac{5}{2}}}{(m^2R^3)^{p-1}s^{(p+3)/2}}$$

Amplitude (T)

$T =$



$p = n_1 + n_2 - 1$

$$I(\mathbf{k}) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{1}{J}\int |T|^2\, d\tau_p \ ,$$

$$J = 2A_1A_2\sqrt{s(s-4m_N^2)}$$

$$d\tau_p \equiv (2\pi)^4 \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2l_{i0}(2\pi)^3}$$

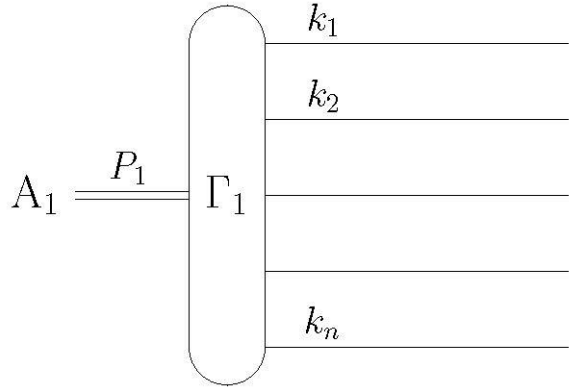
$s = s_{NN}$

$$I(\mathbf{k}) = \frac{1}{J}|T|^2\, \tau_p$$

$$\tau_p = (2\pi)^4 \int \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2l_{i0}(2\pi)^3}$$

Light-cone partonic wave function

S.J. Brodsky, P. Hoyer, A. Mueller, W.-K. Tang, Nucl.Phys. B369 (1992) 519

$J_{n1} =$ 

$$x_i \equiv \frac{k_{i+}}{p_{1+}}\,, \qquad k_{i+} \equiv \frac{k_{i0}+k_{iz}}{\sqrt{2}}$$

$$p_1 = P_1/A_1$$

$$\psi_1(x_i, \mathbf{k}_{i\perp}) \equiv \frac{\Gamma_1(x_i, \mathbf{k}_{i\perp})}{\sum_{i=1}^{n_1} \frac{m^2+\mathbf{k}_{i\perp}^2}{x_i} - A_1 m_N^2}$$

$$J_{n_1} = \int \psi_1(x_i, \mathbf{k}_{i\perp})\, 2\delta(\sum_{i=1}^{n_1} x_i - A_1)\, (2\pi)^3 \delta^{(2)}(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i} \frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3}$$

M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614

$$J_{n_1} \sim \psi_1(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)$$

$$J_{n_1} = \frac{C_1}{m^{(n_1-1)/2} R_1^{3(n_1-1)/2}}$$

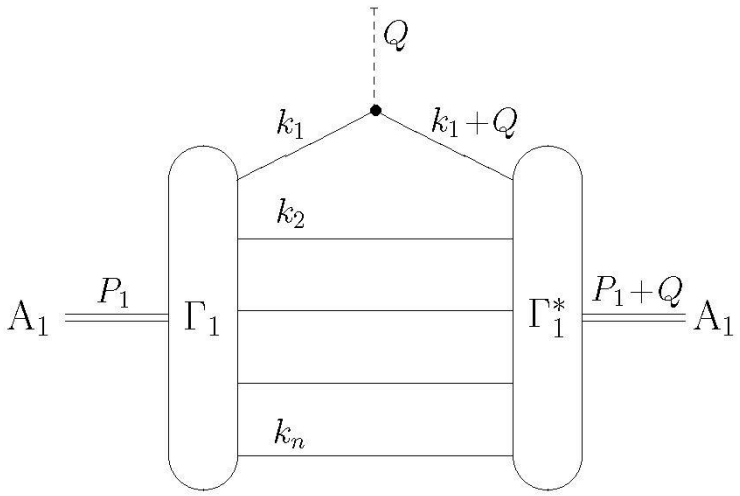
m - mass of the constituent quark
R_1- size of the system (R_1= R_A or R_N)
C_1- dimensionless constant, independent of the dimensional parameters of the model

$$J_n^2 \sim |\psi(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)|^2 \sim \frac{1}{V^{n-1}} \sim \frac{1}{R^{3(n-1)/2}}$$

for a wave function with one dimensional parameter

Normalization of the wave function (Γ)

$F(Q^2) =$



$$Q^2 \to 0 \quad \Rightarrow \quad F(Q^2) \to 1$$

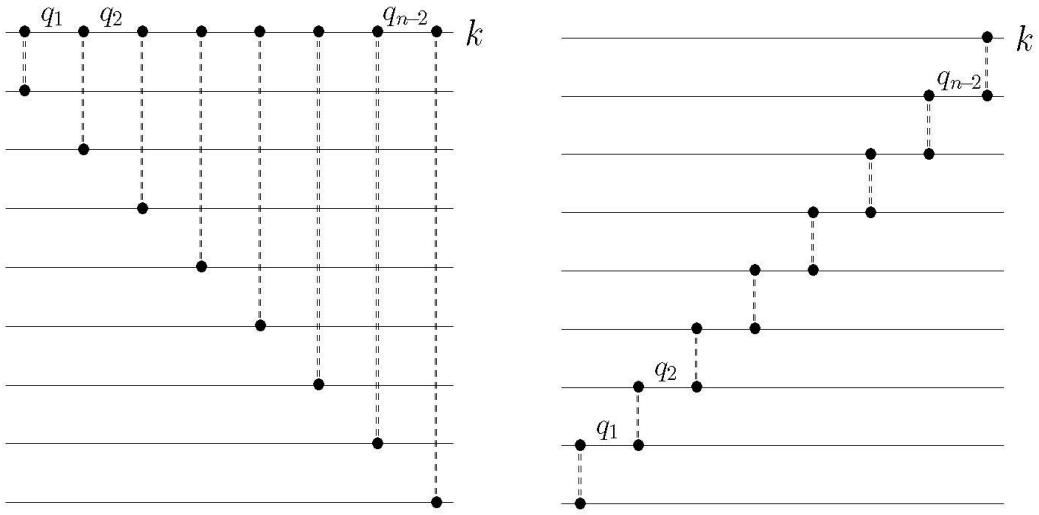
M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614

$$\int |\psi_1(x_i, \mathbf{k}_{i\perp})|^2 \, 2\delta(\sum_{i=1}^{n_1} x_i - A_1) \, (2\pi)^3 \delta^{(2)}(\sum_{i=1}^{n_1} \mathbf{k}_{i\perp}) \prod_{i=1}^{n_1} \frac{dx_i}{2x_i} \frac{d^2\mathbf{k}_{i\perp}}{(2\pi)^3} = A_1$$

$$\psi_1(x_i, \mathbf{k}_{i\perp}) \equiv \frac{\Gamma_1(x_i, \mathbf{k}_{i\perp})}{\sum_{i=1}^{n_1} \frac{m^2 + \mathbf{k}_{i\perp}^2}{x_i} - A_1 m_N^2}$$

Similarly for $\psi_2(y_i, \mathbf{h}_{i\perp})$ $\qquad y_i \equiv \frac{h_{i-}}{p_{2-}}$, $\qquad h_{i-} \equiv \frac{h_{i0} - h_{iz}}{\sqrt{2}}$ $\qquad p_2 = P_2/A_2$

Block of hard exchanges (B)



S.J. Brodsky, B.T. Chertok,
Phys.Rev. D 14 (1976) 3003

$$B = \frac{C_B}{s^{n-2}} = \frac{C_B}{s^{n_1+n_2-2}}$$

$$n = n_1 + n_2$$

$$p = n - 1$$

$$r_B \sim \frac{1}{\sqrt{s}}$$

$$B(k_i, h_i; \mathbf{k}, \mathbf{l}_i) \approx B(P_1/n_1, P_2/n_2; \mathbf{k}_{max}, -\mathbf{k}_{max}/p)$$

$$T = J_{n_1} J_{n_2} B.$$

$$T = \frac{C_1\, C_2\, C_B}{m^{(n-2)/2} R_1^{3(n_1-1)/2} R_2^{3(n_2-1)/2} s^{n-2}} = \frac{C_1\, C_2\, C_B}{m^{(n-2)/2} R^{3(n-2)/2} s^{n-2}}$$

$$I(\mathbf{k}) = \frac{1}{J} |T|^2\, \tau_p$$

$$\tau_p = (2\pi)^4 \int \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2 l_{i0} (2\pi)^3}$$

After Calculation of Phase Volume and Relation with Cumulative Number:

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)\, m]^2}\ .$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p-\frac{5}{2}}}{(m^2R^3)^{p-1}\, s^{(p+3)/2}}$$

$$p = p(A)$$

two (!)
small parameters:
$m/\sqrt{s} \ll 1$
$A - x \ll 1$

A=2 d+d → π+X $\quad I_{dd\to\pi}(x) \sim s^{-7}\,(2-x)^{14}$

A=1 p+p → π+X $\quad I_{pp\to\pi}(x) \sim s^{-4}\,(1-x)^{5}$

$$r_B \sim \frac{1}{\sqrt{s}}$$

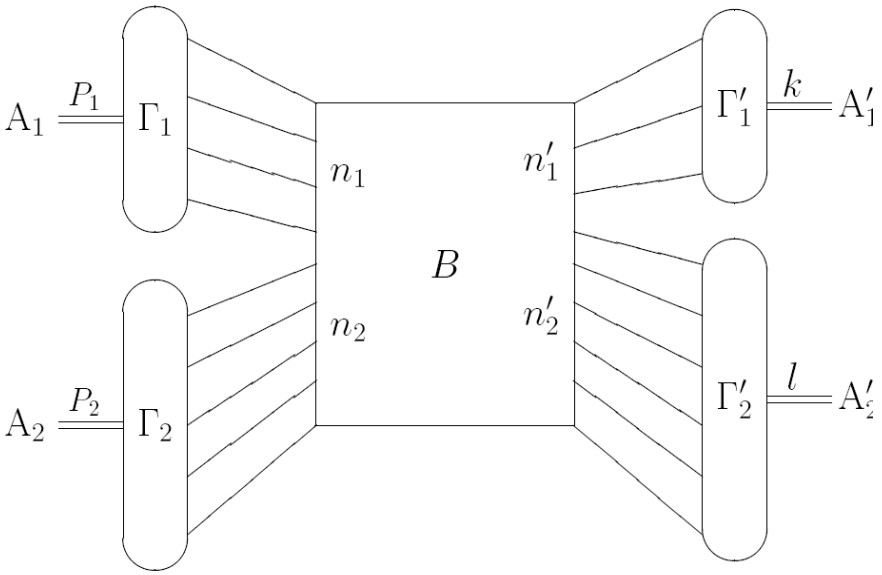
Changes may occur at moderate NICA energy:

$M_X \sim (A-x)\sqrt{s}$

so at rather small NICA energies: partonic phase volume => hadronic phase volume only **d+d → π+NNNN,** neglecting **d+d → π+ πNNNN** and so on.

Comparison with Quark Counting Rules for Quasi-Elastic Processes

one small parameters: $m/\sqrt{s} \ll 1$



$$\frac{d\sigma}{dt} \sim \frac{1}{s^{n_1+n_2+n'_1+n'_2-2}}$$

Matveev V.A., Muradyan R.M., Tavkhelidze A.N., Lett. Nuovo Cimento 7 (1973) 719

Brodsky S., Farrar G., Phys.Rev.Lett. 31 (1973) 1153

Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003

Uzikov Yu. N., JETP Lett. 81 (2005) 303

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\, s^2_{A_1A_2}} |T_{2\to 2}|^2 = \frac{1}{16\pi\, A_1^2 A_2^2 s^2} |T_{2\to 2}|^2$$

$$T_{2\to 2} = J_{n_1}\, J_{n_2}\, B\, J_{n'_1}\, J_{n'_2}$$

$$\frac{d\sigma}{dt} = \frac{C'}{s^{2n-2}\, m^{2n-4} R_1^{3(n_1-1)} R_2^{3(n_2-1)} R_1'^{3(n'_1-1)} R_2'^{3(n'_2-1)}}$$

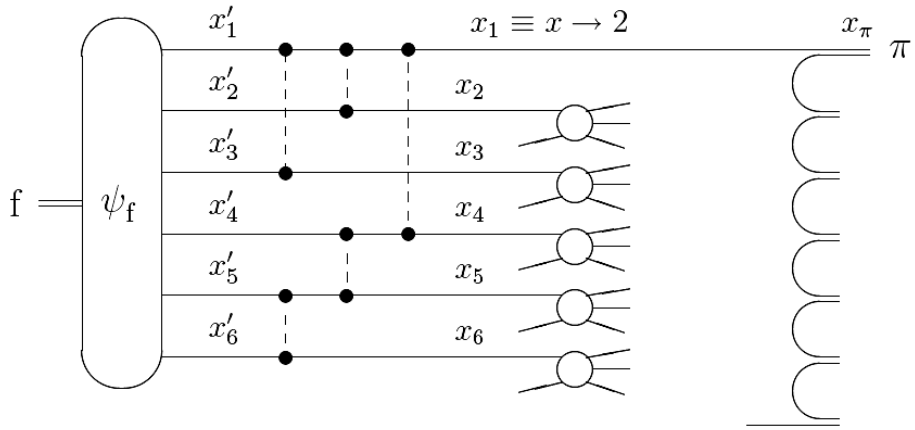
$n = n_1 + n_2$
$= n'_1 + n'_2$

Yu.L. Dokshitzer, QCD Phenomenology, Lectures at the CERN–Dubna School, Pylos, August 2002

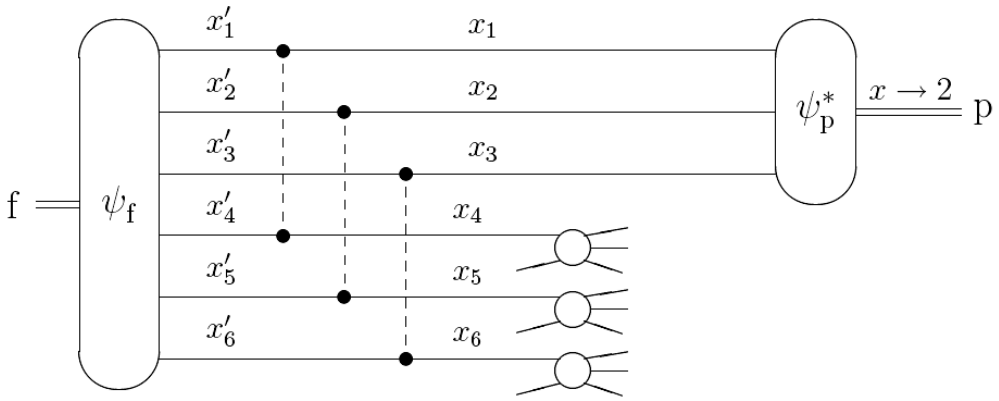
Early validity of QCR in the deuteron break-up by a photon, $\gamma + D \to p + n$

The mechanism of proton production
at central rapidities and large transverse momenta
due to flucton-flucton interaction.

Coherent Quark Coalescence and Production of Cumulative Protons **in Fragmentation Region**



- the cumulative pion production
by hadronization of one fast quark
M.A. Braun, V.V. Vechernin,
*Nucl.Phys.**B 427**, 614 (1994);*
*Phys.Atom.Nucl. **60**, 432 (1997);*
***63**, 1831 (2000)*

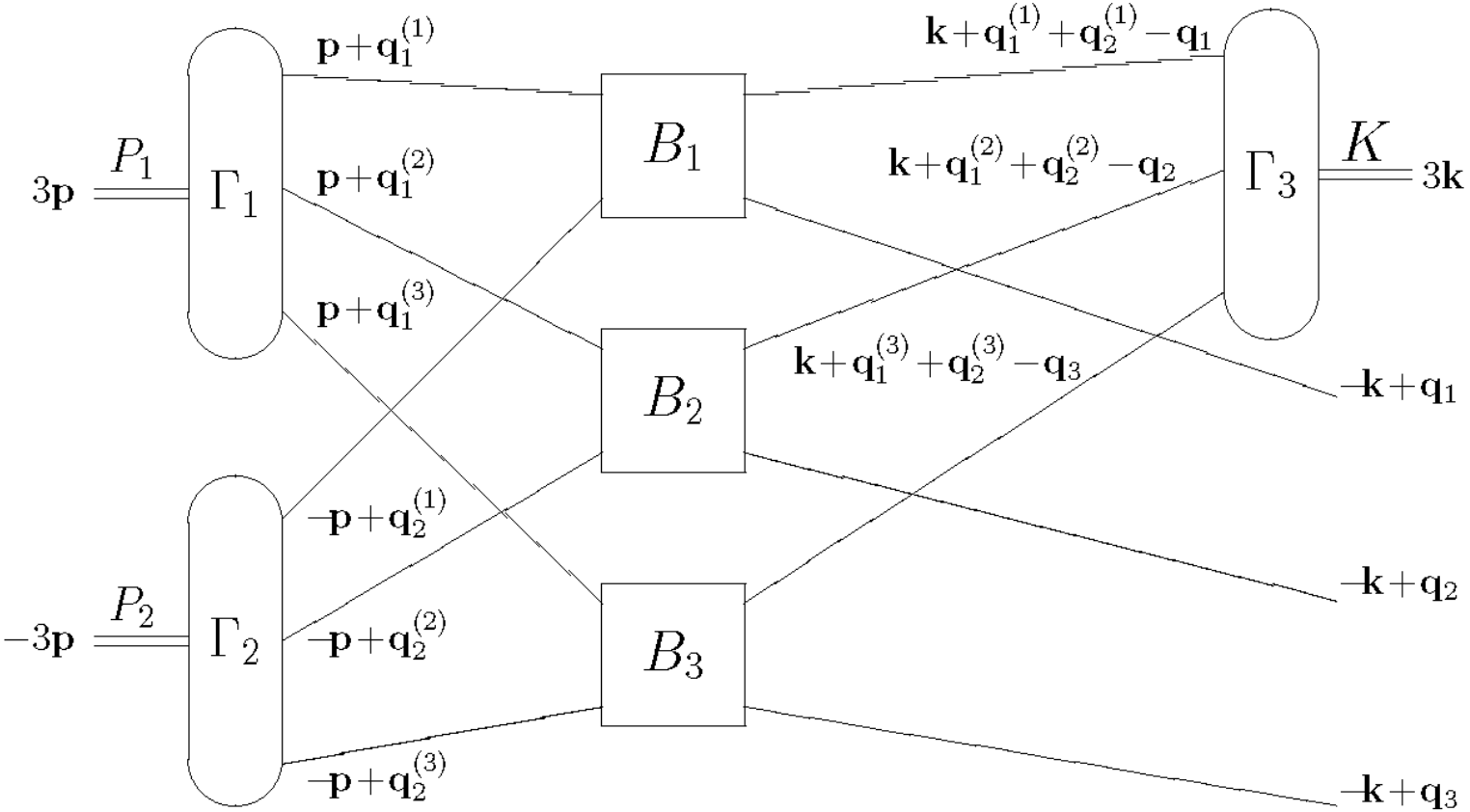


- the cumulative proton production
by **coherent** quark coalescence mechanism:
M.A. Braun, V.V. Vechernin,
*Nucl.Phys.**B 92**, 156 (2001);*
*Theor.Math.Phys **139**, 766 (2004);*
V.Vechernin,
AIP Conf.Proc.1701 (2016) 060020.

The last diagram **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).
But instead of using the relativistic generalization of non-relativistic NN wave function
the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge
*Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. **B369** (1992) 519.*
was developed and applied.

Consider the technique using the simplest example p+p → p+X at

$$x \to 1, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}/2, \quad \theta^* \sim 90^\circ$$



$$\sum_{i=1}^{3} \mathbf{q}_1^{(i)} = 0$$

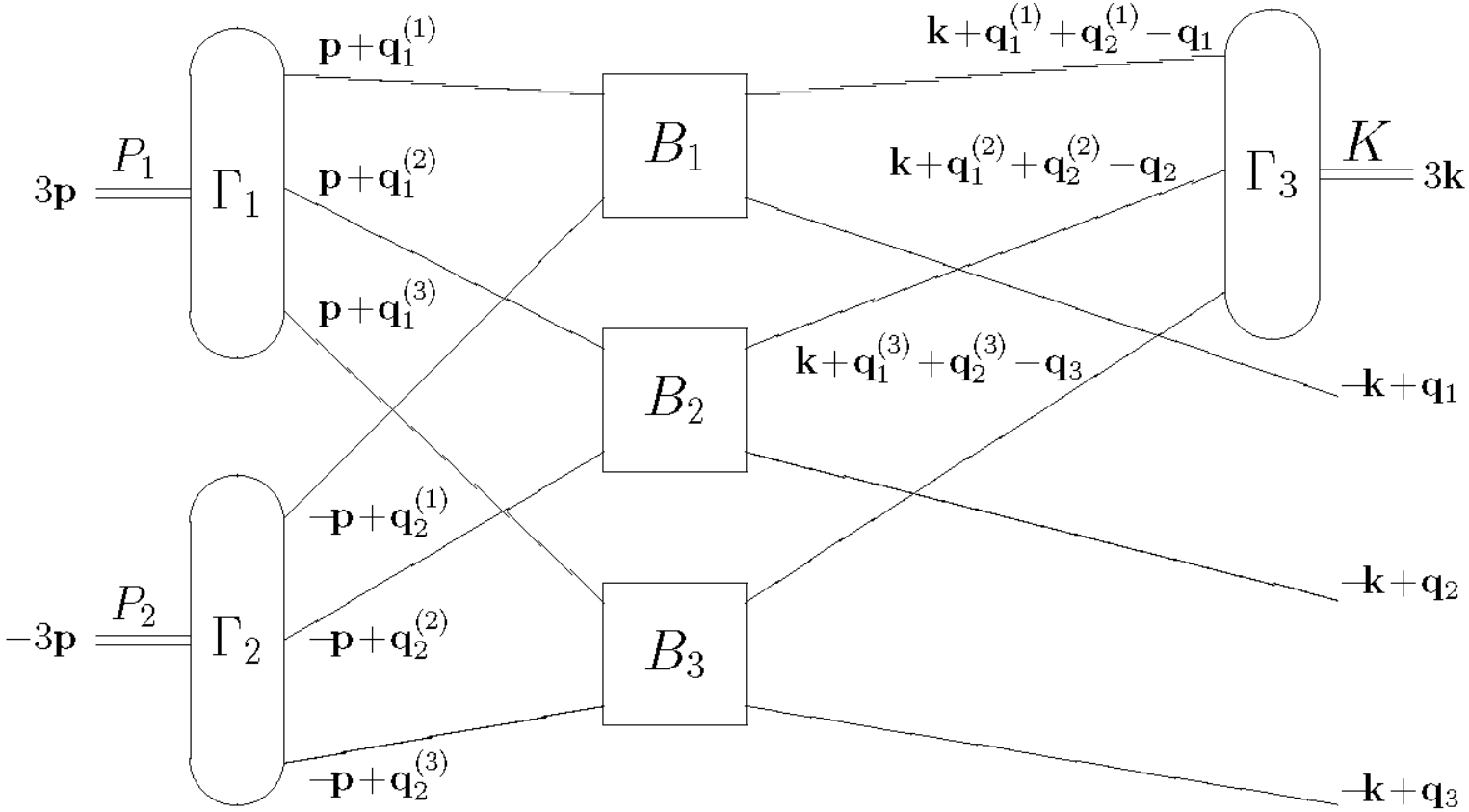
$$\sum_{i=1}^{3} \mathbf{q}_2^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_i = 0$$

Here will be no proportionality $J_{n_1} \sim \psi_1(z_i - z_j = 0, \mathbf{r}_{i\perp} - \mathbf{r}_{j\perp} = 0)$

Separation of hard blocks (n=3)

$$T(n\mathbf{p};n\mathbf{k},\mathbf{q}_i)=$$



$$\sum_{i=1}^{3}\mathbf{q}_1^{(i)}=0$$

$$\sum_{i=1}^{3}\mathbf{q}_2^{(i)}=0$$

$$\sum_{i=1}^{3}\mathbf{q}_i=0$$

$$T(n\mathbf{p};n\mathbf{k},\mathbf{l}_i)=J(n\mathbf{p};n\mathbf{k},\mathbf{q}_i)\prod_{j=1}^{n}B_j(\mathbf{p};\mathbf{k})$$

Used approximation

$$l^2 - m^2 + \mathrm{i}\epsilon = [l_0 + E(\mathbf{l})][l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon] \approx 2E(\mathbf{l})[l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon]$$

Integration over zero components of momenta

It leads to Heitler's (“old fashioned”) perturbation theory:

1) **All particles are considered on mass shell**
2) **Inclusion of "energy denominators" between interactions**
3) **Permutation of interactions in time where possible**
4) **Additional factor 1/[2*E*(l)] from each propagator**

$$\frac{1}{[\sum_i E(\mathbf{l}_i) - E_{init} - \mathrm{i}\epsilon]}$$

Weakly coupled systems

Third small parameter: $\alpha = \sqrt{\frac{\varepsilon}{m}} \ll 1$

$$M = n(m - \varepsilon)$$

$$\left\langle \frac{\mathbf{q}^{*2}}{2m} \right\rangle \simeq \varepsilon \qquad |\mathbf{q}^*| \simeq \sqrt{m\varepsilon} = m\,\alpha$$

- in the rest frame

$$|\mathbf{q}_\perp| \simeq \sqrt{m\varepsilon} = m\,\alpha$$

$$|q_z| \simeq \gamma |q_z^*| \simeq \gamma\sqrt{m\varepsilon} = \frac{E_p}{m}\sqrt{m\varepsilon} = E_p\,\alpha$$

$$E(\mathbf{p}+\mathbf{q}) \equiv \sqrt{(\mathbf{p}+\mathbf{q})^2 + m^2}$$

$$E(\mathbf{p}+\mathbf{q}) = E_p \left\{ 1 + \frac{(\mathbf{qp})}{E_p^2} + \frac{1}{2E_p^2}\left[\mathbf{q}^2 - \frac{(\mathbf{qp})^2}{E_p^2} \right] + O(\alpha^3) \right\}$$

$$E_p \equiv E(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}\ , \quad E_k \equiv E(\mathbf{k})$$

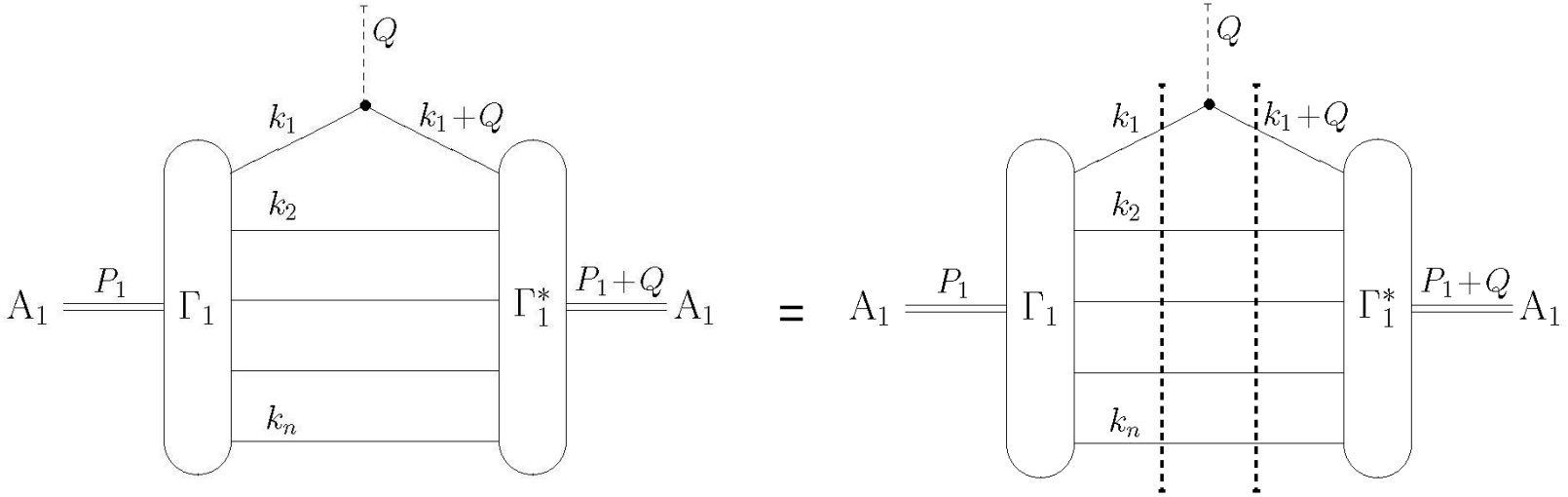
It is **not assumed** here that **|p|>>*m* !**

Valid both in the rest frame of the nucleus (p=0) and in the frame in which the nucleus moves with relativistic momentum *n*p or *n*k (|p|>>*m*, |k|>>*m*)

$$\frac{(\mathbf{qp})}{E_p^2} = \frac{q_z|\mathbf{p}|}{E_p^2} \simeq \frac{E_p\,\alpha\,|\mathbf{p}|}{E_p^2} = \frac{|\mathbf{p}|}{E_p}\alpha$$

Connection of vertices Γ_i with wave functions. Normalization.

$F(Q^2) =$



$$Q^2 \to 0 \quad \Rightarrow \quad F(Q^2) \to 1$$

$$\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)}) = \frac{\Gamma_{n\mathbf{p}}(\mathbf{q}^{(i)})}{\sqrt{n}2E_p\left[\sum_{i=1}^{n} E(\mathbf{p}+\mathbf{q}^{(i)}) - E_{init} - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

Connection of vertices Γ_i with wave functions. Normalization.

In the rest frame of the nucleus (p=0):

$$\varphi_0(\mathbf{q}^{*(i)}) = \frac{\Gamma_0(\mathbf{q}^{*(i)})}{\sqrt{n}2m\left[\sum_{i=1}^{n}\frac{\mathbf{q}^{*(i)2}}{2m} - n\varepsilon - \mathrm{i}\epsilon\right]} = \frac{\Gamma_0(\mathbf{q}^{*(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}\mathbf{q}^{*(i)2} - n2m\varepsilon - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_0(\mathbf{q}^{*(i)})|^2 \prod_{i=1}^{n-1}\frac{d^3\mathbf{q}^{*(i)}}{2m(2\pi)^3} = 1$$

In the frame in which the nucleus moves with relativistic momentum *n*p :

$$\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)}) = \frac{\Gamma_{n\mathbf{p}}(\mathbf{q}^{(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2 - n2m\varepsilon - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1}\frac{d^3\mathbf{q}^{(i)}}{2m\gamma(2\pi)^3} = 1$$

$$\gamma = \frac{E_p}{m}$$

$$\mathbf{q}_\perp^{(i)} = \mathbf{q}_\perp^{*(i)} \simeq \sqrt{m\varepsilon} = m\,\alpha$$
$$q_z^{(i)} = \gamma q_z^{*(i)} \simeq \gamma\sqrt{m\varepsilon} = E_p\,\alpha$$

$$\alpha = \sqrt{\frac{\varepsilon}{m}}$$

Connection of vertices Γ_i with wave functions. Light-cone variables.

Light-cone partonic wave function.

S.J. Brodsky, P. Hoyer, A. Mueller, W.-K. Tang, Nucl.Phys. B369 (1992) 519;
M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614.

$$\varphi_{lc}(x_i, \mathbf{q}_\perp^{(i)}) = \frac{\Gamma_{lc}(x_i, \mathbf{q}_\perp^{(i)})}{\sqrt{Ax_0}\left[\sum_{i=1}^n \frac{m_q^2+\mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_{lc}(x_i, \mathbf{q}_\perp^{(i)})|^2 \prod_{i=1}^{n-1} \frac{dx_i\, d^2\mathbf{q}_\perp^{(i)}}{2x_i(2\pi)^3} = 1$$

$$x_i = \frac{k_+^{(i)}}{p_+}$$

$$\sum_{i=1}^n \mathbf{q}_\perp^{(i)} = 0$$

$$\sum_{i=1}^n x_i = A$$

$$x_0 \equiv A/n$$

$$A = 1 \quad n = 2$$

$$\frac{m_q^2 + \mathbf{q}_\perp^2}{x_1} + \frac{m_q^2 + (-\mathbf{q}_\perp)^2}{x_2} = \frac{m_q^2 + \mathbf{q}_\perp^2}{x(1-x)}$$

Connection of vertices Γ_i with wave functions. Light-cone variables.

Light-cone partonic wave function.

$$M_A^2/A = (nm_q - n\varepsilon_q)^2/A \approx \frac{n}{x_0}(m_q^2 - 2m_q\varepsilon_q)$$

$$x_i = \frac{k_+^{(i)}}{p_{N+}} \approx \frac{k_z^{(i)}}{p_{Nz}} = \frac{p_{Nz}A/n + q_z^{(i)}}{p_{Nz}} = x_0 + \frac{q_z^{(i)}}{p_{Nz}} = x_0 + \frac{q_z^{(i)}}{p_{Nz}}$$

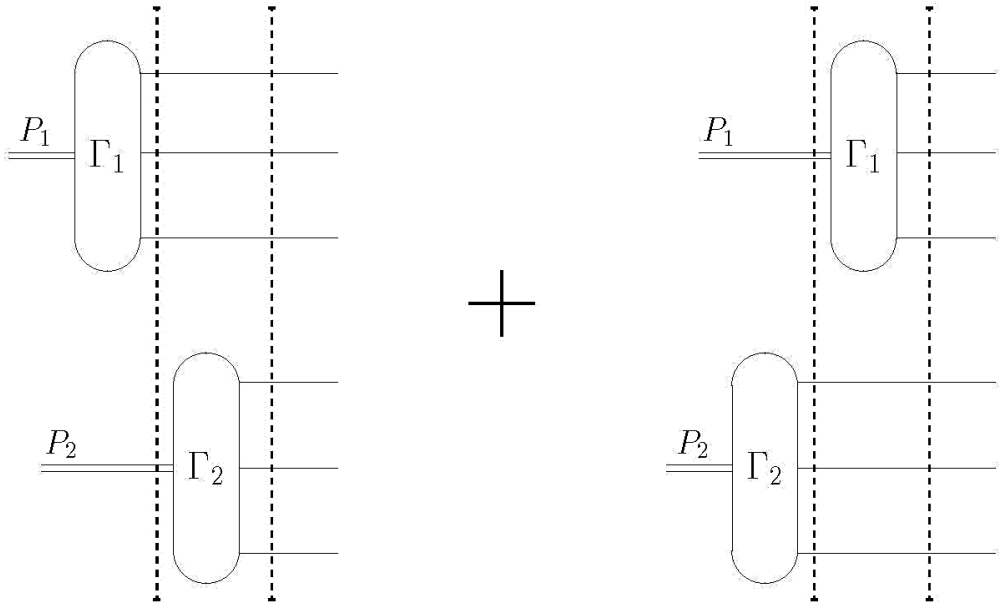
$$\gamma = \frac{E_q}{m_q} \approx \frac{p_{Nz}A/n}{m_q} = \frac{p_{Nz}x_0}{m_q}$$

$$\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} = \sum_{i=1}^{n} \frac{\mathbf{q}_\perp^{(i)2}}{x_0} + \frac{m_q^2}{x_0}\sum_{i=1}^{n}\left[1 - \frac{q_z^{(i)}}{x_0 p_{Nz}} + \frac{q_z^{(i)2}}{(x_0 p_{Nz})^2}\right] - \frac{n}{x_0}(m_q^2 - 2m_q\varepsilon_q)$$

$$\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} = \frac{1}{x_0}\sum_{i=1}^{n}\left[\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2\right] - \frac{2nm_q\varepsilon_q}{x_0}$$

$$\varphi_{lc}(q_z^{(i)}, \mathbf{q}_\perp^{(i)}) = \frac{\Gamma_{lc}(q_z^{(i)}, \mathbf{q}_\perp^{(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}[\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2] - 2nm_q\varepsilon_q - \mathrm{i}\epsilon\right]}$$

Permutations of Γ_i vertices (n=3)



$$\varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)}) \equiv \frac{\Gamma_1}{[E_1 - E_1^{init}]2E_p\sqrt{3}} ,$$

$$\varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)}) \equiv \frac{\Gamma_2}{[E_2 - E_2^{init}]2E_p\sqrt{3}}$$

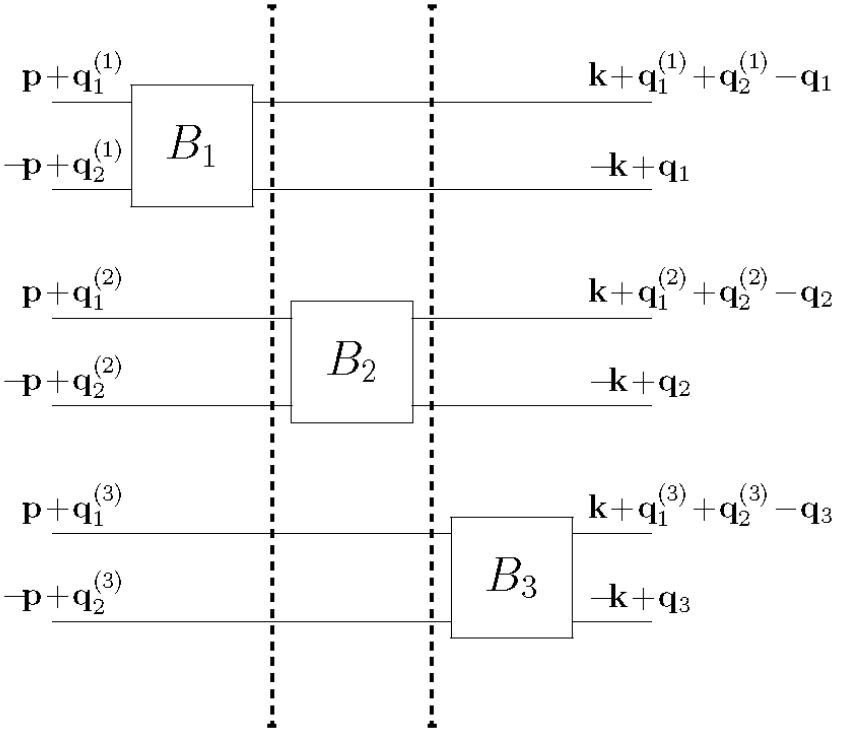
$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=2}^{n} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

$$\left\{ \frac{1}{[E_1 - E_1^{init} - \mathrm{i}\epsilon]} + \frac{1}{[E_2 - E_2^{init} - \mathrm{i}\epsilon]} \right\} \frac{1}{[E_1 + E_2 - E_1^{init} + E_2^{init} - \mathrm{i}\epsilon]} =$$

$$= \frac{1}{[E_1 - E_1^{init}][E_2 - E_2^{init}]}$$

$$E_1 \equiv \sum_{i=1}^{3} E(\mathbf{p} + \mathbf{q}_1^{(i)}) , \quad E_2 \equiv \sum_{i=1}^{3} E(-\mathbf{p} + \mathbf{q}_2^{(i)}) = \sum_{i=1}^{3} E(\mathbf{p} - \mathbf{q}_2^{(i)})$$

Permutations of B blocks (n=3)



$$\frac{E_p^2}{[-a_1-\mathrm{i}\epsilon][a_3-\mathrm{i}\epsilon]}$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)}-\mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-2\mathbf{q}_i)$$

$$\sum_{i=1}^{3} a_i = 0$$

$$\frac{1}{b-\mathrm{i}\epsilon} = \frac{1}{b} + i\pi\delta(b)$$

$$\frac{-E_p^2}{[a_1+\mathrm{i}\epsilon][a_3-\mathrm{i}\epsilon]} + \frac{-E_p^2}{[a_3+\mathrm{i}\epsilon][a_1-\mathrm{i}\epsilon]} = -\frac{2E_p^2}{a_1 a_3} - 2\pi^2 E_p^2 \delta(a_1)\delta(a_3)$$

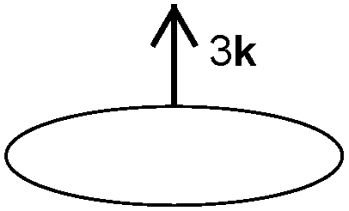
$$\frac{1}{a_1 a_3} + \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} = \frac{a_1+a_2+a_3}{a_1 a_2 a_3} = 0$$

$$D(a_1,a_2,a_3) = -2\pi^2 E_p^2[\delta(a_1)\delta(a_2) + \delta(a_2)\delta(a_3) + \delta(a_1)\delta(a_3)]$$

Inclusive cross section of proton prod. in p+p → p+X (n=3)

$$x \to 1, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}/2, \quad \theta^* \sim 90^\circ$$

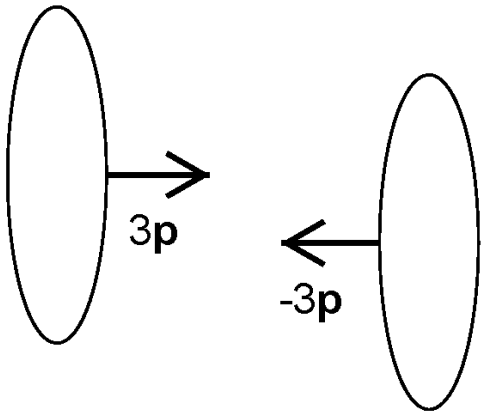
$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}} \int |T(\mathbf{p};\mathbf{k},\mathbf{q}_i)|^2\, d\tau_n$$



$$T(\mathbf{p};\mathbf{k},\mathbf{q}_i) = J(\mathbf{p};\mathbf{k},\mathbf{q}_i) \prod_{j=1}^{3} B_j(\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_i) = \int \prod_{i=1}^{2} \frac{d^3\mathbf{q}_1^{(i)}}{2E_p(2\pi)^3} \frac{d^3\mathbf{q}_2^{(i)}}{2E_p(2\pi)^3} \times$$

$$\times \varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)}) \varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)}) D(a_1,a_2,a_3) \varphi^*_{3\mathbf{k}}(\mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - \mathbf{q}_i)$$



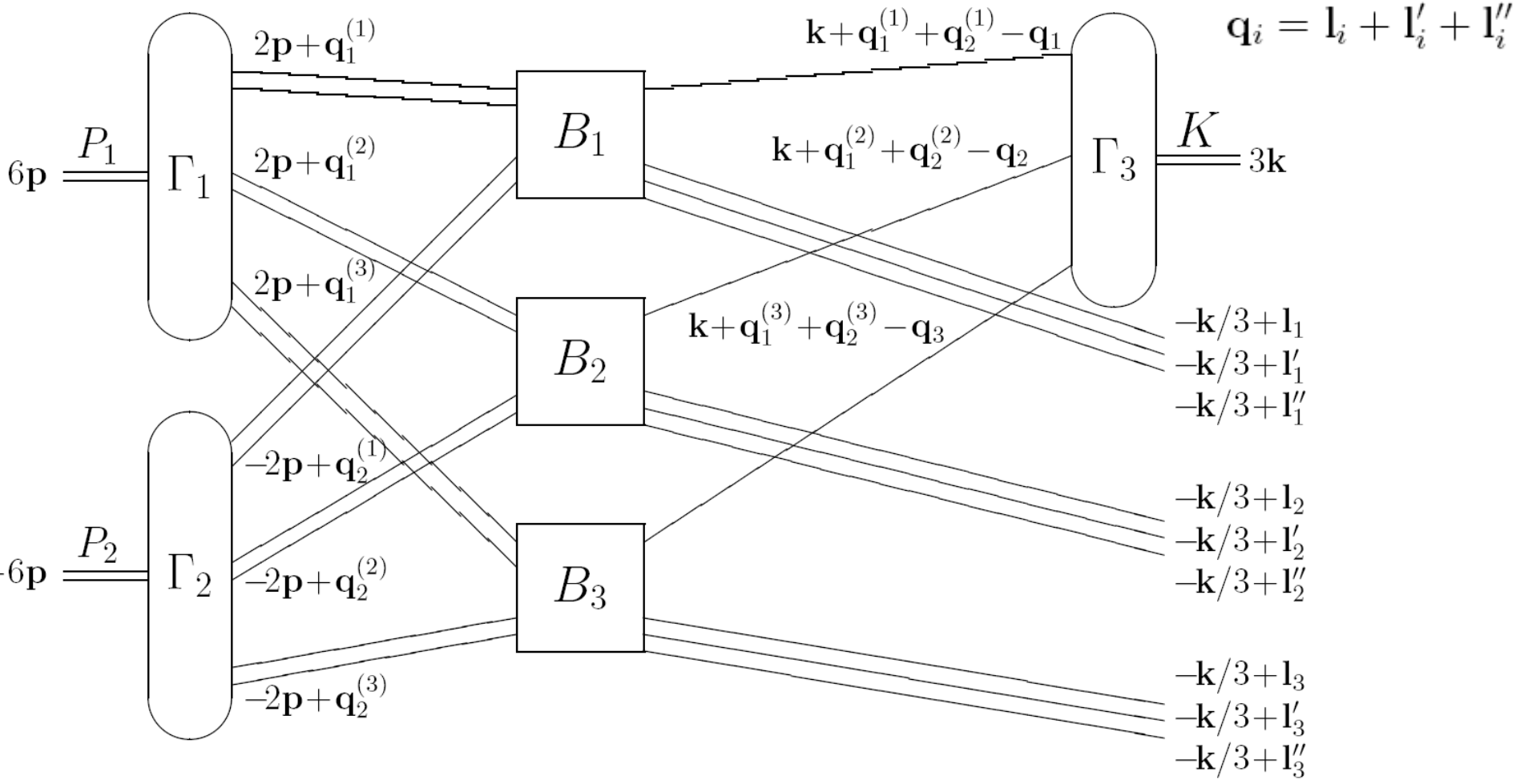
$$D(a_1,a_2,a_3) = -2\pi^2 E_p^2 [\delta(a_1)\delta(a_2) + \delta(a_2)\delta(a_3) + \delta(a_1)\delta(a_3)]$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)} - \mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - 2\mathbf{q}_i)$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

Amplitude of proton production in d+d → p+X at

$$x \to 2, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}, \quad \theta^* \sim 90^\circ$$



$$\mathbf{q}_1^{(i)} = \mathbf{k}_1^{(i)} + \mathbf{k}_1^{\prime(i)}$$

$$\overline{\mathbf{q}}_1^{(i)} = (\mathbf{k}_1^{(i)} - \mathbf{k}_1^{\prime(i)})/2$$

$$\sum_{i=1}^{3} \mathbf{q}_i = \sum_{i=1}^{3} [\mathbf{l}_i + \mathbf{l}'_i + \mathbf{l}''_i] = 0$$

Amplitude of proton production in d+d → p+X at

$$x \to 2, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}, \quad \theta^* \sim 90^\circ$$

$$T(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i) = J(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i)\prod_{j=1}^{3} B_j(2\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i) = \int \prod_{i=1}^{2} \frac{d^3\mathbf{q}_1^{(i)}}{2E_p(2\pi)^3}\frac{d^3\mathbf{q}_2^{(i)}}{2E_p(2\pi)^3}\times$$

$$\times\left[\int \varphi_{6\mathbf{p}}(\mathbf{q}_1^{(i)},\overline{\mathbf{q}}_1^{(i)})\prod_{i=1}^{3}\frac{d^3\overline{\mathbf{q}}_1^{(i)}}{2E_p(2\pi)^3}\right]\left[\int \varphi_{-6\mathbf{p}}(\mathbf{q}_2^{(i)},\overline{\mathbf{q}}_2^{(i)})\prod_{i=1}^{3}\frac{d^3\overline{\mathbf{q}}_2^{(i)}}{2E_p(2\pi)^3}\right]$$

$$\times D(a_1,a_2,a_3)\,\varphi^*_{3\mathbf{k}}(\mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-\mathbf{q}_i)$$

$$D(a_1,a_2,a_3) = -2\pi^2 E_p^2[\delta(a_1)\delta(a_2)+\delta(a_2)\delta(a_3)+\delta(a_1)\delta(a_3)]$$

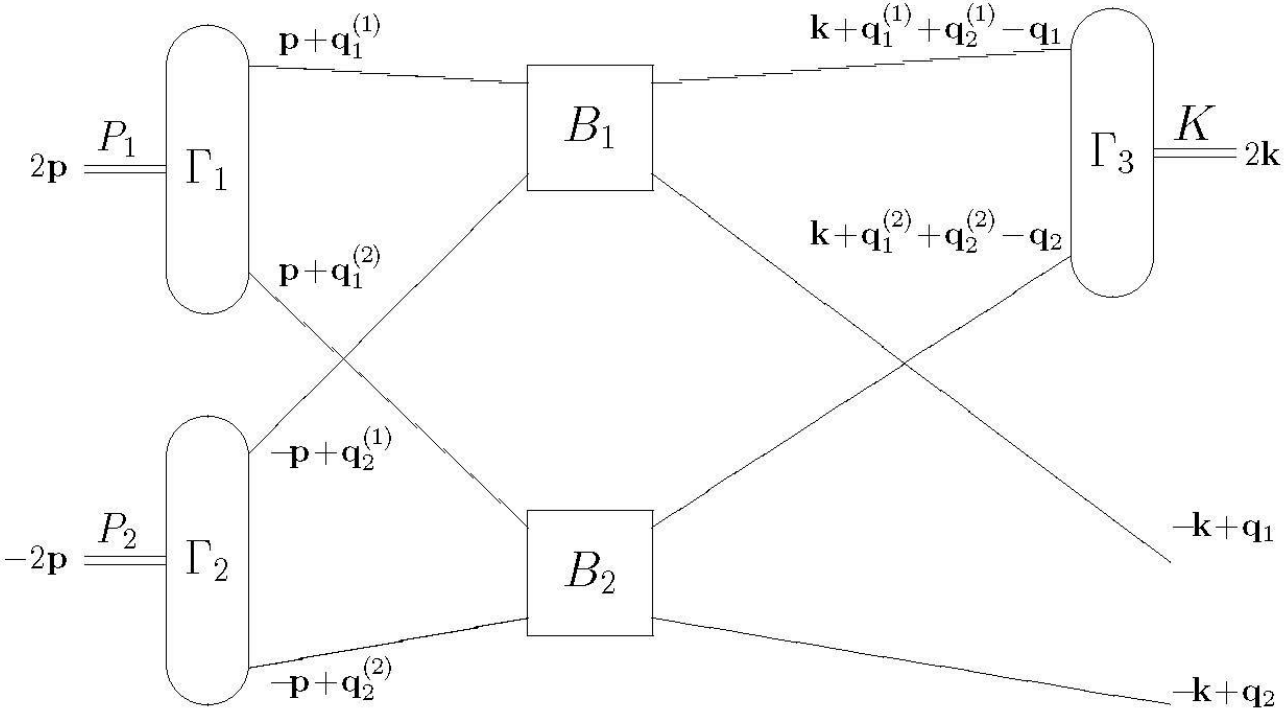
$$a_i \equiv (2\mathbf{p},\,\mathbf{q}_1^{(i)}-\mathbf{q}_2^{(i)}) - (\mathbf{k},\,\mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-2\mathbf{q}_i) \qquad \mathbf{q}_i = \mathbf{l}_i+\mathbf{l}'_i+\mathbf{l}''_i$$

$$\mathbf{q}_1^{(i)} = \mathbf{k}_1^{(i)} + \mathbf{k}_1^{\prime(i)}$$

$$\overline{\mathbf{q}}_1^{(i)} = (\mathbf{k}_1^{(i)} - \mathbf{k}_1^{\prime(i)})/2$$

$$\sum_{i=1}^{3}\mathbf{q}_i = \sum_{i=1}^{3}[\,\mathbf{l}_i+\mathbf{l}'_i+\mathbf{l}''_i] = 0$$

Coherent Coalescence (n=2)



can be used to describe:

1) **d+d → d+p+n**

at the nucleon level

2) **p+p → p+X**

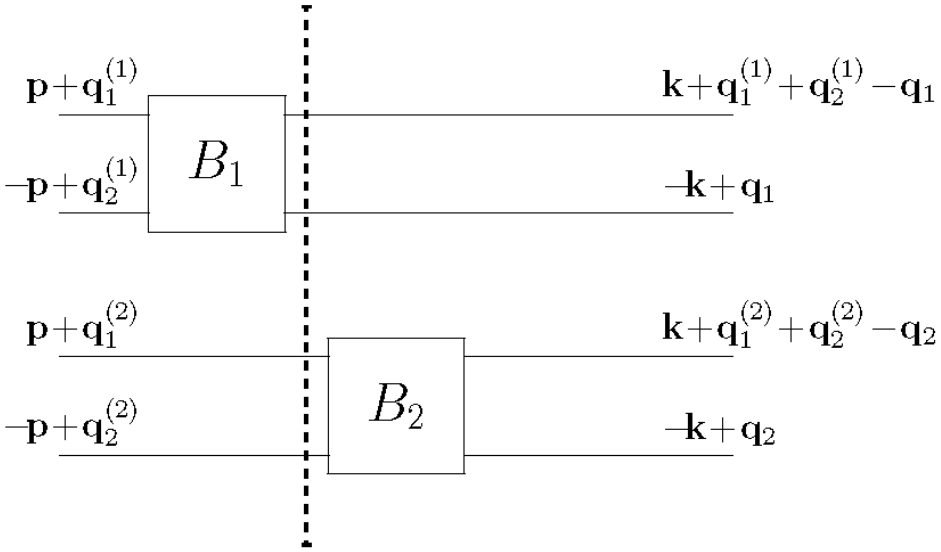
in the quark-diquark

picture of the nucleon

$$\mathbf{q}_1^{(1)}+\mathbf{q}_1^{(2)}=0\ ,\ \mathbf{q}_2^{(1)}+\mathbf{q}_2^{(2)}=0\ ,\ \mathbf{q}_1+\mathbf{q}_2=0$$

Braun M.A., Vechernin V.V.,
Yad.Fiz. 36 (1982) 614; 44 (1986) 784; 47 (1988) 1452;
J. Phys. G 16 (1990) 1615.

Permutations of B blocks (n=2)



$$\frac{E_p}{[-a_1-\mathrm{i}\epsilon]}+\frac{E_p}{[-a_2-\mathrm{i}\epsilon]}$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)}-\mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-2\mathbf{q}_i)$$

$$a_1 + a_2 = 0$$

$$\frac{-E_p}{[a_1+\mathrm{i}\epsilon]}+\frac{-E_p}{[a_2+\mathrm{i}\epsilon]} = i\pi E_p[\delta(a_1)+\delta(a_2)]$$

$$\frac{1}{b-\mathrm{i}\epsilon} = \frac{1}{b} + i\pi\delta(b)$$

$$D(a_1,a_2) = i\pi E_p[\delta(a_1)+\delta(a_2)] = 2\pi i E_p\,\delta(a_1)$$

Inclusive cross section (n=2)

$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}} \int |T(\mathbf{p};\mathbf{k},\mathbf{q}_1)|^2\, d\tau_2$$

$$T(\mathbf{p};\mathbf{k},\mathbf{q}_1) = J(\mathbf{p};\mathbf{k},\mathbf{q}_1) \prod_{j=1}^{2} B_j(\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_1) = \int \frac{d^3\mathbf{q}_1^{(1)}}{2E_p(2\pi)^3} \frac{d^3\mathbf{q}_2^{(1)}}{2E_p(2\pi)^3} \times$$

$$\times 2\pi i E_p\, \delta(a_1)\, \varphi_{2\mathbf{p}}(\mathbf{q}_1^{(1)}) \varphi_{-2\mathbf{p}}(\mathbf{q}_2^{(1)}) \varphi^*_{2\mathbf{k}}(\mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)} - \mathbf{q}_1)$$

$$a_1 \equiv (\mathbf{p},\, \mathbf{q}_1^{(1)} - \mathbf{q}_2^{(1)}) - (\mathbf{k},\, \mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)} - 2\mathbf{q}_1)$$

$$\int |\varphi_{2\mathbf{p}}(\mathbf{q})|^2 \frac{d^3\mathbf{q}}{2E_p(2\pi)^3} = 1 \qquad \int |\varphi_{2\mathbf{k}}(\mathbf{q})|^2 \frac{d^3\mathbf{q}}{2E_k(2\pi)^3} = 1$$

Fourier transform of Inclusive amplitude (n=2)

$$\mathbf{q} = \mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)}$$
$$\widetilde{\mathbf{q}} = (\mathbf{q}_1^{(1)} - \mathbf{q}_2^{(1)})/2$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_1) \sim \int d\mathbf{q}\, d\widetilde{\mathbf{q}}\, \varphi_{2\mathbf{p}}(\mathbf{q}/2+\widetilde{\mathbf{q}})\varphi_{-2\mathbf{p}}(\mathbf{q}/2-\widetilde{\mathbf{q}})\varphi^*_{2\mathbf{k}}(\mathbf{q}-\mathbf{q}_1)\delta(a_1)$$
$$a_1 = (2\mathbf{p},\,\widetilde{\mathbf{q}}) - (\mathbf{k},\,\mathbf{q}-2\mathbf{q}_1) = 2p\,\widetilde{q}_z - (\mathbf{k},\,\mathbf{q}-2\mathbf{q}_1)$$

$$\int |\varphi_{2\mathbf{p}}(\mathbf{q})|^2 \frac{d\mathbf{q}}{(2\pi)^3} = 1$$

$$\int |\varphi_{2\mathbf{k}}(\mathbf{q})|^2 \frac{d\mathbf{q}}{(2\pi)^3} = 1$$

$$\int |\psi_{2\mathbf{p}}(\mathbf{r})|^2 d\mathbf{r} = 1$$

$$\int |\psi_{2\mathbf{k}}(\mathbf{r})|^2 d\mathbf{r} = 1$$

$$\psi_{2\mathbf{p}}(\mathbf{r}_1) \equiv \int \varphi_{2\mathbf{p}}(\mathbf{q}) e^{-i\mathbf{r}_1\mathbf{q}} \frac{d\mathbf{q}}{(2\pi)^3}$$

$$\psi_{-2\mathbf{p}}(\mathbf{r}_2) \equiv \int \varphi_{-2\mathbf{p}}(\mathbf{q}) e^{-i\mathbf{r}_2\mathbf{q}} \frac{d\mathbf{q}}{(2\pi)^3}$$

$$\psi_{2\mathbf{k}}(\mathbf{r}) \equiv \int \varphi_{2\mathbf{k}}(\mathbf{q}) e^{-i\mathbf{r}\mathbf{q}} \frac{d\mathbf{q}}{(2\pi)^3}$$

Inclusive amplitude in coordinate space (n=2)

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_1) \sim \int d\mathbf{r}_\perp dz_1 dz_2\, \psi_{2\mathbf{p}}(z_1,\mathbf{r}_\perp)\psi_{-2\mathbf{p}}(z_2,\mathbf{r}_\perp)\psi^*_{2\mathbf{k}}(\mathbf{r}_K)$$
$$\times e^{-i\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}-\frac{z_1+z_2}{2}\frac{\mathbf{k}}{p},\mathbf{q}_1\right)}$$

$$\mathbf{r}_K = \frac{\mathbf{r}_1+\mathbf{r}_2}{2} + \frac{z_1+z_2}{2}\frac{\mathbf{k}}{p}$$

$$\mathbf{r}_{1\perp} = \mathbf{r}_{2\perp} = \mathbf{r}_\perp$$

The importance of taking into account the coherence of the coalescence process

$$T(\mathbf{p};\mathbf{k},\mathbf{q}_1) = J(\mathbf{p};\mathbf{k},\mathbf{q}_1)\prod_{j=1}^{2} B_j(\mathbf{p};\mathbf{k})$$

$$(2\pi)^3\, 2K_0\frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}}\int |T(\mathbf{p};\mathbf{k},\mathbf{q}_1)|^2\, d\tau_2$$

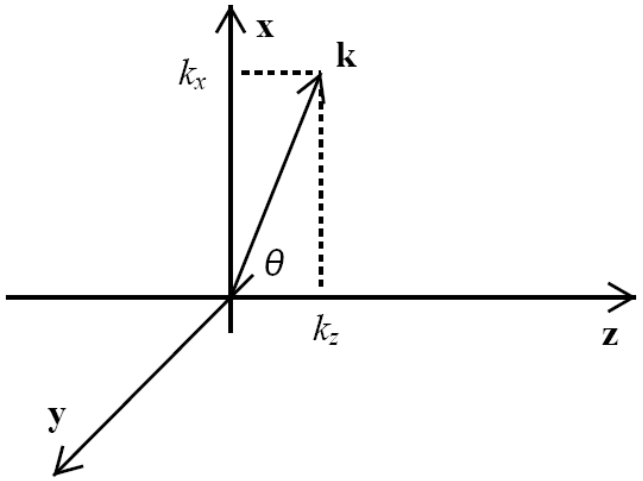
Gaussian approximation for wave functions

$$\psi_{2\mathbf{p}}(z,\mathbf{r}_\perp) = C_p \exp -\frac{(z\gamma_p)^2 + \mathbf{r}_\perp^2}{2R^2}$$

$$\gamma_p = \frac{E_p}{m} \qquad \mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|} \qquad \gamma_k = \frac{E_k}{m}$$

$$\psi_{2\mathbf{k}}(\mathbf{r}) = C_k \exp -\frac{[(\mathbf{nr})\gamma_k]^2 + [\mathbf{r} - \mathbf{n}(\mathbf{nr})]^2}{2R^2} = C_k \exp -\frac{(\mathbf{nr})^2[\gamma_k^2 - 1] + \mathbf{r}^2}{2R^2}$$

$$\mathbf{k} = (k_x, 0, k_z) \quad k_x = |\mathbf{k}_\perp| \qquad (\mathbf{nr}) = xn_x + zn_z = x\sin\theta + z\cos\theta$$



Asymptotic behaviour of inclusive cross sections

n=2:

Coalescence:

$$I_{\mathrm{pp}\to\mathrm{p}}(x) = C\frac{m^2}{R}\frac{(1-x)^{0.5}}{s^{3.5}}F(\theta)$$

Quark fragmentation:

$$I_{\mathrm{pp}\to\pi}(x) = C'\frac{1}{(m^2R^3)^2}\frac{(1-x)^2}{s^3}$$

n=3:

Coalescence:

$$I_{\mathrm{pp}\to\mathrm{p}}(x) \sim (1-x)^2$$

Quark fragmentation:

$$I_{\mathrm{pp}\to\pi}(x) = C''\frac{1}{(m^2R^3)^4}\frac{(1-x)^5}{s^4}$$

Angular dependence of the Coalescence

$$F(\theta) = \frac{1}{\frac{12}{\gamma^2} + \sin^2\theta\left(1 - \frac{1}{\gamma^2}\right)\left(2 - \frac{3}{\gamma^2}\right)}$$

$$\gamma = \frac{E_p}{m}$$

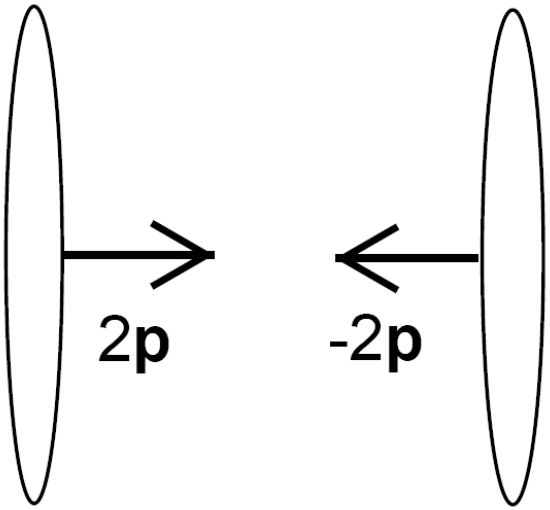
$$\gamma = 1 \;\Rightarrow\; F(\theta) = \frac{1}{12} = const$$

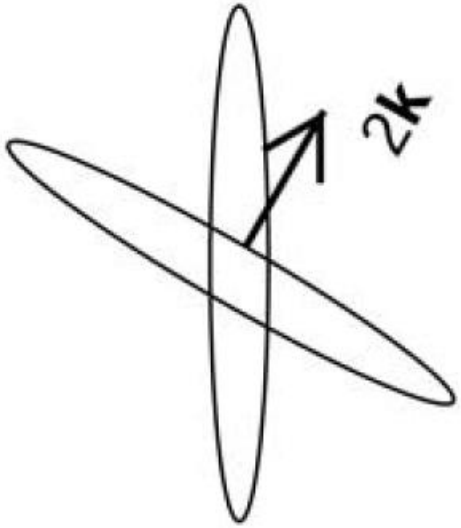
$$\gamma \gg 1 \;\Rightarrow\; F(\theta) = \frac{1}{2\sin^2\theta} = \frac{1}{2}\mathrm{ch}^2\eta$$

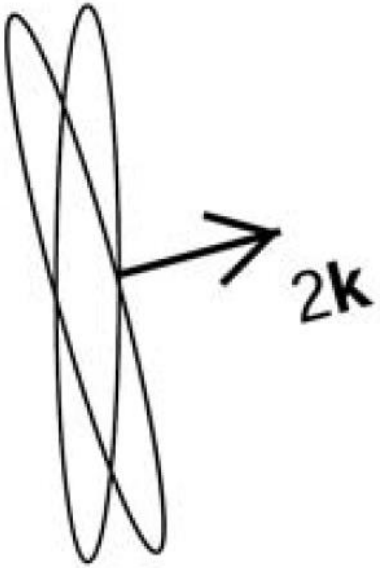
$\eta = \ln \mathrm{tg}(\theta/2)$ - pseudorapidity

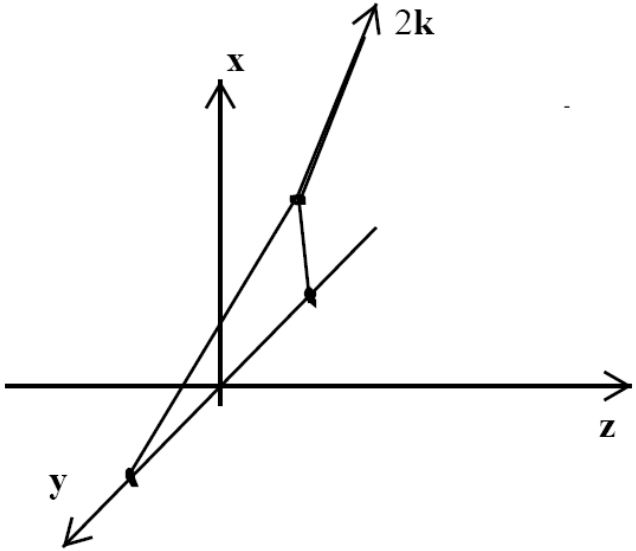
Increase in the coalescence coefficient for small angles in the relativistic case

Physical interpretation in the center of mass system









Historical note

Coherent Coalescence Mechanism at nucleon level:

Braun M.A., Vechernin V.V., Yad.Fiz. 47 (1988) 1452; J. Phys. G 16 (1990) 1615.

$p+A \to d, Tr, {}^3He + X$ - **in central and fragmentation regions**

- using the Feynman diagram technique
- taking into account the stretching of the wave function of the resulting fragment in momentum space

$$\kappa_2(\mathbf{k}) \sim \int d^2\mathbf{b}\, dz_1\, dz_2\; \rho_{n_1}(\mathbf{b}, z_1)\, \rho_{n_2}(\mathbf{b}, z_2)\, |\psi(y^*)|^2.$$

$$k \equiv |\mathbf{k}|$$

$$y^* \equiv |\mathbf{y}^*| = \frac{k_-}{m}|z_1 - z_2|$$

$$k_- = \sqrt{k^2 + m^2} - k\cos\theta$$

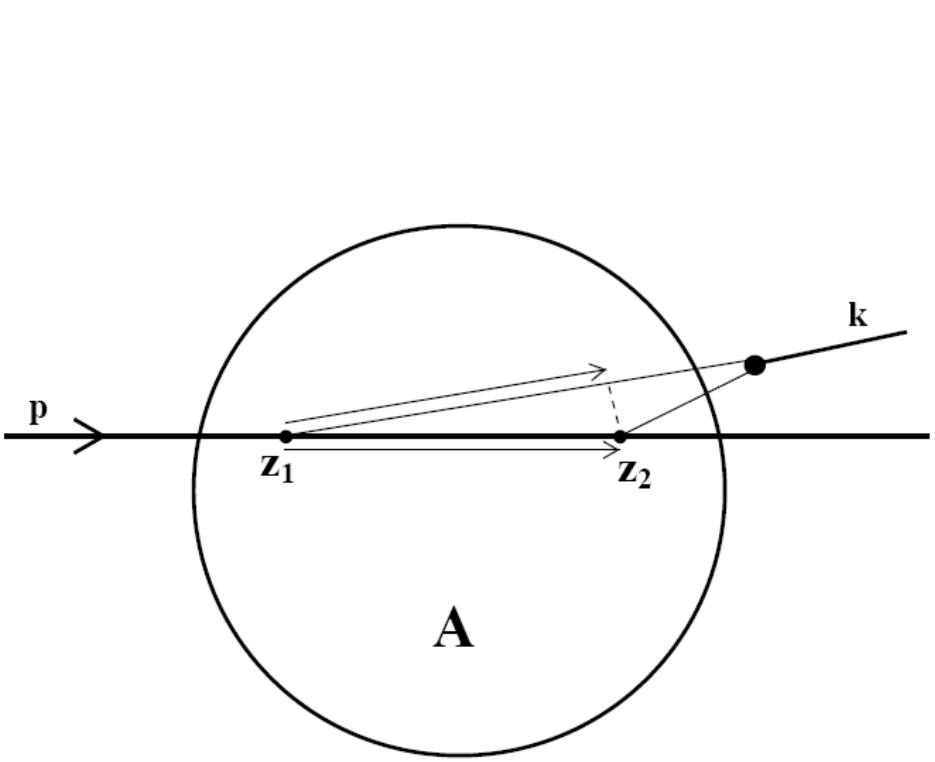
$$\kappa_2(\mathbf{k}) \sim \frac{1}{k_-}$$

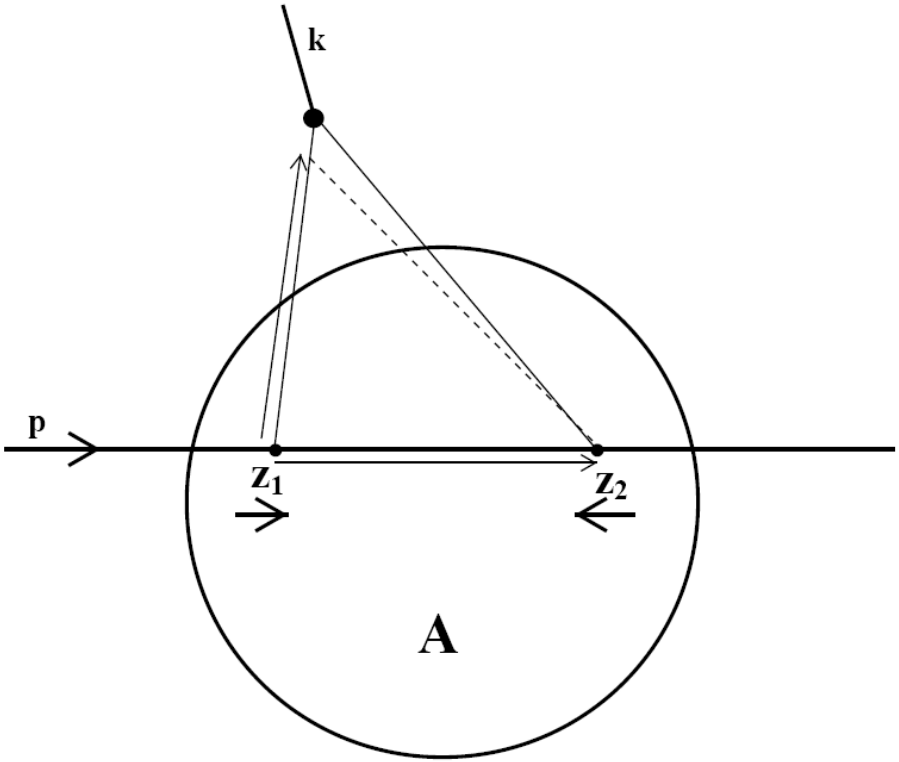
$$\theta \simeq 0^\circ \quad k_- \to 0$$
$$\theta \simeq 90^\circ \quad k_- \simeq |\mathbf{k}|$$
$$\theta \simeq 180^\circ \quad k_- \simeq 2|\mathbf{k}|$$

- increase of the Coalescence Coefficient

$$z_1 \longrightarrow z_2$$

Physical interpretation in the laboratory frame





Braun M.A., Vechernin V.V., Yad.Fiz. 47 (1988) 1452; J. Phys. G 16 (1990) 1615.

Gavrilov V.B., Kornienko N.L., Leksin G.A., Semenov S.V., Sov. J. Nucl. Phys. 41 (1985) 540;

Preprint ITEP-69 Moscow, 1985.

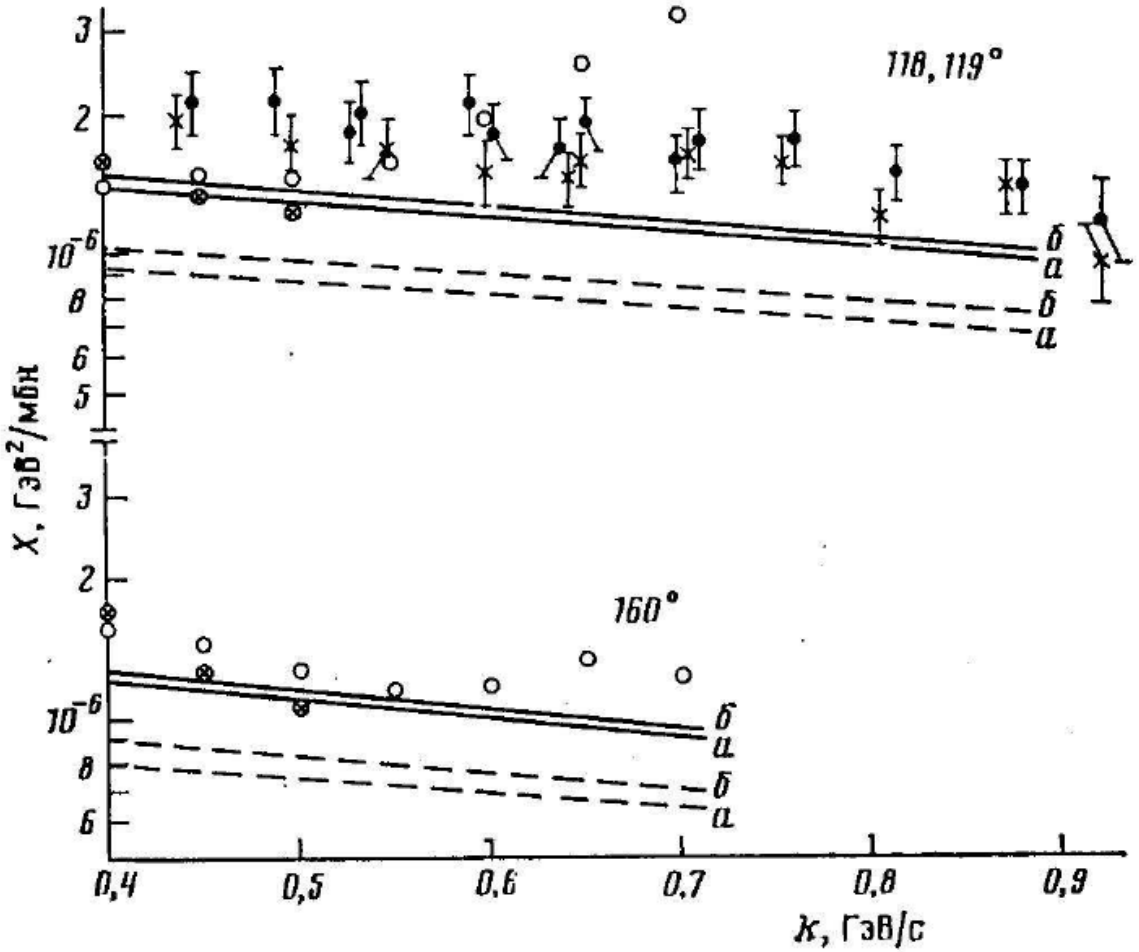


Рисунок 2.15: Сравнение в кумулятивной области теоретических расчетов величины $X = (I_A^{(d)}/A)/(I_A/A)^2$ по формуле (2.66) с экспериментальными данными [23], полученными при облучении ядер-мишеней ($\otimes$ - Cu и $\circ$ - Ta) протонами с начальным импульсом p=400 ГэВ/c и с экспериментальными данными [132], полученными при облучении тех же мишеней ($\times$ - Cu и $\bullet$ - Ta) протонами с энергией 10 ГэВ

[23] Bayukov Yu.D. et al. Phys. Rev. C20 (1979) 764; 2257,
[132] Бояринов С.В. и др. Препринт ИТЭФ 86-165, М., 1986.

Characteristic features of the amplitude in mechanism of Coherent Quark Coalescence

- no proportionality to the wave function value at zero
- smaller number of hard exchanges compared to fragmentation mechanism
- the absence of the need for all quarks to gather in an ever-smaller region as energy increases
- the coherence - the convolution of wave functions (not probabilities!)
- MC simulations cannot be applied (just like for Gribov screening)
- usual factorization assumption is invalid
- calculation of Feynman graphs automatically leads to the correct space-time picture of the process

$$r_d \gg r_N \sim r_f \gg r_B \sim \frac{1}{\sqrt{s}}$$

Summary

1. Mechanism of Coherent Quark Coalescence leads to the increase of proton production in considered region, compared to the mechanism of fragmentation of one quark into proton, which was used for description of pion production, due to fewer number of hard exchanges and the absence of the need for all quarks to gather in an ever-smaller region as energy increases.

2. Taking into account the stretching of the wave function of the resulting relativistic particle in momentum space is essential. In this case the probability of Quark Coalescence occurs lower for angles about 90°. The physical interpretation of this phenomenon is presented.

Future studies

1) Explicit hadronization of recoil partons. At moderate NICA energy:
partonic phase volume => hadronic phase volume
(only **d+d → π+NNNN,** neglecting **d+d → π+ πNNNN** and so on).

$$M_X \sim (A - x)\sqrt{s}$$

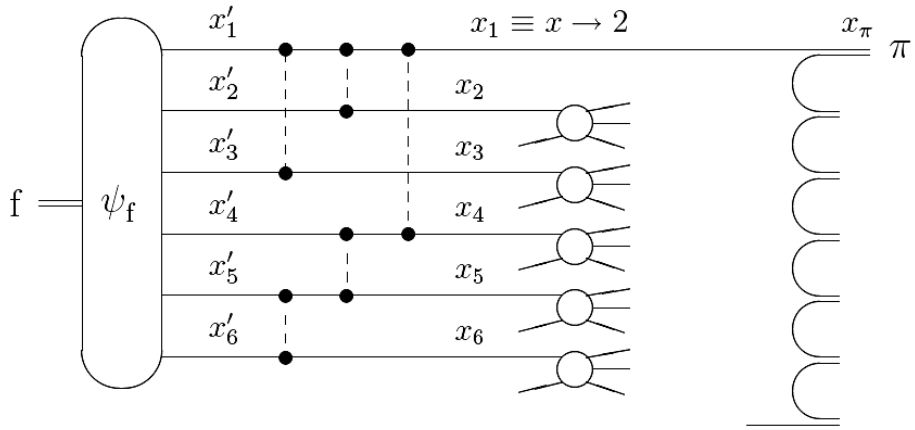
2) Inclusion of diquarks in the scheme. *V.T. Kim, Modern Phys. Lett. A 3 (1988) 909*
Two options: - diquark fragmentation or
- coherent coalescence of quark and diquark (n=2)
M.A. Braun, V.V. Vechernin, Nucl.Phys.B 427 (1994) 614

The authors are grateful to Saint-Petersburg State University for research project 103821868 and travel grant 137734523.

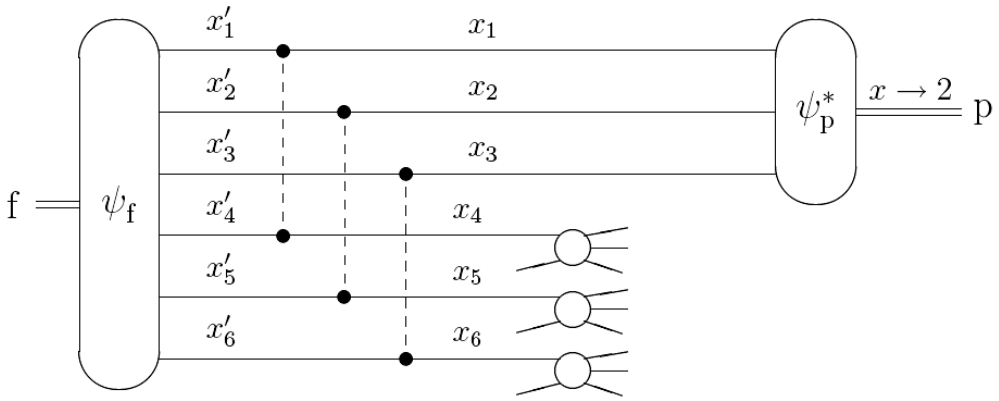
Backup slides

Using the experience in describing the production of pions and protons in the traditional cumulative region of fragmentation of one of the colliding nuclei.

Coherent Quark Coalescence and Production of Cumulative Protons in Fragmentation Region

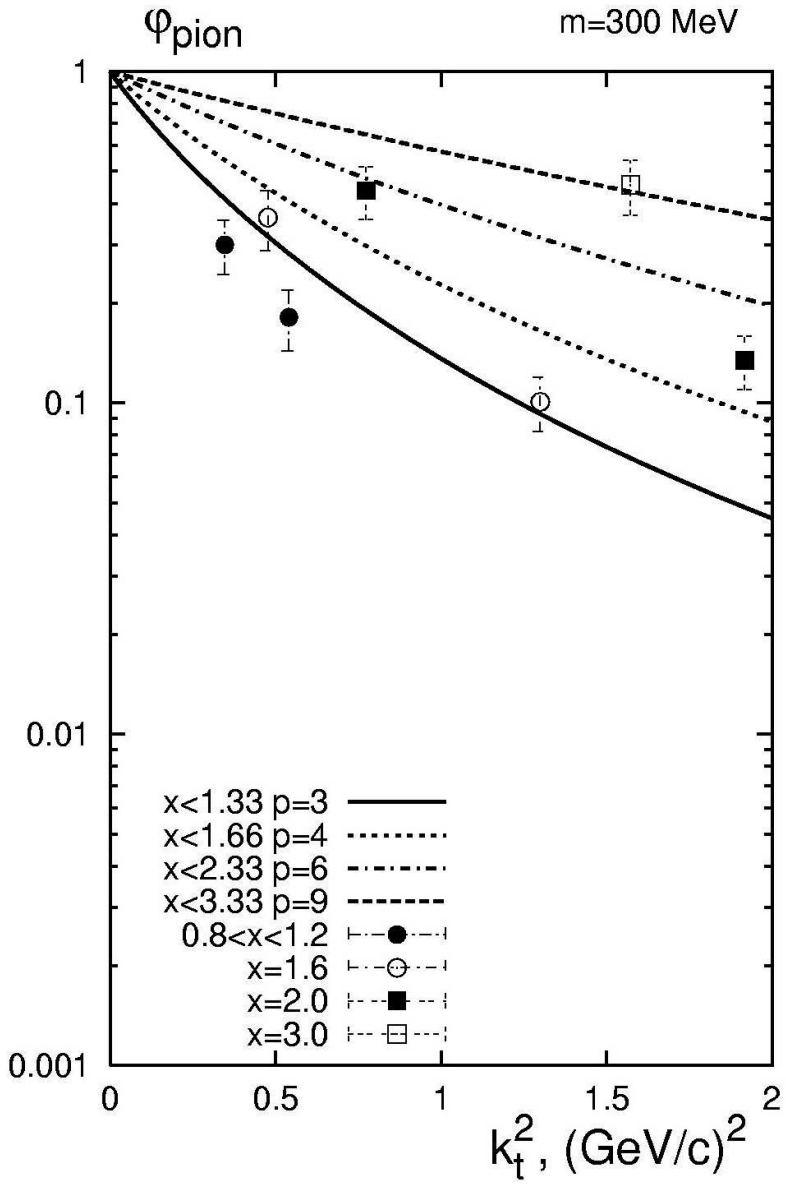


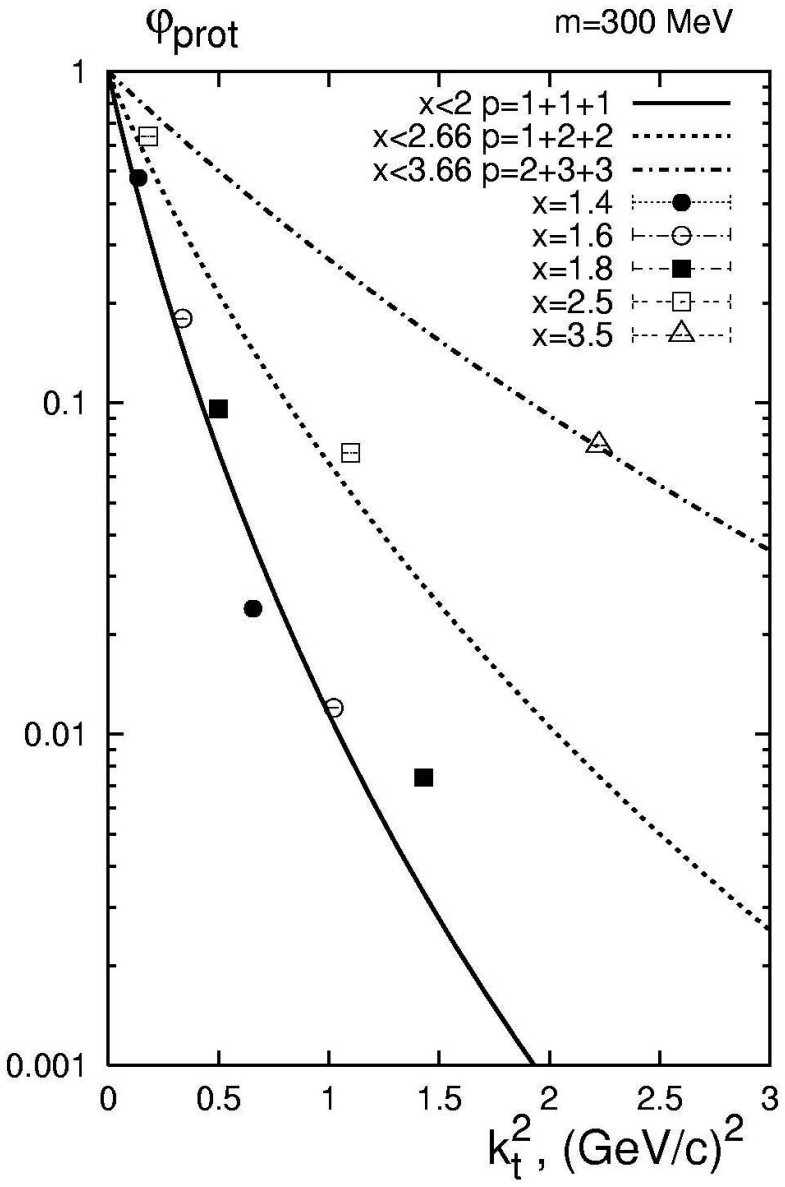
- the cumulative pion production
by hadronization of one fast quark
*M.A. Braun, V.V. Vechernin, Nucl.Phys.**B 427**, 614 (1994); Phys.Atom.Nucl. **60**, 432 (1997); **63**, 1831 (2000)*



- the cumulative proton production
by **coherent** quark coalescence mechanism:
*M.A. Braun, V.V. Vechernin, Nucl.Phys.**B 92**, 156 (2001); Theor.Math.Phys **139**, 766 (2004); V.Vechernin, AIP Conf.Proc.1701 (2016) 060020.*

The last diagram **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).
But instead of using the relativistic generalization of non-relativistic NN wave function
the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge
*Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. **B369** (1992) 519.*
was developed and applied.

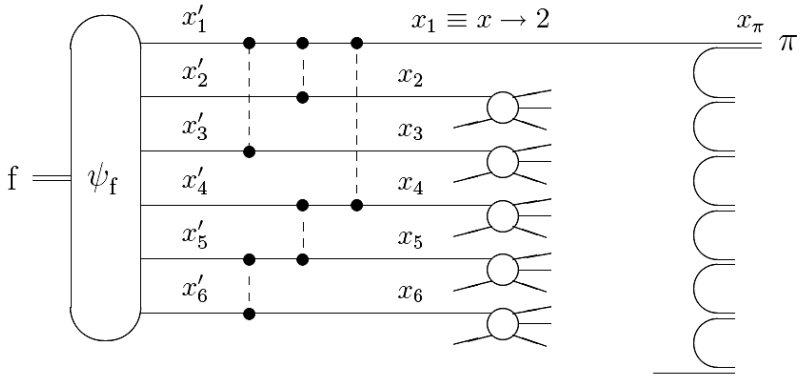


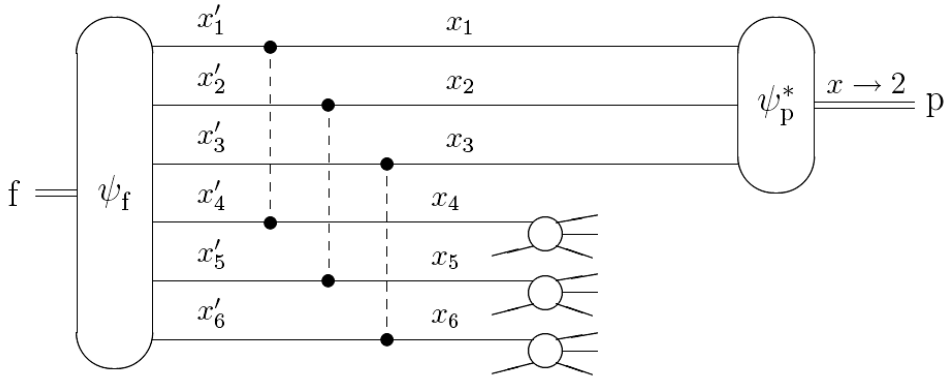


V.Vechernin, AIP Conference Proceedings 1701 (2016) 060020.

*S.V. Boyarinov et al., Sov.J.Nucl.Phys. **46**, 871 (1987)*
*S.V. Boyarinov et al., Physics of Atomic Nuclei **57**, 1379 (1994)*
*S.V. Boyarinov et al., Sov.J.Nucl.Phys. **55**, 917 (1992)*

Comparison of the mechanisms of pion and proton production in dd collisions in traditional cumulative region of fragmentation of one of the colliding nuclei





$$f_\pi(x,k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C_\pi (2-x)^9\, \Phi_5\left(\frac{k_\perp}{m_q}\right) / \Phi_5(0)$$

$$f_{\rm p}(x,k_\perp) \equiv \frac{k_0\, d^3\sigma_{\rm p}}{d^3\mathbf{k}} = C_{\rm p} (2-x)^5\, \Phi_1^3\left(\frac{k_\perp}{3m_q}\right) / \Phi_1^3(0)$$

$$\Phi_p(t) = 2\pi \int_0^\infty dz\, z J_0(tz) [z K_1(z)]^p$$

$$\Phi_1(t) = \frac{4\pi}{(t^2+1)^2}$$

Energy scaling of cumulative inclusive cross section in the flucton fragmentation region:

$$f_\pi(x,k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C\, s^0\, (f-x)^{2p-1}\, \Phi_p\left(\frac{k_\perp}{m_q}\right)$$

Pion and Proton Yields:
Estimation of Production Rates at Different Initial Energies

Problem with modeling using MC event generators:

Not done yet, as **this mechanism is absent in existing MC event generators**. It is necessary to introduce into the MC generator some **admixture of 6-quark flucton** into deuteron and **theoretically calculated inclusive cross-sections of production of various particles** in the new cumulative region **from flucton-flucton interaction.** **(in progress with Semyon Yurchenko)**.

Estimates based on extrapolation from the region of fragmentation of a nucleus:

Vechernin V.V., Belokurova S.N., Yurchenko S.V. Phys Part Nucl, 2024, Vol. 55, pp. 889–894.

$$\langle n\rangle_{\mathrm{dd}} \cdot \sigma^{tot}_{\mathrm{dd}} = 4\pi \int_{0.5}^{1} dy \int_{k_\perp^{min}(y)}^{k_\perp^{max}(y)} dk_\perp \, k_\perp \, f(\, x(y,k_\perp),\, k_\perp)$$

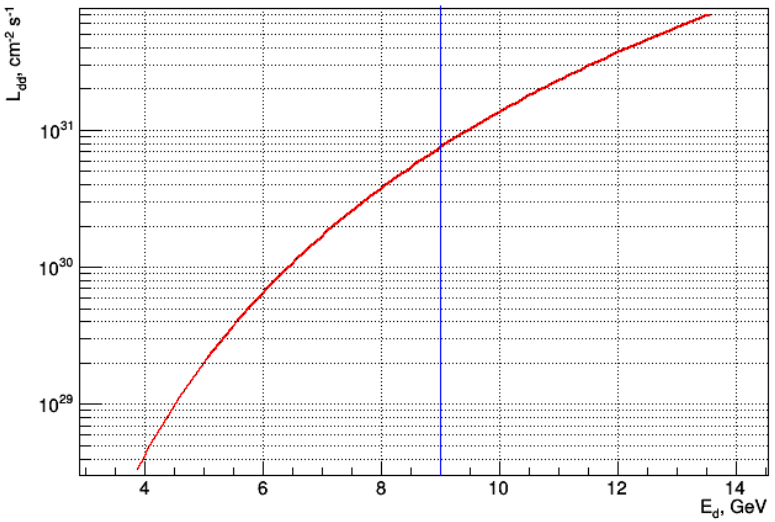
Table 1. The magnitude of the transverse momentum of cumulative pions and protons in dd scattering, corresponding to values of the variable $x = 1$ and 2 ($k_\perp^{min}$ and $k_\perp^{max}$) at a given value of the rapidity y, for two values of the initial energy.

$\sqrt{s_{NN}}$	4 GeV			8 GeV		
	y	$k_\perp^{min}$	$k_\perp^{max}$	y	$k_\perp^{min}$	$k_\perp^{max}$
dd → π	0.5	1.728	2.752	0.5	4.197	6.672
dd → π	1.0	1.102	2.002	1.0	2.687	4.86
dd → p	0.5	1.741	2.999	0.5	4.218	6.803
dd → p	1.0	0.852	2.089	1.0	2.605	4.915

Pion and Proton Yields:
Estimation of Production Rates in dd collisions at $\sqrt{s_{NN}}$ = 4 and 8 GeV

Table 2. The results of calculations of the multiplicity of pions and protons in the cumulative region by the formula (9), using the inclusive cross sections (1) and (2), for rapidities in the interval $0.5 < |y| < 1$ formed in dd scattering due to the interaction of a nucleon with a 6-quark fluctuon.

	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$\langle n_{\pi^-}\rangle_{\mathrm{dd}}$	$x > 1.0$	$9{\cdot}10^{-4}$	$1.9{\cdot}10^{-4}$
	$x > 1.2$	$6.6{\cdot}10^{-5}$	$1.2{\cdot}10^{-5}$
	$x > 1.5$	$3.6{\cdot}10^{-7}$	$5.8{\cdot}10^{-8}$
$\langle n_p\rangle_{\mathrm{dd}}$	$x > 1.0$	$2.3{\cdot}10^{-2}$	$9{\cdot}10^{-6}$
	$x > 1.2$	$1.2{\cdot}10^{-3}$	$4.6{\cdot}10^{-7}$
	$x > 1.5$	$1.04{\cdot}10^{-5}$	$4.2{\cdot}10^{-9}$



$$Y_{\mathrm{dd}} = 0.1 L_{\mathrm{dd}} \sigma_{\mathrm{dd}}^{\mathrm{tot}} \langle n \rangle_{\mathrm{dd}} t.$$

depends only on product $\langle n \rangle_{\mathrm{dd}} \sigma_{\mathrm{dd}}^{\mathrm{tot}}$ (see previous slide)

The SPD Collab. (V. Abazov et al.), Natural Science Review 1 1 (2024); arXiv:2404.08317 [hep-ex]

Pion and Proton Yields:
Estimation of Production Rates in dd collisions at $\sqrt{s_{NN}}$ = 4 and 8 GeV

Table 3. Estimates of the yields ($Y_{\rm dd}$) of cumulative pions and protons in the rapidity interval $0.5 < |y| < 1$ in dd collisions in one hour of data acquisition at the SPD facility of the NICA collider, calculated by the formulas (9) and (11) using the inclusive cross sections (1) and (2) and taking into account the luminosity reduction at energy 4 GeV [21] (see text).

	$\sqrt{s_{NN}}$	4 GeV	8 GeV
$Y_{\rm dd} \to \pi^-$	$x > 1$	400	8 000
	$x > 1.2$	30	500
	$x > 1.5$	0.16	2.5
$Y_{\rm dd} \to {\rm p}$	$x > 1$	10 000	400
	$x > 1.2$	500	20
	$x > 1.5$	4.5	0.18

Inclusive cross sections for the production of pions and protons in dd-collisions, integrated over rapidity intervals 0.5 < |y| < 1

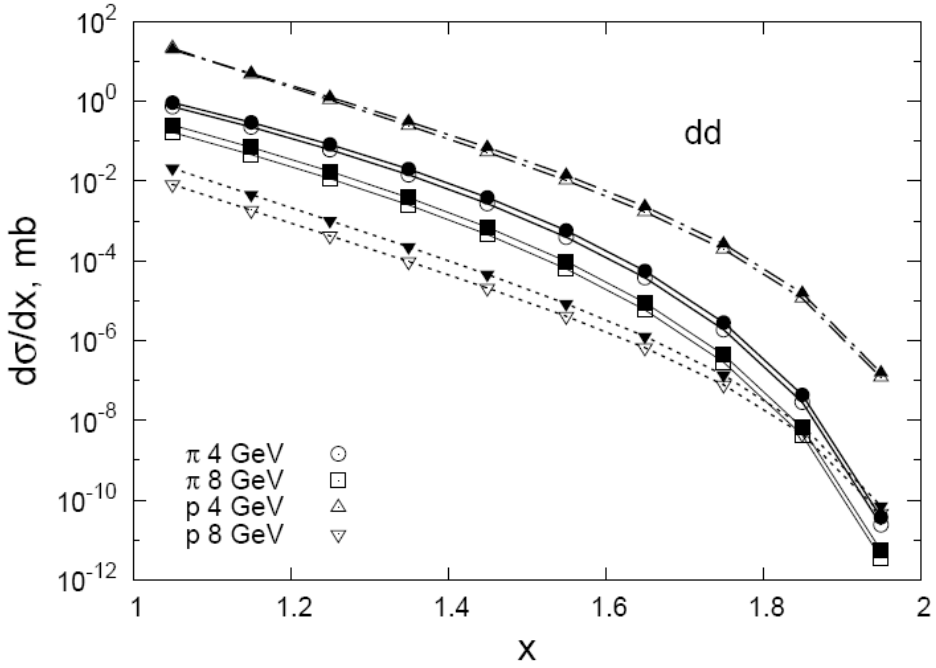


Figure 2. Inclusive cross sections for the production of pions (○, □) and protons (△,▽) in dd collisions, integrated over rapidity intervals $0.5 < |y| < 1$ and available for study with NICA SPD, respectively, for two initial energies $\sqrt{s_{NN}} = 4$ and 8 GeV, as a function of the light-cone cumulative variable $x = 2x_+$ (open simbols) and the cumulative number $x = x_M$ (solid symbols). Model calculations by (9) using (1) and (2). (Curves serve to guide the eye.)

$$\frac{d\sigma}{dx} = \frac{\langle n \rangle_{\rm dd}^{\Delta x}}{\Delta x} \sigma_{\rm dd}^{tot} = \frac{2\pi}{\Delta x} \int_{0.5}^{1} dy \int_{k_\perp^x(y)}^{k_\perp^{x+\Delta x}(y)} dk_\perp \, k_\perp \times$$

$$\times f(x(y,k_\perp), k_\perp) \, , \qquad (9)$$

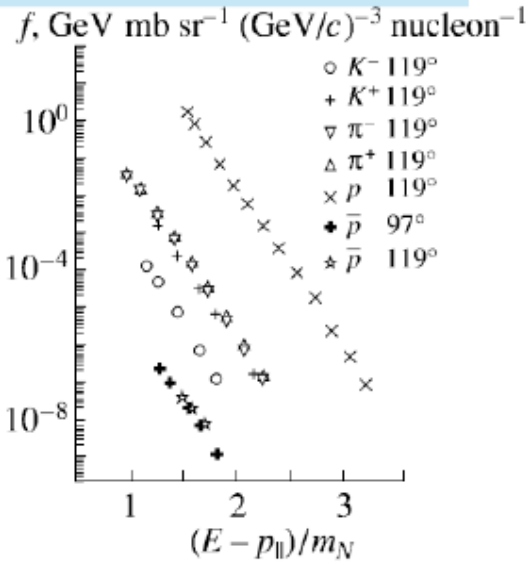
Vechernin V.V., Yurchenko S.V. Cumulative production at central rapidities and large transverse momenta in the quark model of flucton fragmentation, Moscow University Physics Bulletin, 2024, Vol. 79, Suppl. 1, pp. S174–S178.

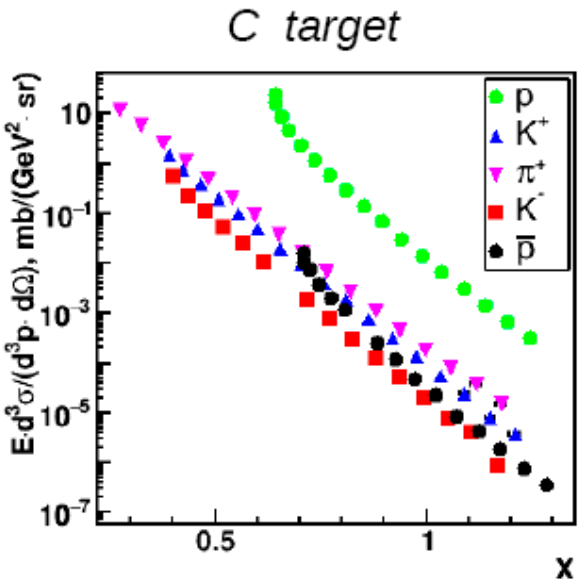
$f + 2N$

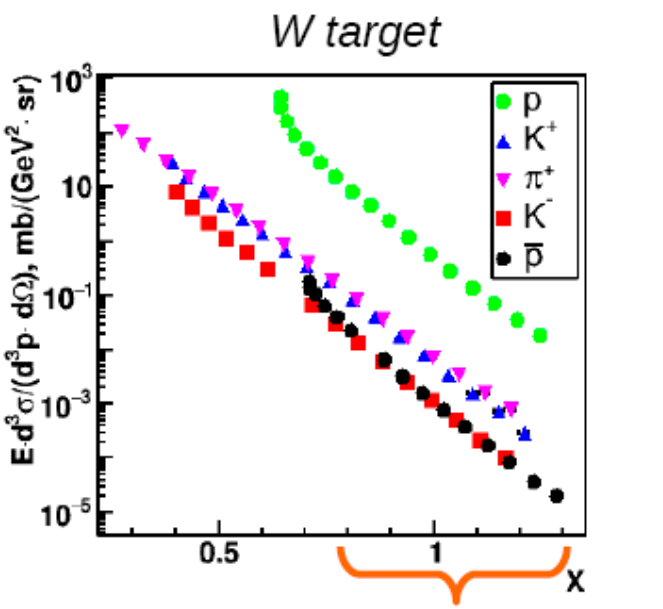
$x_1 = x = x_M$

$x_2 = 2$

Г.А. Лексин, ЯФ, т. 65, № 11, 2002, стр. 2042 — 2051







$\sqrt{s_{NN}}$ = 9.8 GeV

N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhanov, V. Viktorov "High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c"
(National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino)
LXX International Conference "NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies", St Petersburg, October 11-17, 2020.

Flucton-flucton interaction in dd collisions

- **It can be studied only in new cumulative region of large transverse momenta in mid-rapidity region at NICA (not in the traditional cumulative region of fragmentation of one of the nuclei).**
- **There are no additional interactions in dd collision, compared with collisions of heavier nuclei, if both deuterons are in flucton configuration at the moment of collision.**
- **Higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD (important for a registration of rare cumulative events).**
- **The studies in new cumulative region becomes possible due to the moderate energy of the NICA collider and is completely impossible at ultrahigh energies of the RHIC and LHC.**

Calculation of Phase Volume

$$\tau_p = (2\pi)^{4-3p} \int \prod_{i=1}^{p} \frac{d^3\mathbf{l}'_i}{2l_{i0}} \delta^{(3)}\left(\sum_{i=1}^{p} \mathbf{l}'_i\right) \times$$

$$\times \delta\left(\sum_{i=1}^{p} \left[\sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2} - \sqrt{(\mathbf{k}/p)^2 + m^2}\right] - \Delta\right)$$

$$\mathbf{l}'_i = -\mathbf{k}/p + \mathbf{l}_i$$

$$l_{i0} = \sqrt{(\mathbf{k}/p + \mathbf{l}'_i)^2 + m^2}$$

$$p = n_1 + n_2 - 1$$

$$\Delta = A\sqrt{s} - \sqrt{k^2 + m^2} - \sqrt{k^2 + p^2 m^2}$$

$$k \to k_{max} \Rightarrow \Delta \to 0$$

$$\sqrt{k_{max}^2 + m^2} + \sqrt{k_{max}^2 + (p\,m)^2} = A\sqrt{s}$$

$$\tau_p = \frac{1}{2^p m^{p-1} p^{\frac{3}{2}}} \frac{\left(\frac{E_p}{2\pi}\Delta\right)^{\frac{3}{2}p - \frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$E_p \equiv \sqrt{k^2/p^2 + m^2}$$

Relation with Cumulative Number

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)\, m]^2}\ .$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$\Delta = (A - x)[\sqrt{s} + O(1/\sqrt{s})]$$

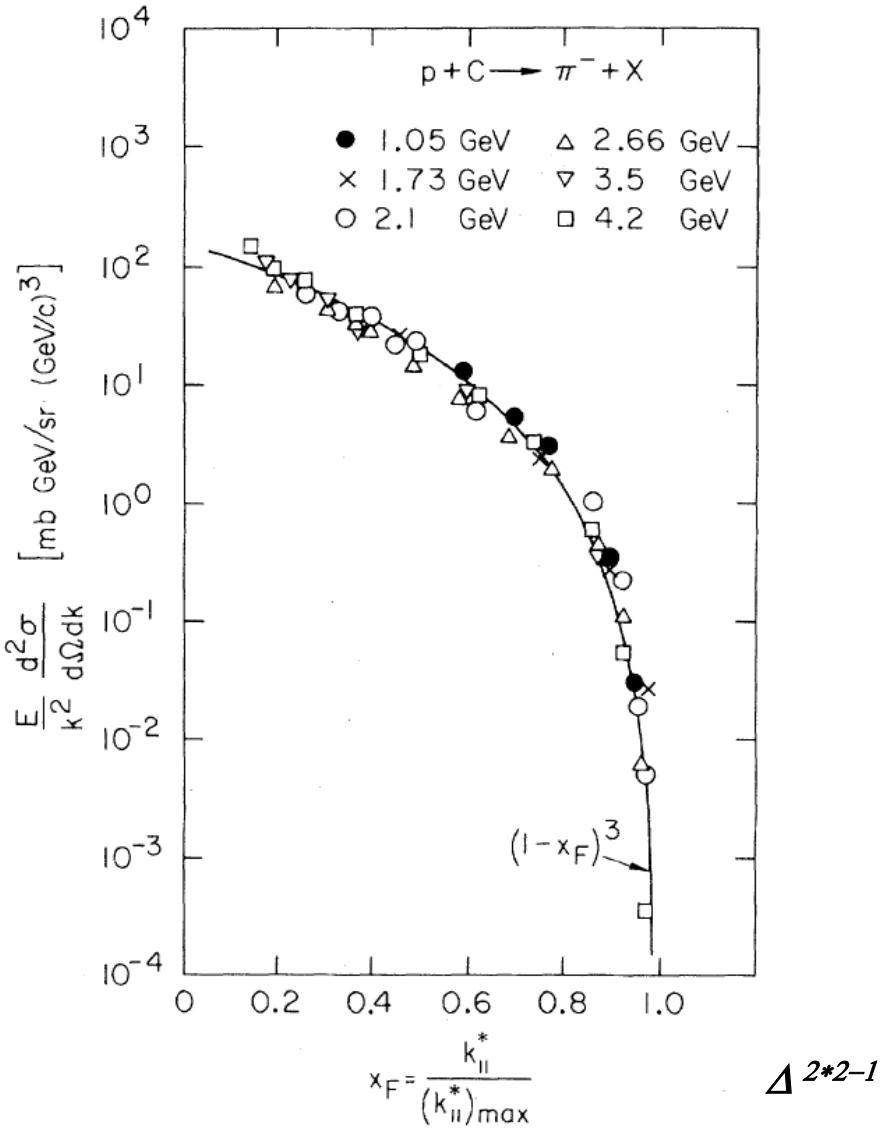
$$\tau_p = \frac{1}{2^{4p-5} p^{3p/2-1} m^{p-1}} \frac{\left[\frac{A}{\pi} s(A-x)\right]^{\frac{3}{2}p-\frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$p = p(A)$$

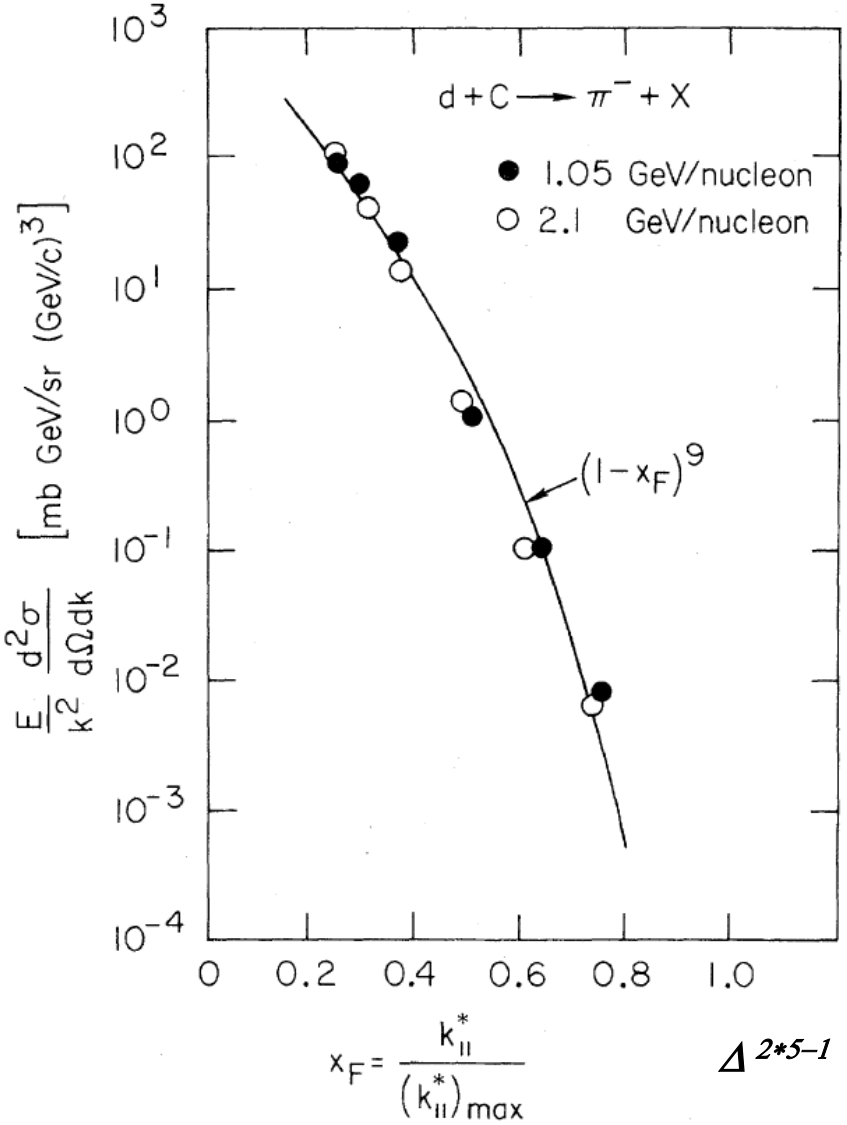
$$I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p-\frac{5}{2}}}{(m^2 R^3)^{p-1} s^{(p+3)/2}}$$

two (!)
small parameters:
$m/\sqrt{s} \ll 1$
$A - x \ll 1$

Schmidt I.A., Blankenbecler R. Phys.Rev. ***D15*** *(1977) 3321*



Δ 2*2–1



Δ 2*5–1

Threshold behaviour of ***inclusive cross sections*** (quark counting rules) at |t|<<s.

The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).

Quark counting rules for *elastic and quasi elastic* reactions with nuclei

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719
Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

$s \to \infty, \quad t/s$ fixed

$(d\sigma/dt)_{\pi p \to \pi p} \sim s^{-8}, \quad (d\sigma/dt)_{pp \to pp} \sim s^{-10}, \quad (d\sigma/dt)_{\gamma p \to \pi p} \sim s^{-7}, \quad (d\sigma/dt)_{\gamma p \to \gamma p} \sim s^{-6}$

$\sim s^{-n}$ $A+B \to C+D$ $n=n_A+n_B+n_C+n_D-2$ $n_p=3$ $n_\pi=2$ $n_\gamma=1$

$$\frac{d\sigma}{dt}(A+B \to C+D) \to \frac{1}{t^{N-2}} f(t/s) \qquad N=n_A+n_B+n_C+n_D$$

Yu.L. Dokshitzer, QCD Phenomenology,
Lectures at the CERN–Dubna School, Pylos, August 2002

the deuteron break-up by a photon, $\gamma + D \to p + n$

$$\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \qquad \frac{t}{s} = \text{const}, \qquad K-2=1+6+3+3-2=11$$

For light nuclei:

Yu.N. Uzikov, Indication of Asymptotic Scaling in the Reactions
dd->p^{3}H, dd->n^{3}He and pd->pd, JETP Letters 81 (2005) 303.

$\sim s^{-22}$ (6+6+3+9-2=22) and $\sim s^{-16}$ (3+6+3+6-2=16)

Incorporating diquarks

V.T. Kim, Diquarks and Dynamics of Large P(T) Baryon Production, Mod.Phys.Lett.A 3 (1988) 909.

p/π^+ - ratio explanation, using that the diquark distribution function is harder: $(1-x)^1$ vs $(1-x)^3$ for quarks [$(1-x)^{2p-1}$].

Yu.L. Dokshitzer, QCD Phenomenology, Lectures at the CERN–Dubna School, Pylos, August 2002

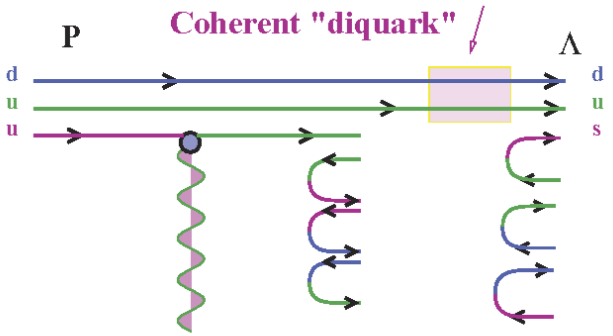


Fig. 4a: Gluon exchange produces a leading baryon.

M.A. Braun, V.V. Vechernin, Nuclear Structure Functions and Particle Production in the Cumulative Region in the Parton Model, Nucl.Phys. B427 (1994) 614

Can string junction carries the baryon number?

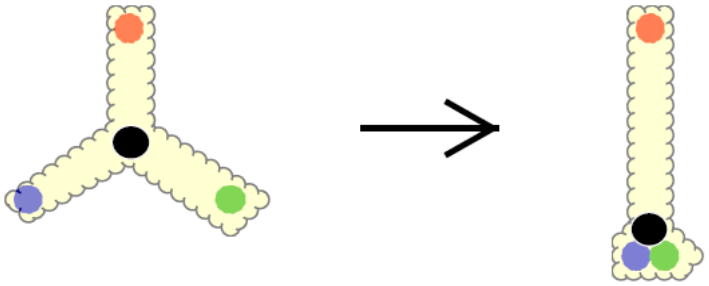
L. Montanet, G. C. Rossi, and G. Veneziano, “Baryonium Physics,”
Phys. Rept. 63, 149–222 (1980).

D. Kharzeev, “Can gluons trace baryon number?”
Phys.Lett. B 378, 238–246 (1996), arXiv:nucl-th/9602027.
Can be verified experimentally by studing of
baryon stopping in central pp and AA collisions.

Yu.M. Shabelski,
String Junction and Diffusion of Baryon Charge in Multiparticle Production Processes,
arXiv: 0705.0947 [hep-ph], (2007).
F. Bopp, Yu.M. Shabelski,
String junction effects for forward and central baryon production in hadron-nucleus collisions
Eur.Phys.J.A 28 (2006) 237-243

G.Pihan, A.Monnai, B.Schenke, Chun Shen,
Unveiling baryon charge carriers through charge stopping in isobar collisions
arXiv:2405.19439v1 [nucl-th] (2024).

Connection with diquarks:
Now B=1 corresponds to diquark



Comparison with low angle data

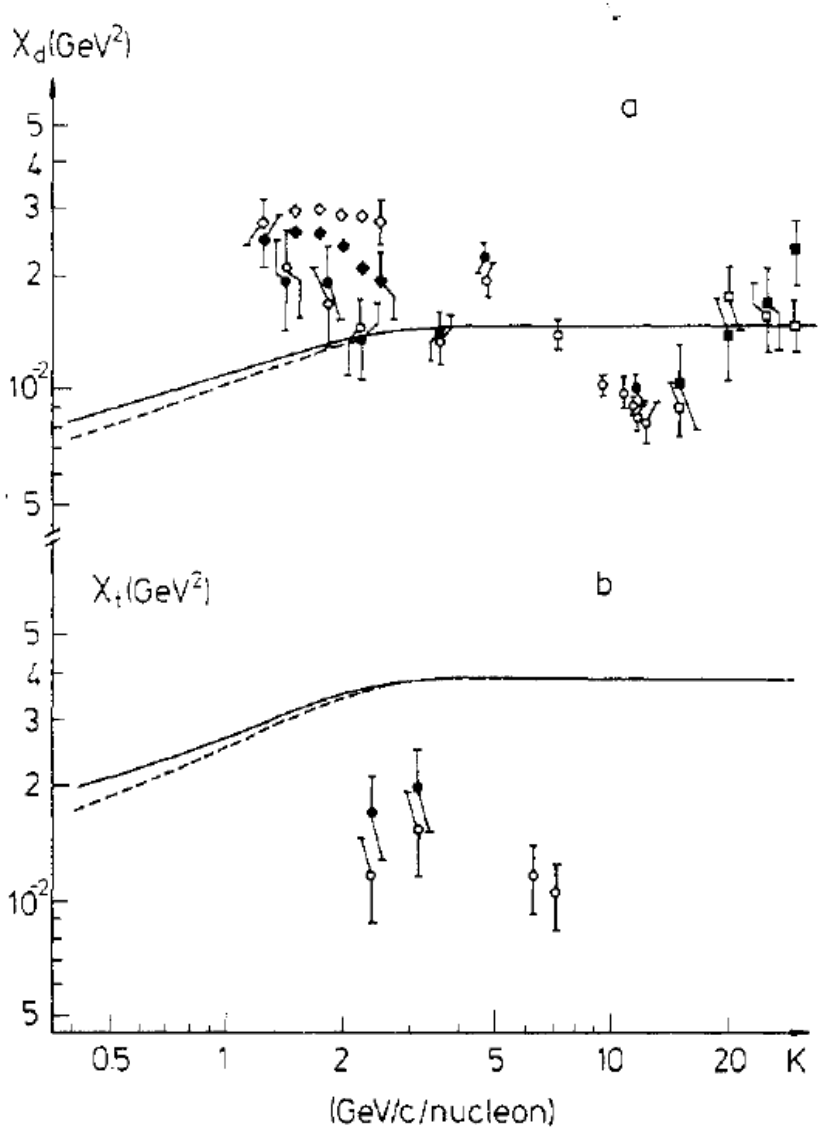


Figure 4. The coalescence coefficient χ_F (9) for the production of (a) deuterons and (b) tritium in the central kinematic region of the reaction (1), (i.e. for the small angles in the lab. system and with the high momenta k per nucleon). The full and broken curves are the results of our calculations with the formula (29) for the target nuclei Cu and Pb, respectively. The experimental data points correspond to the production of fragments in the following reactions: ● and ○, on Cu and Pb for the angle 9° at the initial energy 70 GeV [3]; ■ and □, on Ti and W for the angle 4.4° at the initial energy 300 GeV [7]; ◆ and ◇, on Cu and Ta for the angle 3.5° at the initial energy 9.2 GeV [4]. Here k is the momentum per one nucleon of fragment.