Double longitudinal spin asymmetry of direct photons in collisions of longitudinally polarized protons in the NLO approximation taking into account electromagnetic corrections

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This work was supported by the State Contract for Fundamental Research (subject code FSEG2025-0009).



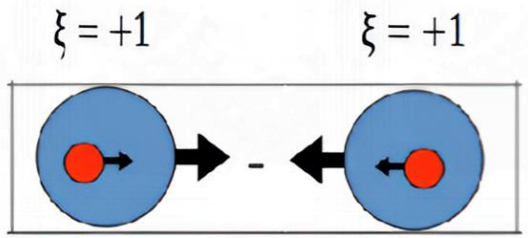




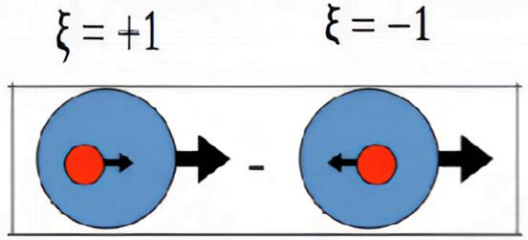
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Introduction
(collision of longitudinally polarized protons)

$$\xi = \frac{\vec{P}\cdot\vec{S}}{|P||S|}$$



$$\varepsilon = \frac{\vec{p}\vec{s}}{|p||s|} = \pm 1$$



$$a_{\xi=+1}^{\varepsilon=+1} + b_{\xi=+1}^{\varepsilon=-1}$$

$$a_{\xi=+1}^{\varepsilon=-1} + b_{\xi=+1}^{\varepsilon=+1}$$

$$a_{\xi=-1}^{\varepsilon=-1} + b_{\xi=-1}^{\varepsilon=+1}$$

$$a_{\xi=+1}^{\varepsilon=+1} + b_{\xi=+1}^{\varepsilon=-1}$$

$$a_{\xi=-1}^{\varepsilon=+1} + b_{\xi=-1}^{\varepsilon=-1}$$

$$a_{\xi=-1}^{\varepsilon=-1} + b_{\xi=-1}^{\varepsilon=-1}$$

$$a_{\xi=+1}^{\varepsilon=-1} + b_{\xi=-1}^{\varepsilon=+1}$$

$$a_{\xi=-1}^{\varepsilon=+1} + b_{\xi=-1}^{\varepsilon=+1}$$

$$\Rightarrow \sigma^{\xi_1\xi_2} \sim \sum_{\varepsilon_a\varepsilon_b} f_{a(\varepsilon_a)}^{\xi_1} \otimes f_{b(\varepsilon_b)}^{\xi_2} \otimes \sigma^{a(\varepsilon_a)b(\varepsilon_b)\to c\gamma}$$

Introduction (DLSA)

$$\Delta f_a = f^{+1}_{a(+1)} - f^{+1}_{a(-1)} = f^{-1}_{a(-1)} - f^{-1}_{a(+1)} \quad (1)$$

$$A^{\gamma}_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} \sim \frac{\sum_{a,b} \Delta f_a \otimes \Delta f_b \otimes \Delta\sigma^{ab->c\gamma}}{\sum_{a,b} f_a \otimes f_b \otimes \sigma^{ab->c\gamma}} \quad (2)$$

$$\Delta\sigma^{ab->c\gamma} = \frac{1}{2}(\sigma^{a(+)b(+)->c\gamma} - \sigma^{a(+)b(-)->c\gamma}) \quad (3)$$

$$\sigma^{ab->c\gamma} = \frac{1}{2}(\sigma^{a(+)b(+)->c\gamma} + \sigma^{a(+)b(-)->c\gamma}) \quad (4)$$

$$f_{a/b} = f^{+1}_{a/b(+1)} + f^{+1}_{a/b(-1)} = f^{-1}_{a/b(-1)} + f^{-1}_{a/b(+1)} \quad (5)$$

Introduction
(proton spin problem)

Contribution to proton spin from quark helicities : $\Delta\Sigma$

Contribution from gluon helicities : ΔG

Contribution from photon helicities : $\Delta\Gamma$

Contributions from orbital angular momenta of quarks, gluons and photons respectively : $l_{q,z}$, $l_{g,z}$, $l_{\gamma,z}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \Delta\Gamma + l_{q,z} + l_{g,z} + l_{\gamma,z} \quad (5)$$

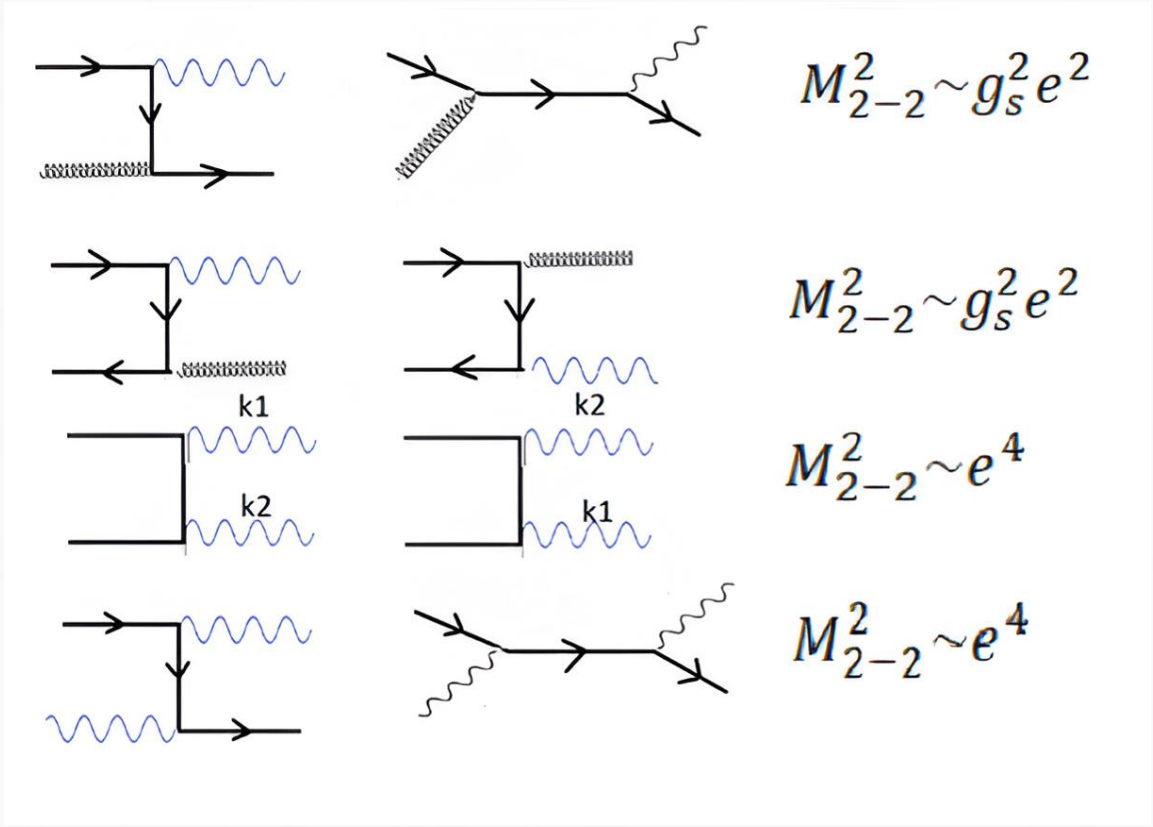
$$\frac{1}{2}\sum_{q}\int_{0}^{1}\Delta f_q(x,Q^2)dx = \frac{1}{2}\Delta\Sigma \quad (6)$$

$$\int_{0}^{1}\Delta f_g(x,Q^2)dx = \Delta G \quad (7)$$

$$\int_{0}^{1}\Delta f_\gamma(x,Q^2)dx = \Delta\Gamma \quad (8)$$

Leading order

The considered subprocesses : $qg \to q\gamma$, $q\bar{q} \to g\gamma$, $q\bar{q} \to \gamma\gamma$, $q\gamma \to q\gamma$ – two-vertex diagrams for direct photon production.



Next to leading order

$$qg \to q\gamma,\ q\bar{q} \to g\gamma$$

$$M_{2-2} = M_{2-2}^{LO} + M_{2-2}^{NLO_QCD} + M_{2-2}^{NLO_QED} + O(g_s e,\ g_s^3 e,\ g_s \mathrm{e}^3)$$

$$M_{2-2}^2 = |M_{2-2}^{LO}|^2 + 2\,\mathrm{Re}(M_{2-2}^{LO*} M_{2-2}^{NLO_QCD}) + 2\,\mathrm{Re}(M_{2-2}^{LO*} M_{2-2}^{NLO_QED}) + O(g_s^2 e^2,\ g_s^4 e^2,\ g_s^2 e^4)$$

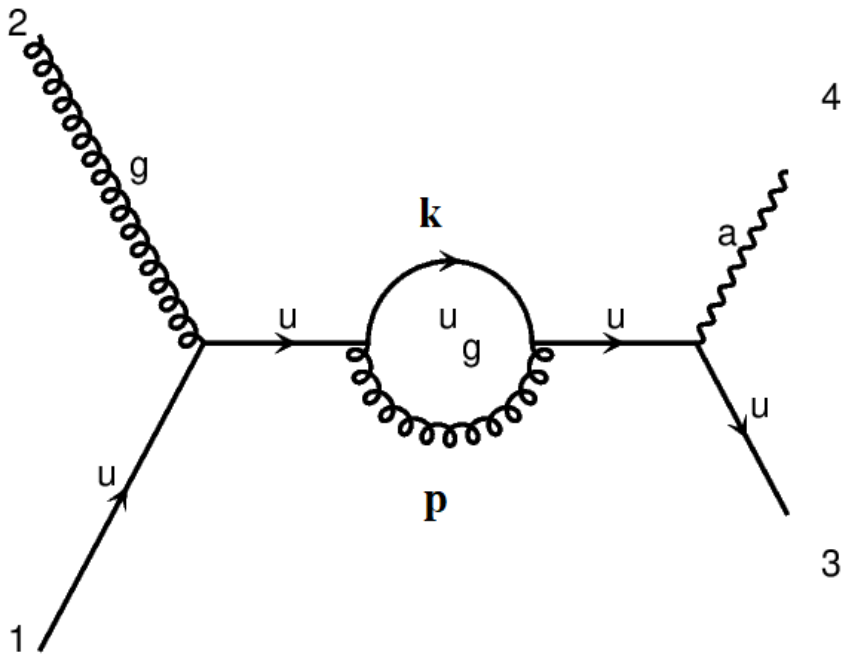
$$q\gamma \to q\gamma,\ q\bar{q} \to \gamma\gamma$$

$$M_{2-2} = M_{2-2}^{LO} + M_{2-2}^{NLO_QCD} + M_{2-2}^{NLO_QED} + O(e^2,\ g_s^2 e^2,\ e^4)$$

$$M_{2-2}^2 = |M_{2-2}^{LO}|^2 + 2\,\mathrm{Re}(M_{2-2}^{LO*} M_{2-2}^{NLO_QCD}) + 2\,\mathrm{Re}(M_{2-2}^{LO*} M_{2-2}^{NLO_QED}) + O(e^4,\ g_s^2 e^2,\ e^6)$$

Divergences

Virtual divergences

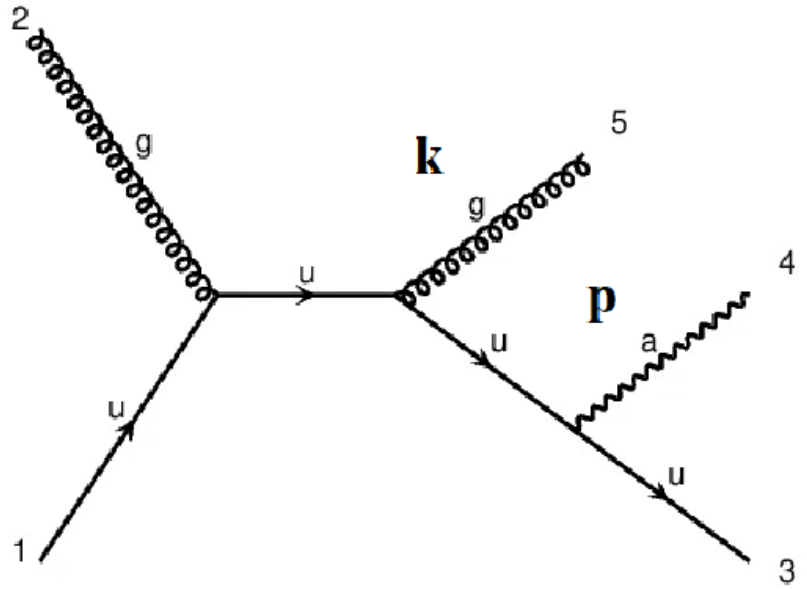


Infrared (IR) soft divergence: p = 0

Infrared (IR) collinear divergence k || p

KLN

Same real divergences



Infrared (IR) soft divergence: p = 0

Infrared (IR) collinear divergence k || p

Real emissions

Real QCD ($|M^{LO}_{2-3}|^2 \sim g_s^4 e^2$)

1) $qg \to q\gamma g$

2) $q\bar{q} \to g\gamma g$

3) $q\gamma \to q\gamma g$

4) $q\bar{q} \to \gamma\gamma g$

Real QED ($|M^{LO}_{2-3}|^2 \sim g_s^2 e^4$)

5) $qg \to q\gamma\gamma$

6) $q\bar{q} \to g\gamma\gamma$

7) $q\gamma \to q\gamma\gamma$

8) $q\bar{q} \to \gamma\gamma\gamma$

Additional partonic channels ($|M^{LO}_{2-3}|^2 \sim g_s^4 e^2$)

9) $gg \to q\bar{q}\gamma$

10) $qq \to q\gamma q$

11) $\bar{q}\bar{q} \to \bar{q}\gamma\bar{q}$

12) $q_1 q_2 \to q_1 \gamma q_2$

13) $\bar{q}_1\bar{q}_2 \to \bar{q}_1\gamma\bar{q}_2$

14) $q\bar{q} \to q\gamma\bar{q}$

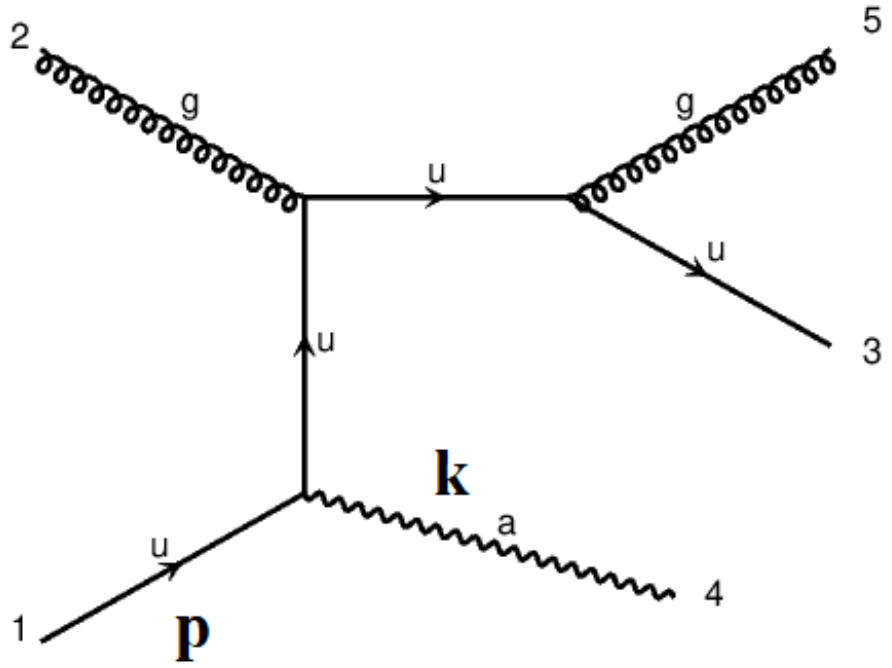
15) $q\bar{q} \to \bar{q}\gamma q$

16) $q_1\bar{q}_2 \to \bar{q}_1\gamma q_2$

17) $q_1\bar{q}_1 \to q_2\gamma\bar{q}_2$

Real emissions

Initial collinear divergences



Inital-State collinear divergence: p || k

$$d\sigma_{2-3}\Big|_{\text{initial } collinear} = d\sigma_{2-2} \otimes \left[\frac{\alpha_s}{2\pi}\frac{1}{\varepsilon}P_{q\to q\gamma}(z)\right]$$

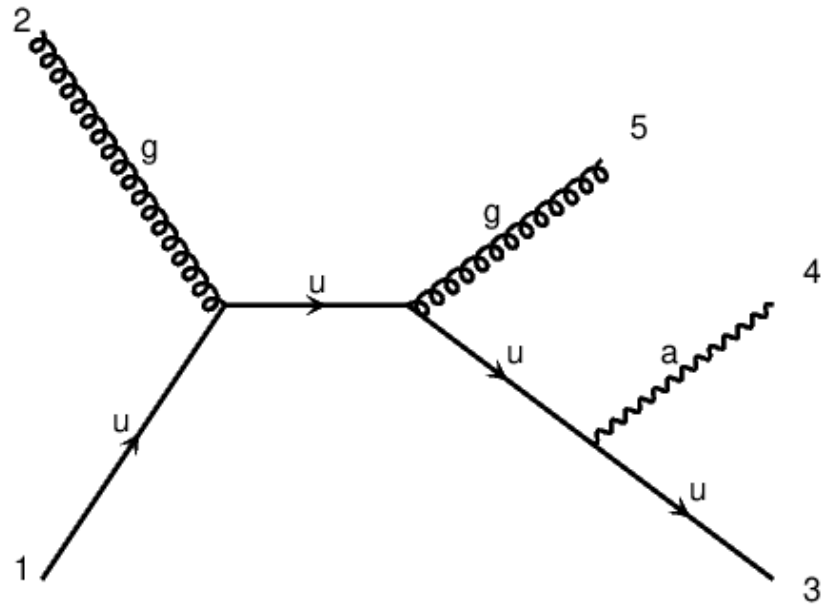
$$d\sigma\Big|_{\text{initial } collinear} \sim f_0(x) \otimes d\sigma_{2-2} \otimes \left[\frac{\alpha_s}{2\pi}\frac{1}{\varepsilon}P_{q\to q\gamma}(z)\right]$$

$$f_0(x) = f_{MS-bar}(x) - \left[\frac{\alpha_s}{2\pi}\frac{1}{\varepsilon}P_{q\to q\gamma}(z)\right] \otimes f_{MS-bar}(x)$$

$$d\sigma\Big|_{\text{initial } collinear} \sim 0$$

Real emissions

Final collinear divergences



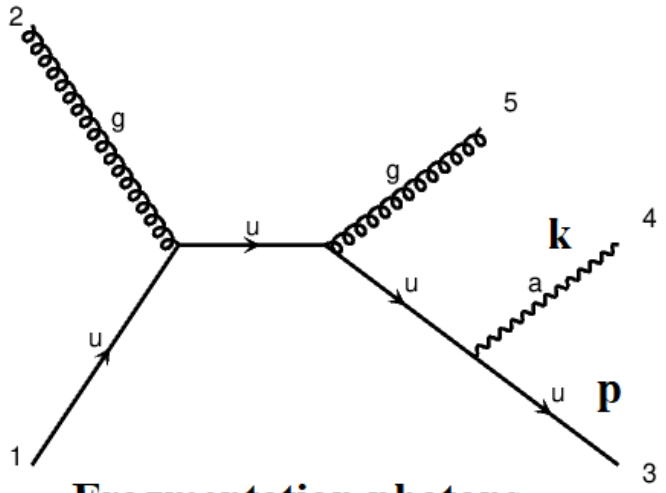
Final state collinear divergence p || k $\sim \frac{1}{\varepsilon}$

$$d\sigma\Big|_{final\ collinear} \sim d\sigma_{2-2} \otimes D_{\gamma\ \mathrm{MS-bar}}^{collinear}(z)$$

$$D_{\gamma\ \mathrm{MS-bar}}^{collinear}(z) \sim e^2 D^0(z) + g_s^2 \ln\left(\frac{M_f^2}{\mu_0^2}\right) P_{qq} D^0(z) + \cdots$$

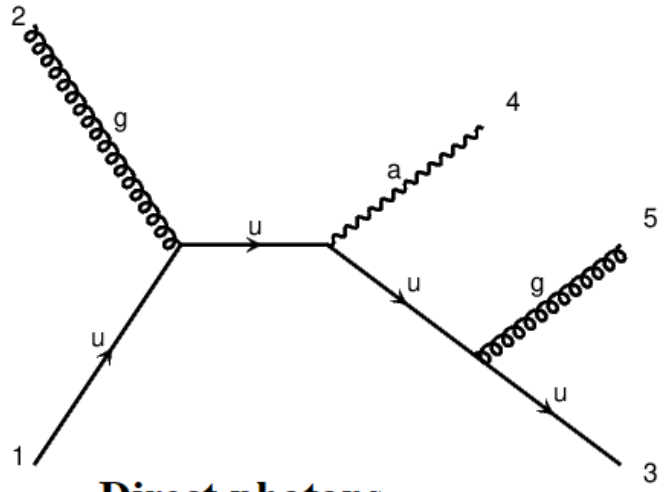
$$d\sigma\Big|_{final\ collinear} \sim \left|M_{2-2}^{LO}\right|^2 \otimes D_{\gamma\ \mathrm{MS-bar}}^{collinear}(z) = O(g_s^2 e^4, g_s^4 e^2, e^6)$$

Prompt photons (inclusive photons)



Fragmentation photons

(k || p)



Direct photons

Bremsstrahlung photons

(k ⫽ p)

Prompt photons (inclusive photons)

LO - Direct photons ⇨ NLO - Prompt photons

Hard process cross section

$$\frac{d\sigma_{a(\varepsilon_1)b(\varepsilon_2)->c\gamma}}{dt} = \frac{|M^{LO}_{a(\varepsilon_1)b(\varepsilon_2)->c\gamma}|^2 + 2\,\mathrm{Re}(M^{LO}_{a(\varepsilon_1)b(\varepsilon_2)->c\gamma}{}^{*} M^{NLO_QCD}_{a(\varepsilon_1)b(\varepsilon_2)->c\gamma}) + 2\,\mathrm{Re}(M^{LO}_{a(\varepsilon_1)b(\varepsilon_2)->c\gamma}{}^{*} M^{NLO_QED}_{a(\varepsilon_1)b(\varepsilon_2)->c\gamma})}{16\pi^2 s} \quad (9)$$

$$\frac{d\sigma_{a(\varepsilon_1)b(\varepsilon_2)->cd\gamma}}{dt} = \frac{1}{1024\pi^5 s}\int d\Phi_{2-3}\,|M^{LO}_{a(\varepsilon_1)b(\varepsilon_2)->cd\gamma}|^2 = \frac{1}{1024\pi^5 s}\int_{-1}^{1} d\cos\vartheta \int_0^{2\pi} d\varphi\,|M^{LO}_{a(\varepsilon_1)b(\varepsilon_2)->cd\gamma}|^2 =$$

$$= \frac{1}{1024\pi^5 s}\int_{-1}^{1} d\cos\vartheta \int_0^{2\pi} d\varphi\,|M^{LO}_{a(\varepsilon_1)b(\varepsilon_2)->cd\gamma}|^2\,\theta^{collinear} + \frac{1}{16\pi^2 s}\int_0^1 dz\,|M^{LO}_{a(\varepsilon_1)b(\varepsilon_2)->c\gamma}|^2\,D_\gamma^{collinear}(z) \quad (10)$$

$$\theta^{collinear} = \theta(\Delta R(p, p_{\gamma/g}) - R_{cut}) - \text{Heviside function}$$

$$\Delta R(p, p_{\gamma/g}) = \sqrt{(y - y_{\gamma/g})^2 + (\varphi - \varphi_{\gamma/g})^2}$$

Recola - REcursive Computation of One-Loop Amplitudes

Invariant prompt (inclusive) photon cross section

$$E^{\gamma}\frac{d^3\sigma}{dp_{\gamma}^3}(\xi_1,\xi_2)\Big|_{y_0}=\sum_{a,b,c}\sum_{\varepsilon_1\varepsilon_2}\iint_{x_1x_2}dx_1dx_2 f_{a,\varepsilon_1}^{\xi_1}(x_1,Q^2)f_{b,\varepsilon_2}^{\xi_2}(x_2,Q^2)J\left(\frac{d\sigma^{a(\varepsilon_1)b(\varepsilon_2)\to c\gamma}}{dt}+\frac{d\sigma^{a(\varepsilon_1)b(\varepsilon_2)\to cd\gamma}}{dt}\right) \quad (11)$$

$$E^{\gamma}\frac{d^3\sigma}{dp_{\gamma}^3}\Big|_{y_0}=\frac{1}{4}\sum_{\xi_1,\xi_2}E^{\gamma}\frac{d^3\sigma}{dp_{\gamma}^3}(\xi_1,\xi_2)\Big|_{y_0}=\sum_{a,b,c}\iint_{x_1x_2}dx_1dx_2 f_a(x_1,Q^2)f_b(x_2,Q^2)J\left(\frac{d\sigma^{ab\to c\gamma}}{dt}+\frac{d\sigma^{ab\to cd\gamma}}{dt}\right) \quad (12)$$

$$\left[E^{\gamma}\frac{d^3\sigma}{dp_{\gamma}^3}(++)-E^{\gamma}\frac{d^3\sigma}{dp_{\gamma}^3}(+-)\right]\Big|_{y_0}=E^{\gamma}\frac{d^3\Delta\sigma}{dp_{\gamma}^3}=\sum_{a,b,c}\iint_{x_1x_2}dx_1dx_2\Delta f_a(x_1,Q^2)\Delta f_b(x_2,Q^2)J\left(\frac{d\Delta\sigma^{ab\to c\gamma}}{dt}+\frac{d\Delta\sigma^{ab\to cd\gamma}}{dt}\right) \quad (13)$$

Helicity distribution of the photon

Within our model, photons are emitted only by valence quarks. This model for unpolarized photon distribution present at work (Carl Scmidt, Jon Pumplin, Daniel Stump, C.P. Yuan «CTEQED PDFs from isolated Photon Production in Deep Inelastic Scattering»):

$$f_\gamma(x,Q_0)=\frac{\alpha}{2\pi}\left[A_u e_u{}^2(P_{\gamma q}\circ f_u{}^0)(x)+A_d e_d{}^2(P_{\gamma q}\circ f_d{}^0)(x)\right] \quad (14)$$

$$A_u=\ln\left(\frac{Q_0^2}{m_u^2}\right) \quad (15)$$

$$A_d=\ln\left(\frac{Q_0^2}{m_d^2}\right) \quad (16)$$

Helicity distribution of photons:

$$\Delta f_\gamma(x,Q_0)=\frac{\alpha}{2\pi}\left[A_u e_u{}^2(P_{\gamma q}\circ \Delta f_u{}^0)(x)+A_d e_d{}^2(P_{\gamma q}\circ \Delta f_d{}^0)(x)\right] \quad (17)$$

Helicity distribution of the photon

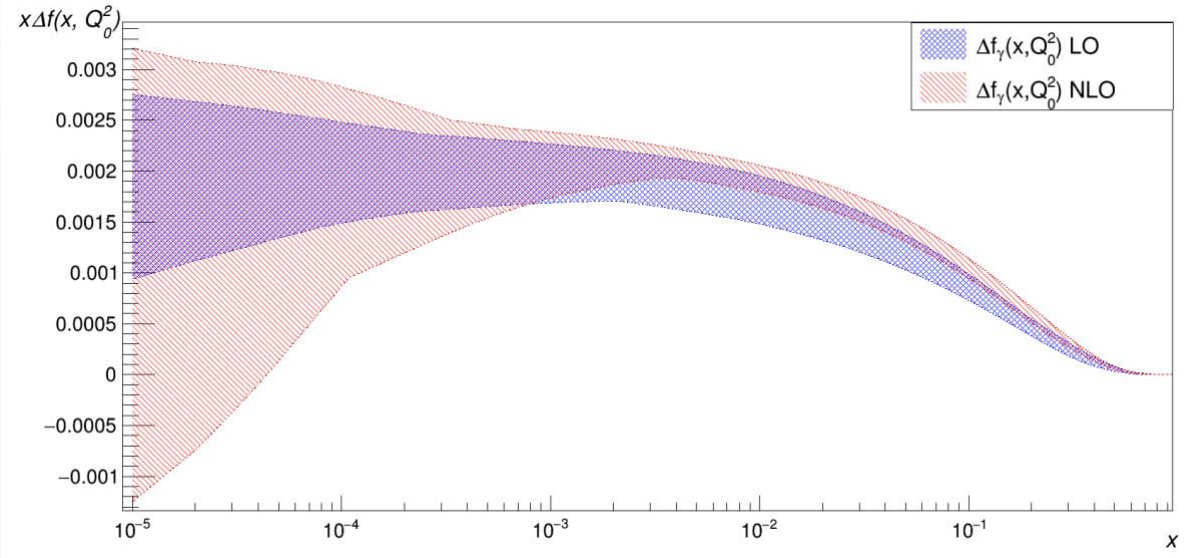


Fig. 1. Helicity distribution of the photon in LO approximation (blue curve) at scale $Q_0 = 5$ GeV with uncertainties – built on u and d quark distributions from the set «NNPDFpol20_lo_as_01180_1000» for 1000 replicas; and the helicity distribution of the photon in NLO approximation (red curve) at the same scale with uncertainties – built on u and d quark distributions from the set «NNPDFpol11_100» for 100 replicas.

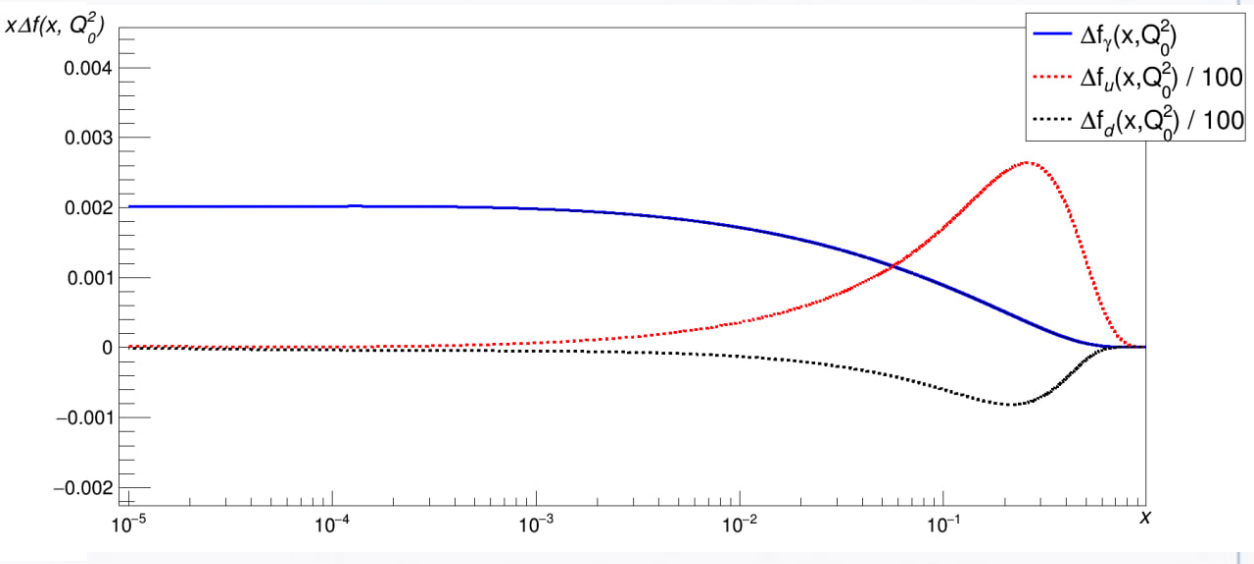


Fig.2. Helicity distribution of the photon in LO approximation (blue curve) compared with the LO helicity distributions of u (red dashed) and d (black dashed) quarks, which were used to construct the photon distribution (formula (10)), divided by 100 at scale $Q_0 = 5$ GeV.

Contribution of photon helicities to proton spin

Using formula (4), the contribution of photon helicities to the proton spin $\Delta\Gamma$ was computed at scale $Q_0 = 5$ GeV:

$$\Delta\Gamma_{LO} = 0.0186158 \pm 0.00119392$$

$$\Delta\Gamma_{NLO} = 0.0204215 \pm 0.00165448$$

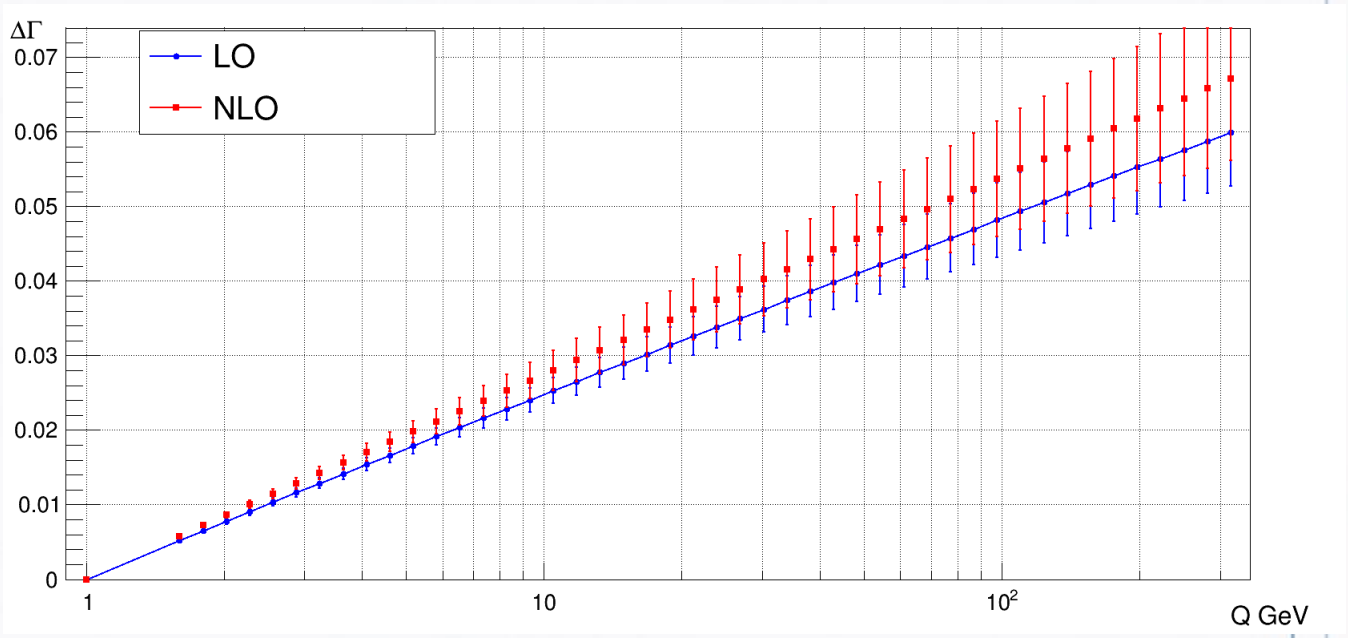
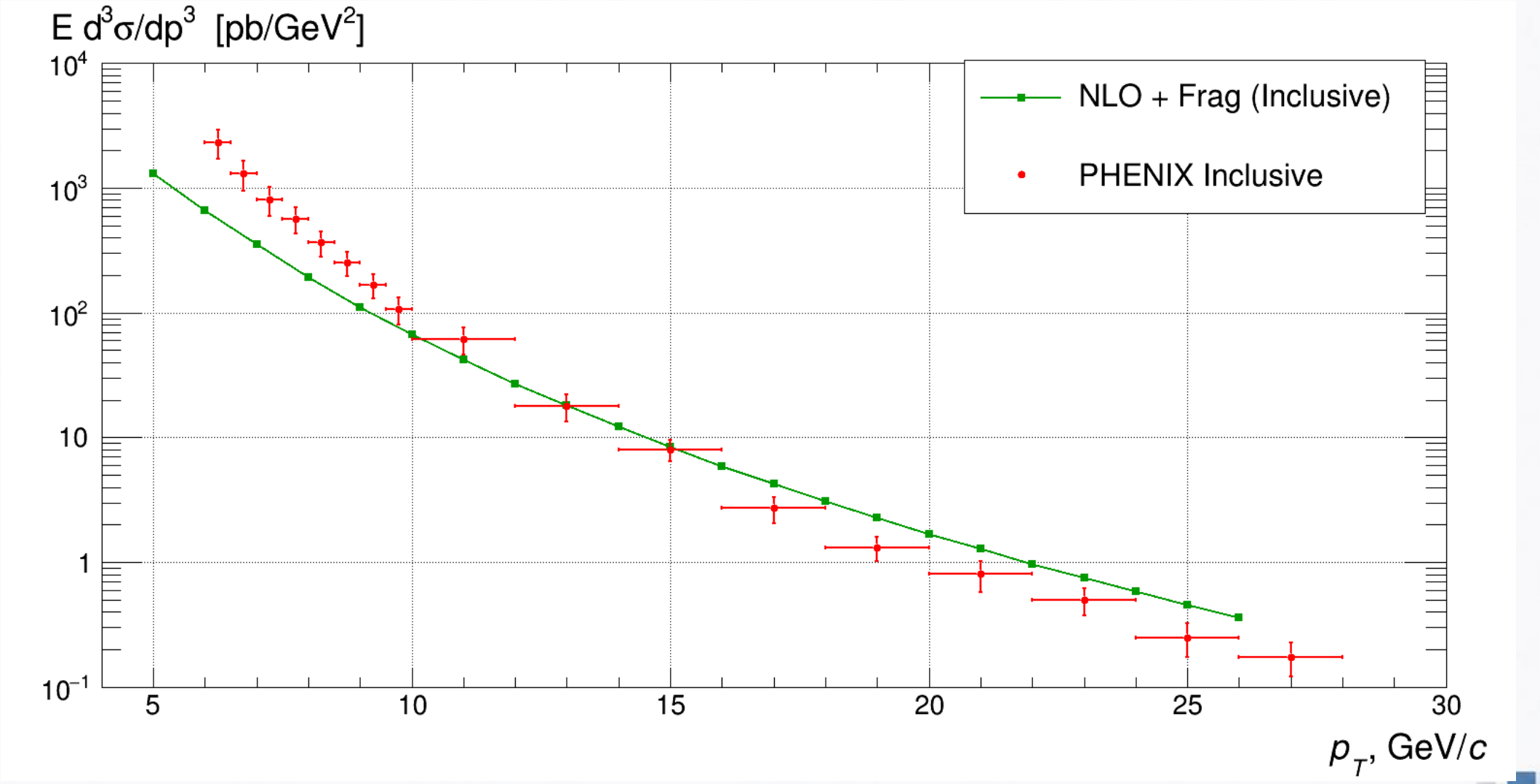
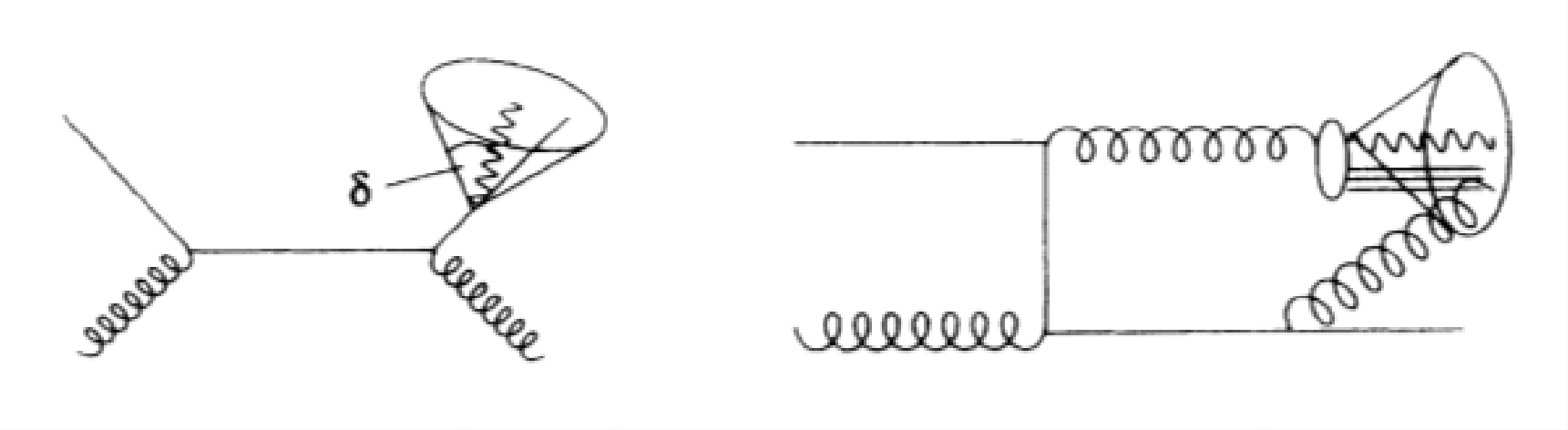


Fig.3. Dependence of the photon helicity contribution $\Delta\Gamma$ on the scale Q. Calculations with LO (blue curve) and NLO (red curve) distribution functions.

Prompt photon (inclusive) invariant cross section, 510 GeV



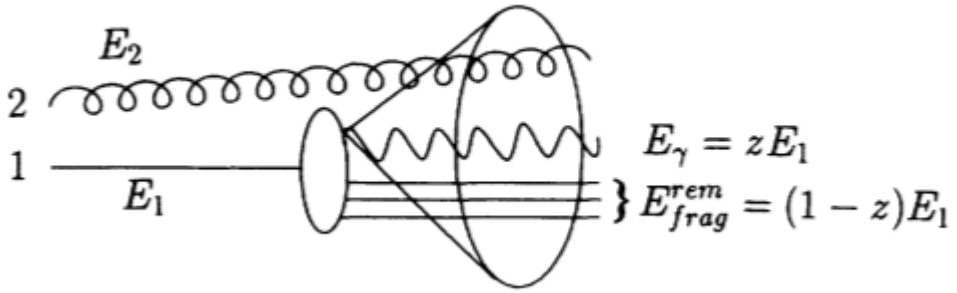
Isolated photons



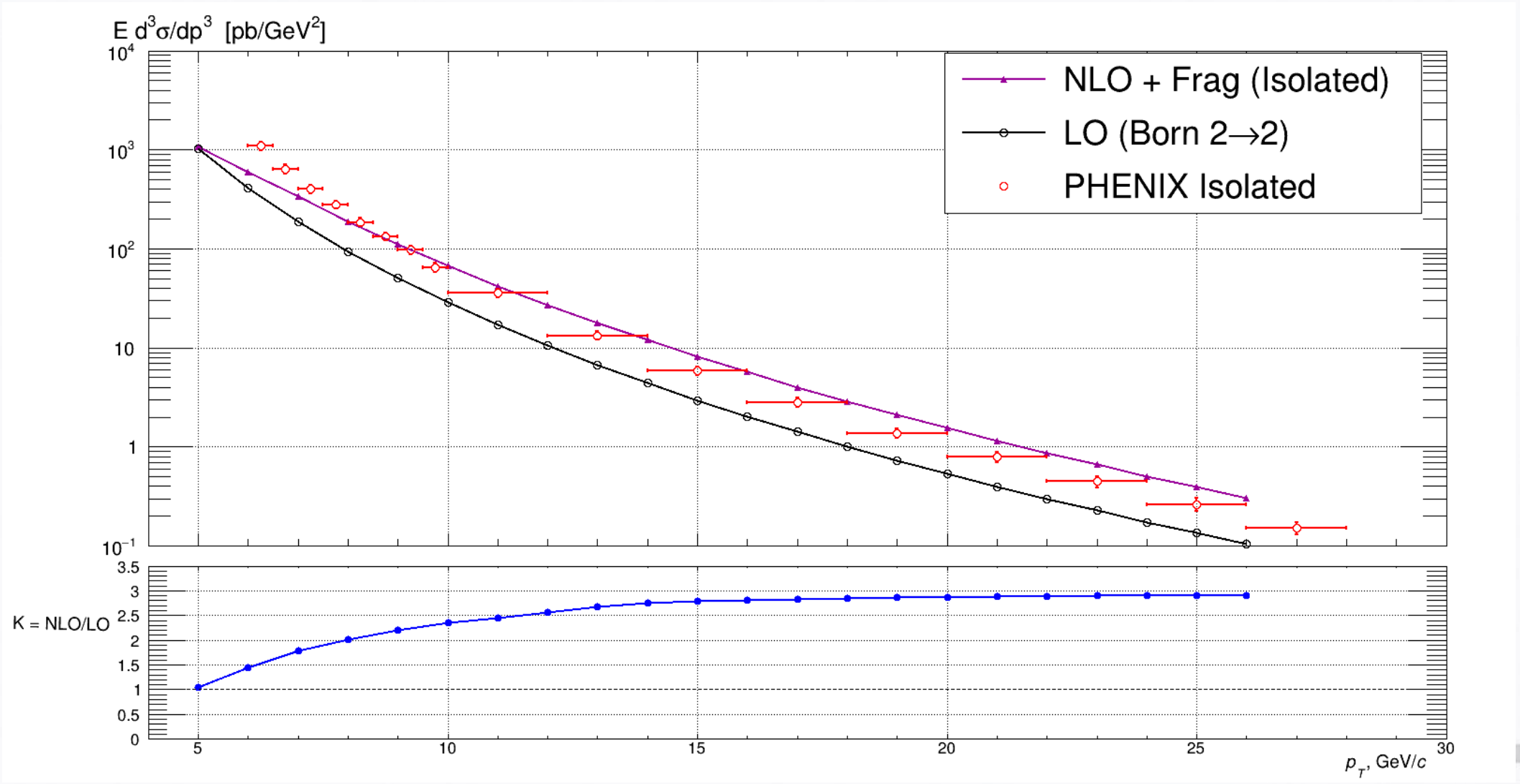
Isolation cut condition

$$r_{cone} = \sqrt{(\delta\eta)^2 + (\delta\phi)^2} = 0.5$$

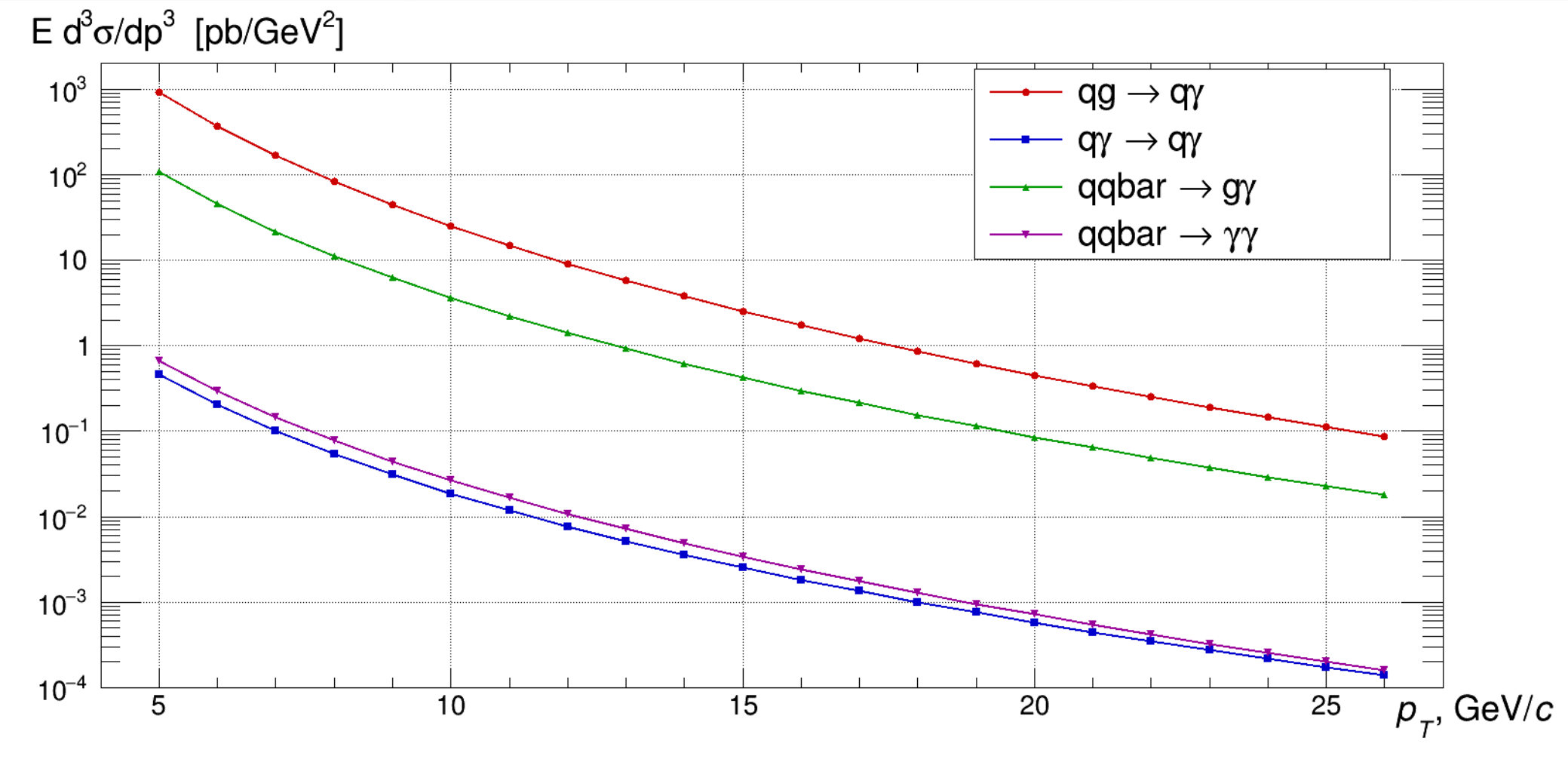
$$E_{cone} < 0.1E_{\gamma}$$



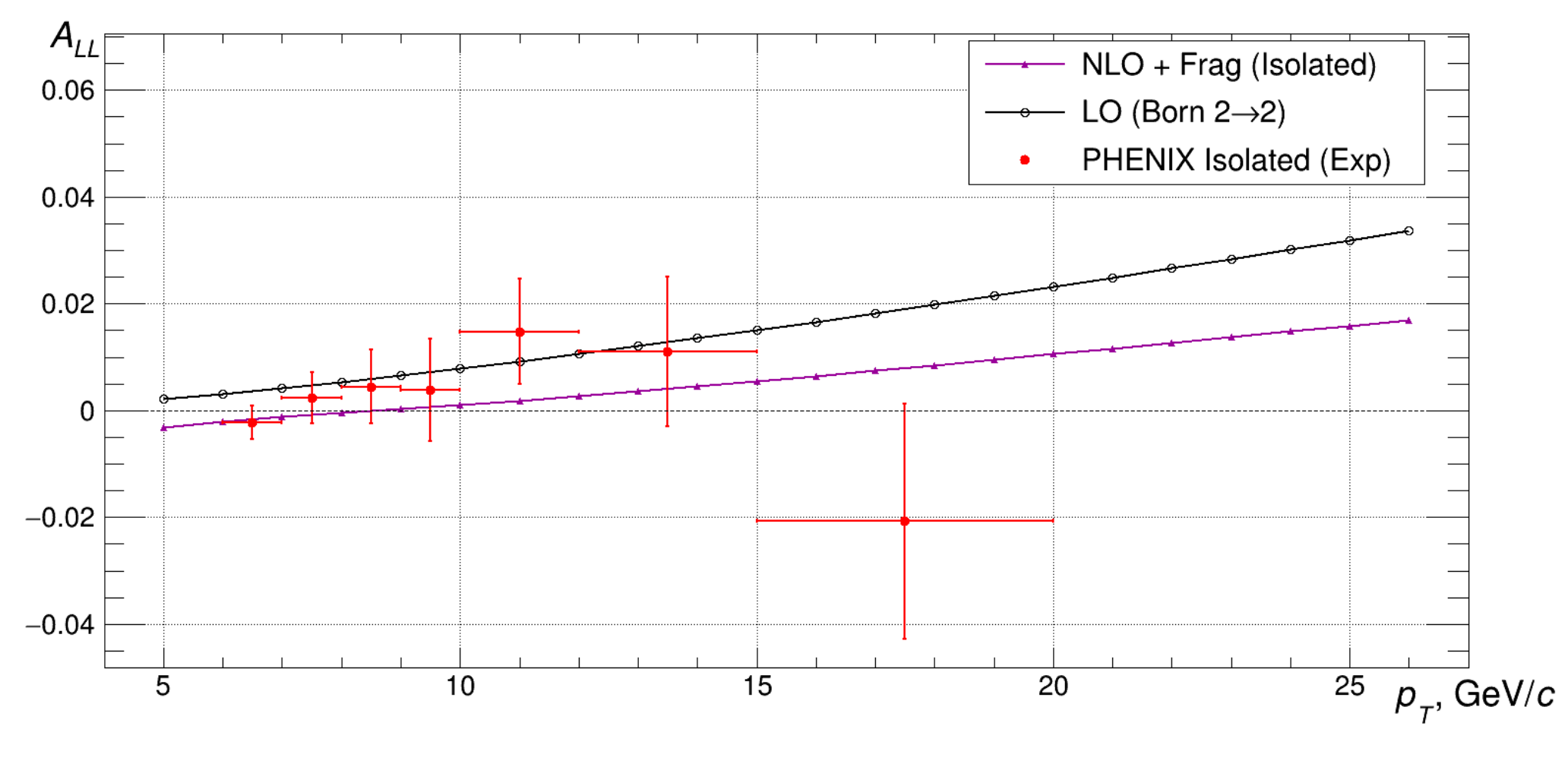
Isolated (direct) photons invariant cross section, 510 GeV



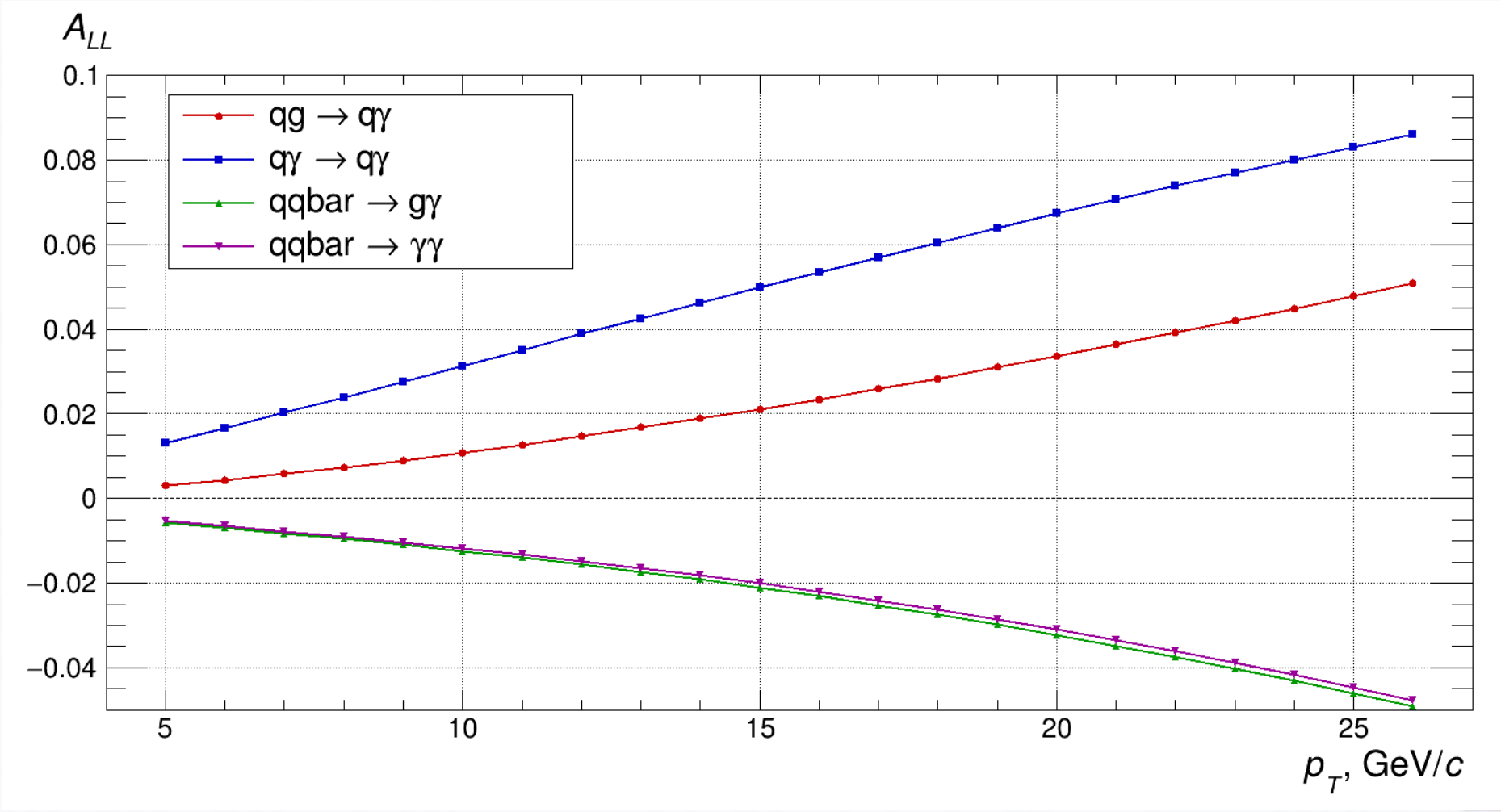
LO direct photons invariant spectrum for different subprocesses, 510 GeV



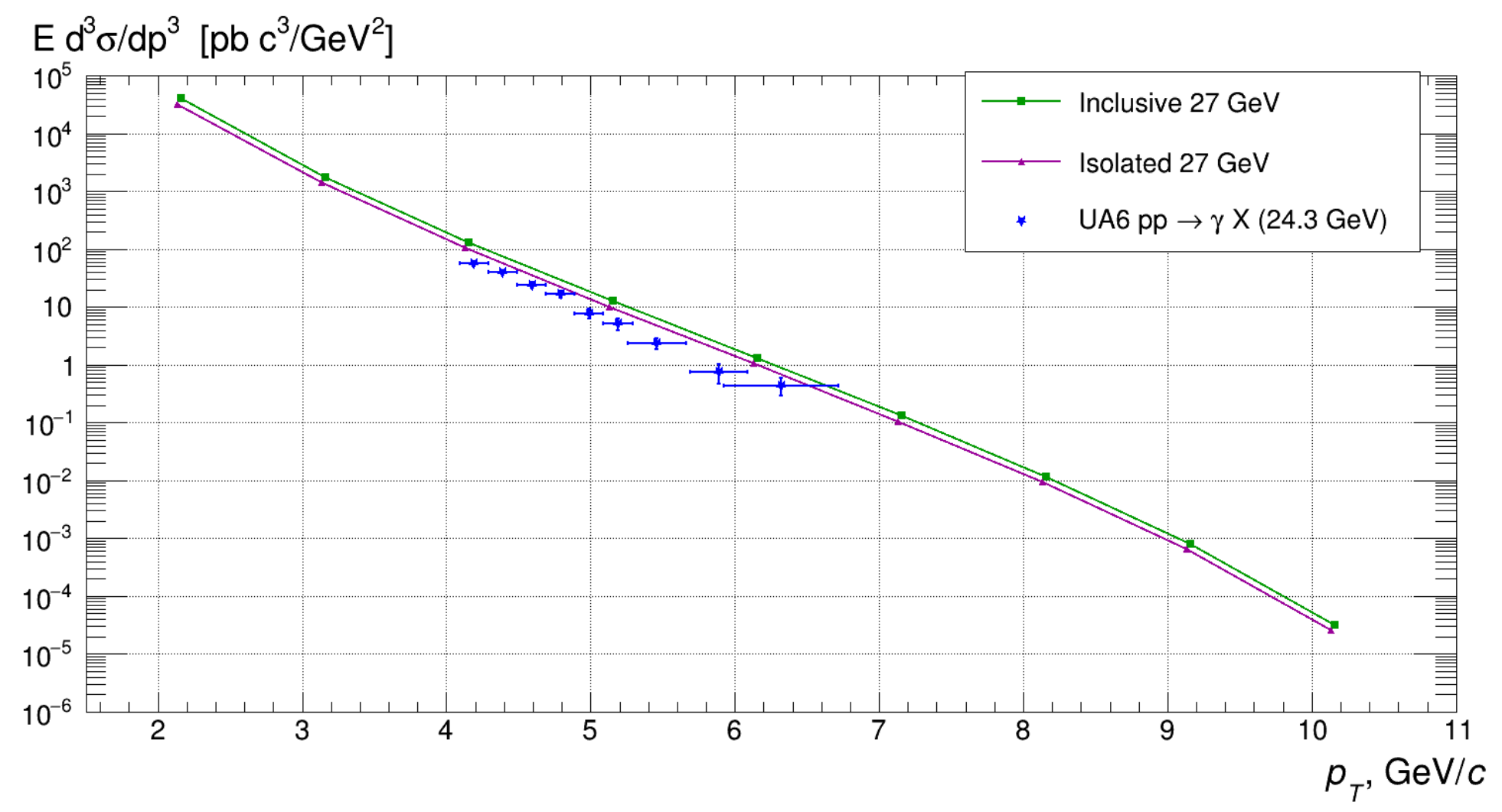
DLSA Isolated and direct (LO) photons, 510 GeV



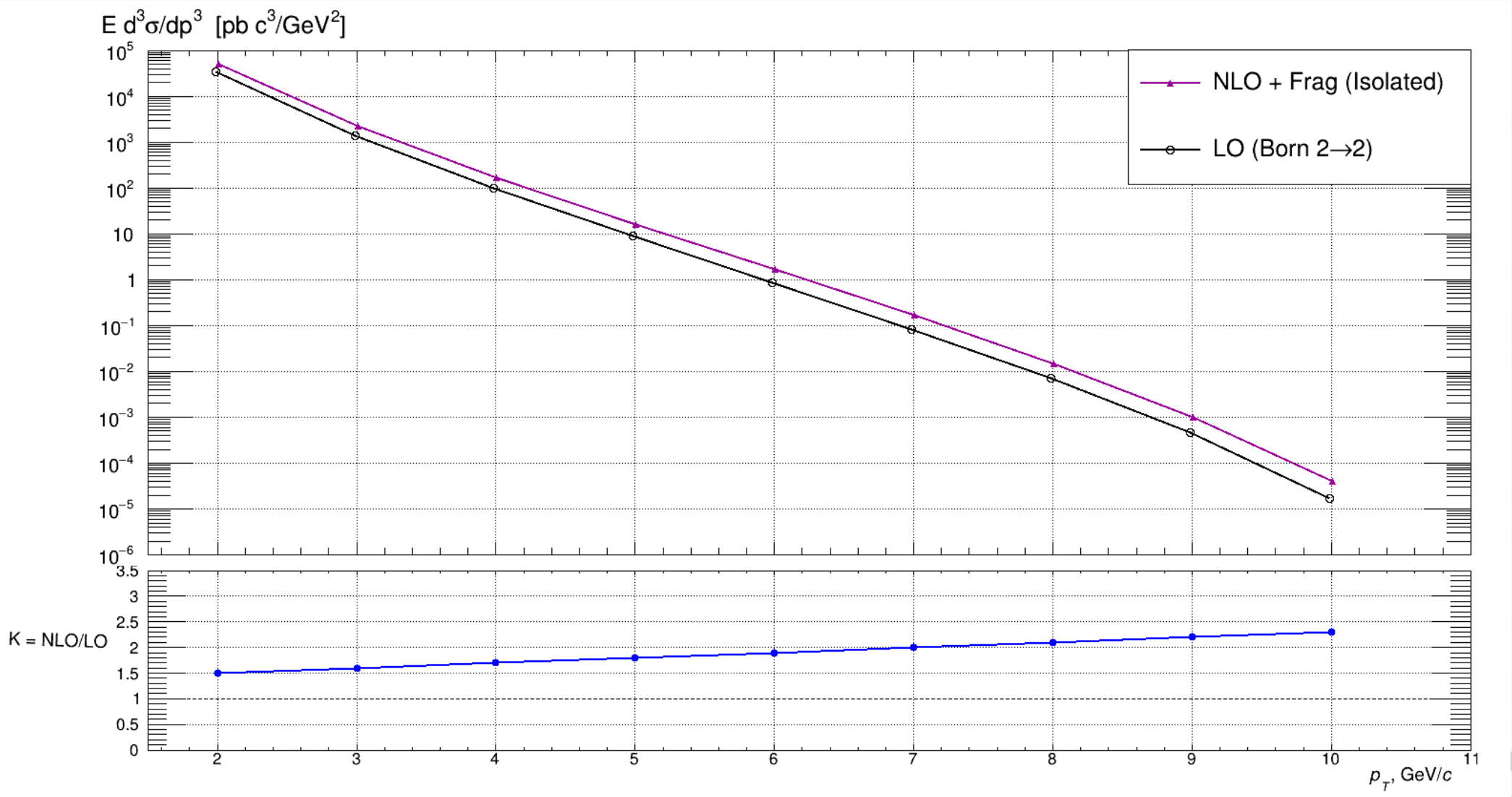
DLSA direct (LO) photons for different subprocesses, 510 GeV



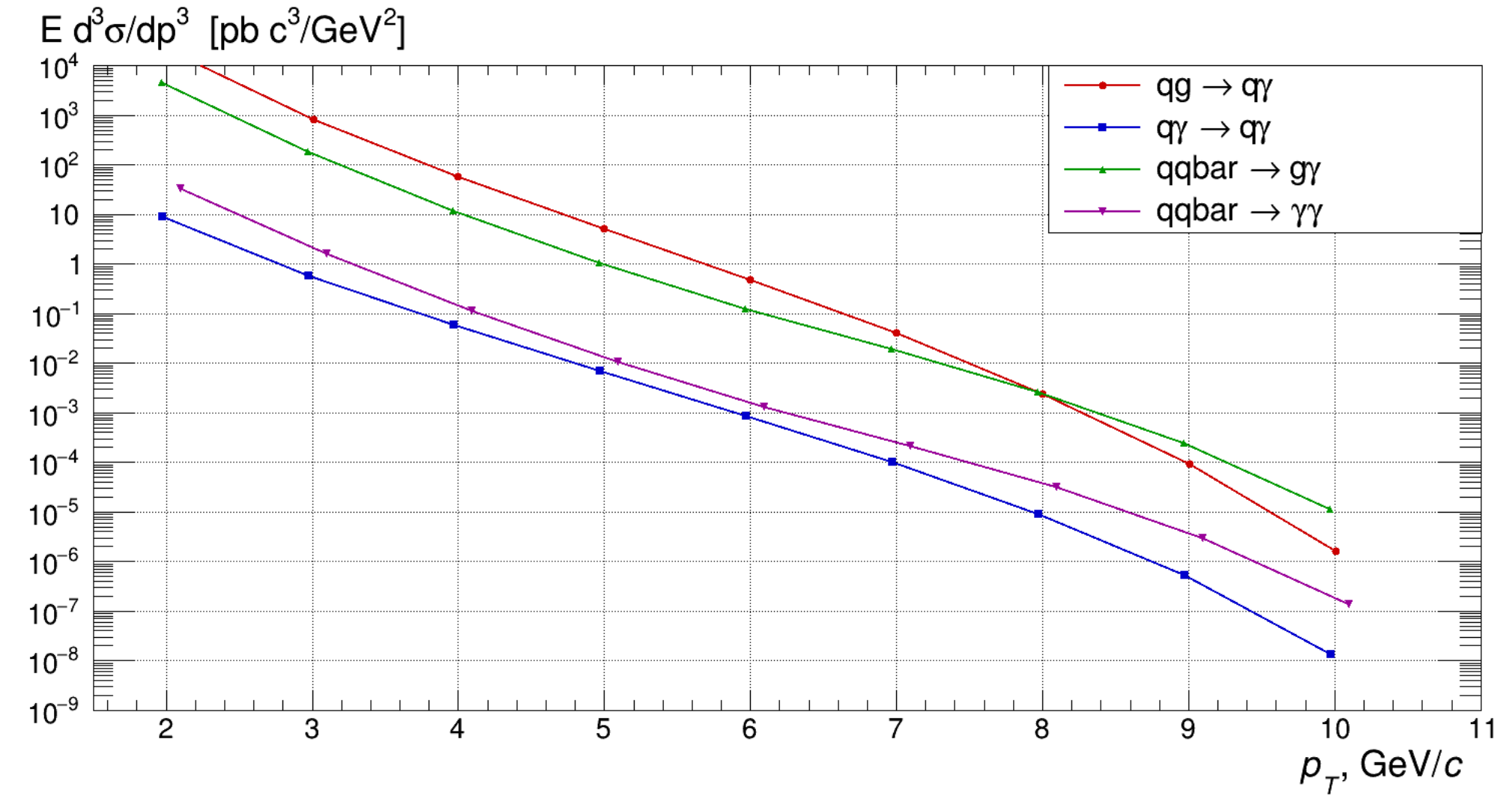
Prompt and isolated photons invariant cross section, 27 GeV



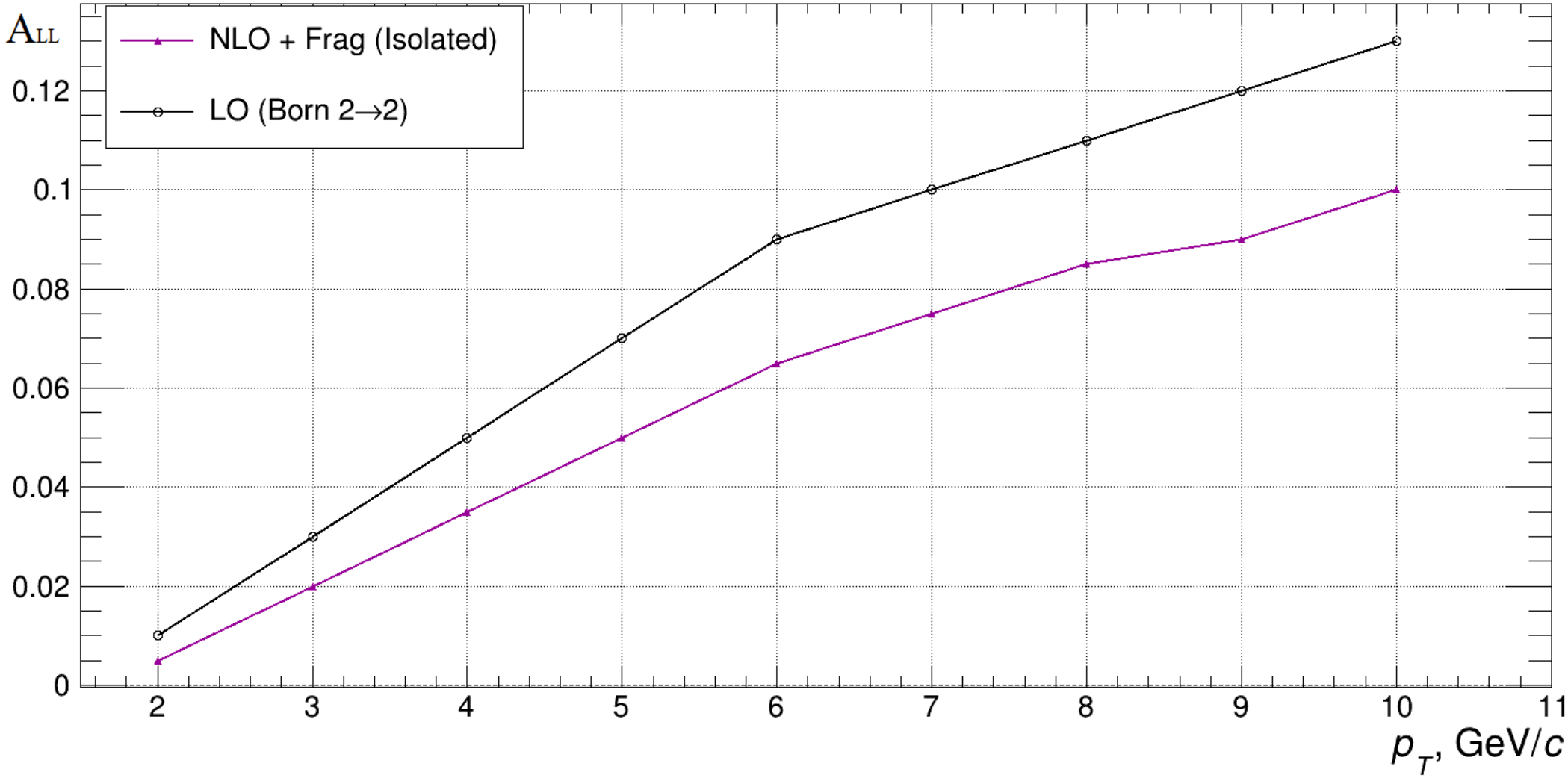
Isolated and direct (LO) photons invariant cross section, 27 GeV



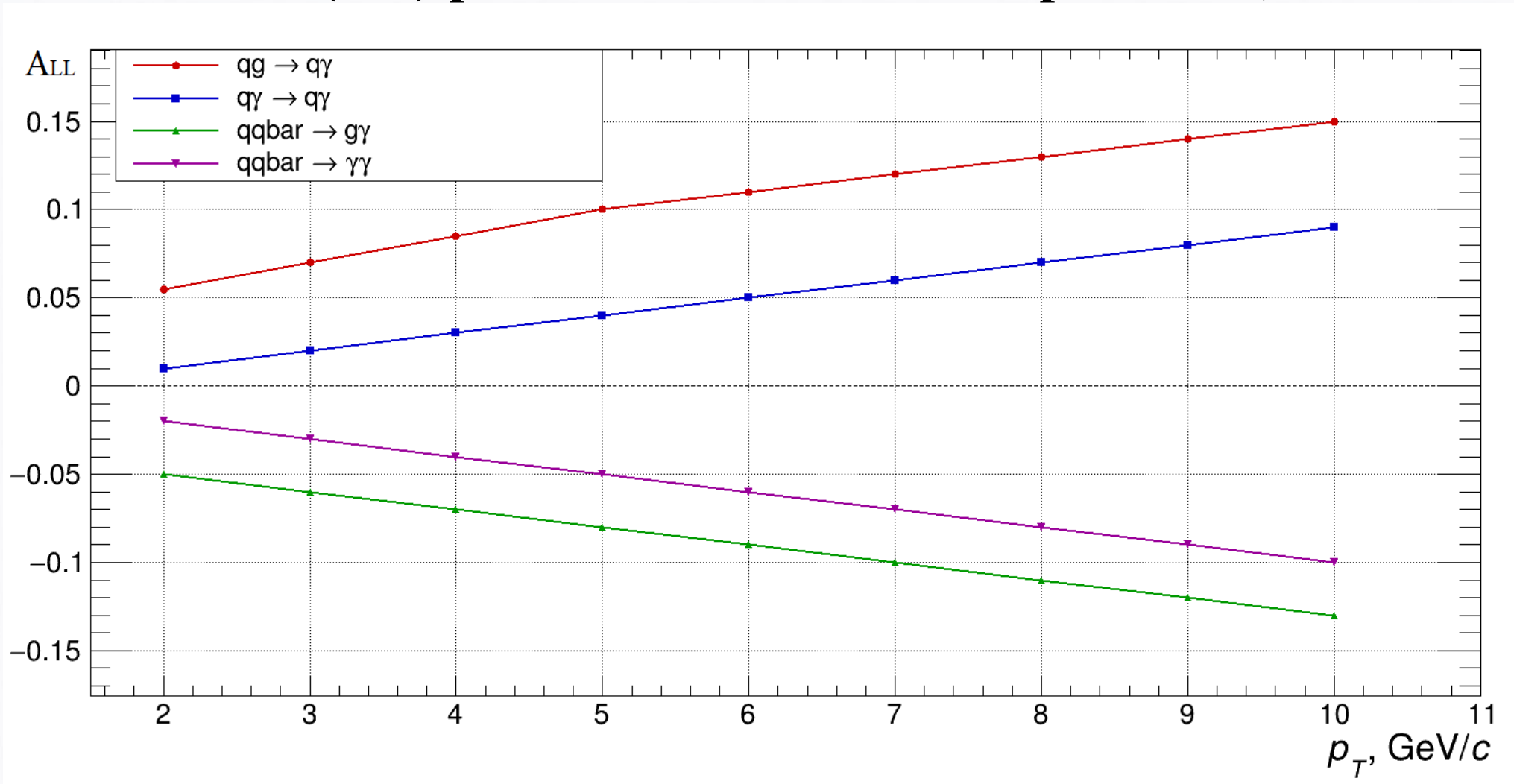
LO direct photons invariant spectrum for different subprocesses, 27 GeV



DLSA Isolated and direct (LO) photons, 27 GeV



DLSA direct (LO) photons for different subprocesses, 27 GeV



Thank you for your attention!