Code for simulations of light nucleus-nucleus elastic interactions.

A. Galoyan, A. Ribon, N. Chalyi, V. Uzhinsky

30.10.2026

First of all, for **elastic dd**-interactions, we try to use **Geant4 elastic models**. At comparison with experimental data, we conclude **Geant4 elastic models** can be used only for a **simulation of small angle scatterings.**

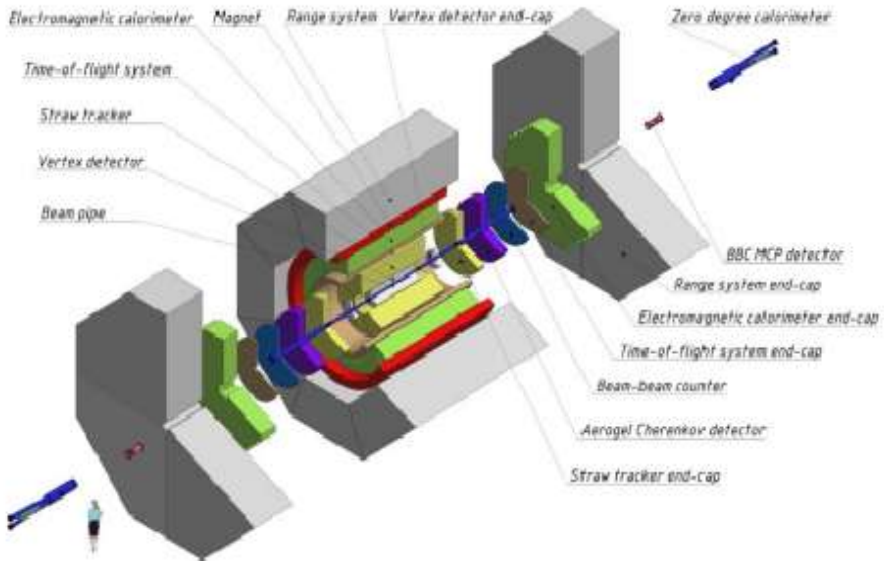
An application of **Glauber model** to **elastic dd**-interactions is considered. Main assumptions of **Glauber model are presented** is done.

A new code on C++ based on Glauber approach is created. Code is working in ROOT framework. Some applications of the code to **elastic dd**-interactions will be presented.

Results of simulations of **elastic He-He and p-He** interactions will be done in comparison with of experimental data.

Motivation of simulation of elastic and inelastic deuteron-deuteron interactions

Spin Physics Detector (**SPD**) will be created at **NICA** accelerator facility. One of the aims of the experiment is a study of nucleon spin structure in polarized proton-proton (*pp*) and deuteron-deuteron (*dd*) collisions at $\sqrt{S}$ up to 27 GeV and luminosity up to $10^{32} cm^{-2} s^{-1}$.



A. Arbuzov et al, On the physics potential to study the gluon content of proton and deuteron at NICA SPD, *arXiv:2011.15005*, *Prog. Part. Nucl. Phys.* 119 (2021) 103858
" Probe gluon distributions in production of charmonia, open charm and prompt photons other very important aim is an extraction distribution function of nucleons".

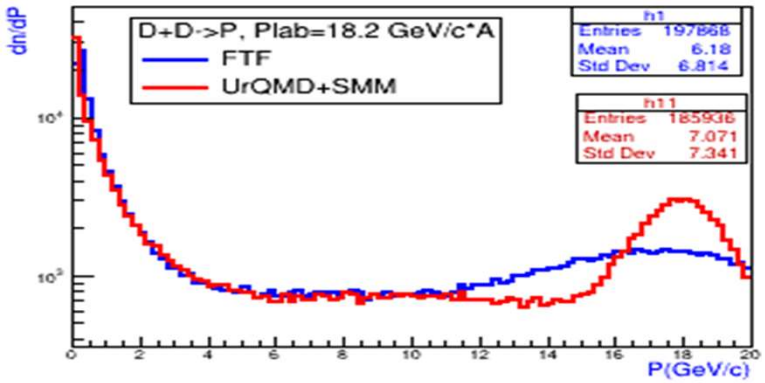
V. Abramov et al, **Possible studies at the first stage of the NICA collider operation with polarized and unpolarized proton and deuteron beams**, *arXiv:2102.08477, Phys .Part. Nucl.* 52 (2021) 6, 1044-1119
Spin effects in elastic *pp*- and *dd*- scatterings, hyperon production in *pp*- and *dd*-interactions, study of multi-quark correlations, dibaryon resonances, exclusive reactions, open charm and charmonia near threshold. Minimal bias interactions will be studied at first stage of SPD.

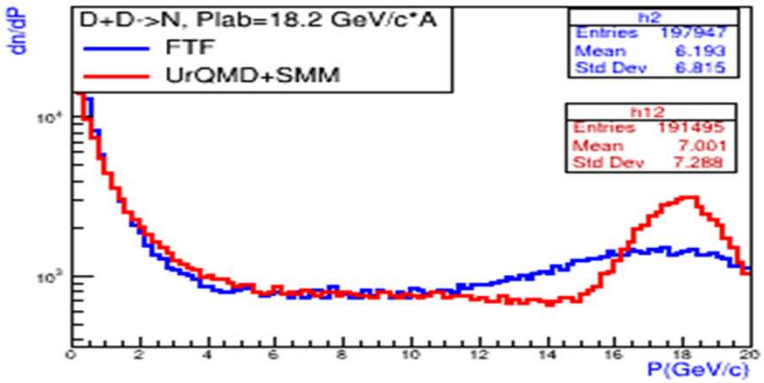
Polarized and un-polarized *dd*-interactions will be studied at SPD. Thus, Monte Carlo simulations of the *dd*-collisions are needed for design and development of the experiment.

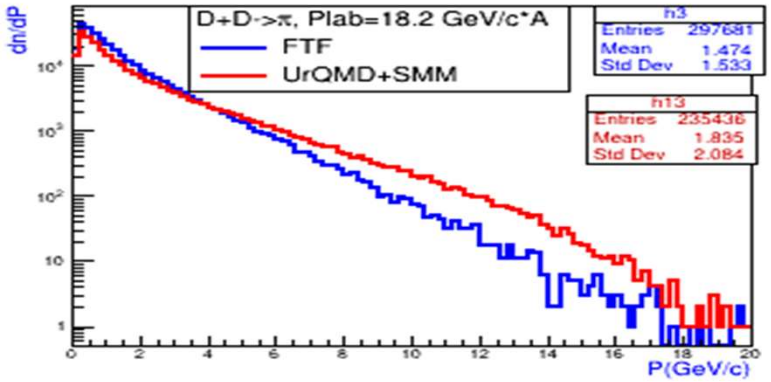
Simulation of inelastic deuteron-deuteron interactions using G4 FTF and UrQMD +SMM models.

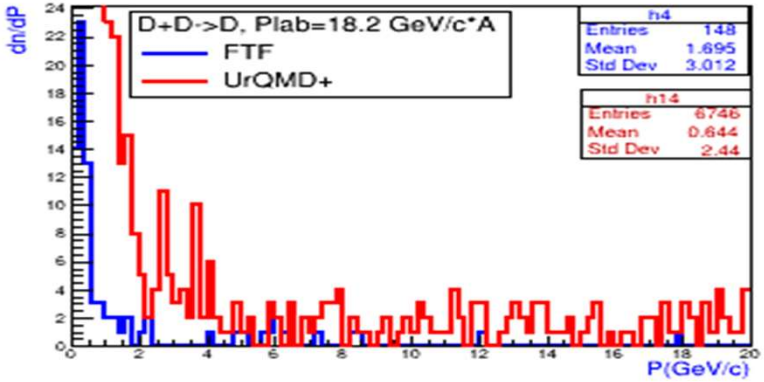
Momentum distribution of particles produced in ***dd*** - interactions at Plab=18.2 GeV/c/N (100000 Events)

Calculations were done with Geant4/FTF (Geant4 11.4) and UrQMD (3.4)+clustering+SMM models.









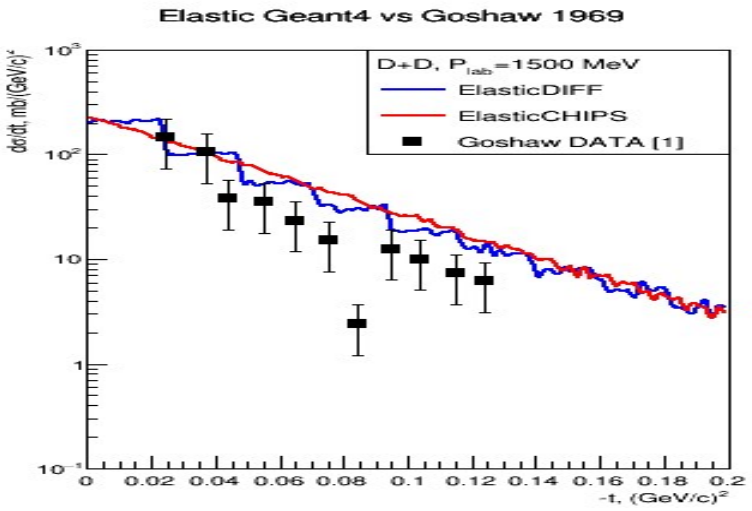
Initial Plab=18.2 GeV/c corresponds to $\sqrt{(S_{nn})}$= 6 GeV expected for ***dd***-interactions at SPD/NICA. Now, Geant4/FTF is installed in SPDRoot and SAMPO frameworks for SPD experiment.

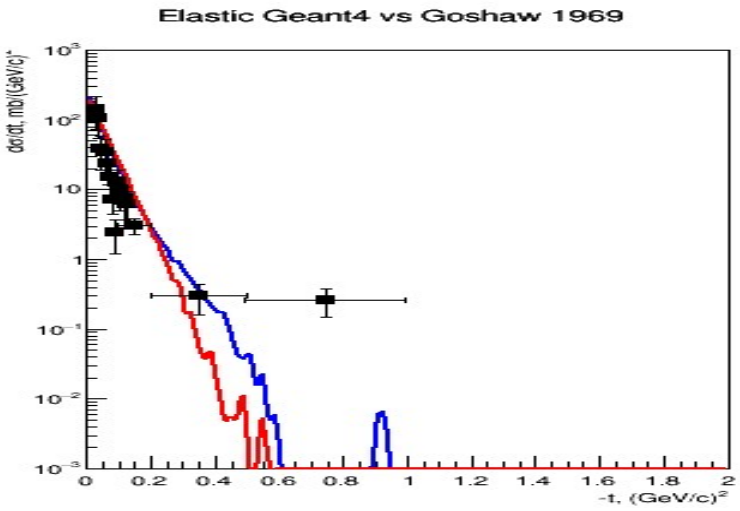
Elastic models in Geant4

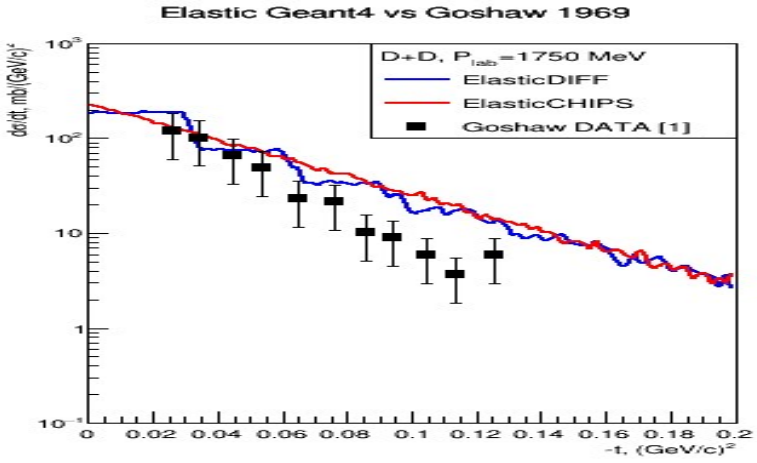
– At the moment in Geant4 there are five models of elastic interactions:
  - Elastic – most simple standard model
  - ElasticLEnp – for np, pp, nn in energy range 10-1200 MeV
  - ElasticCHIPS – based CHIPS (chiral invariant phase space) model
  - ElasticDIFF – created for protons, kaons and pions
  - ElasticHE (hadron elastic) – only for hadron-hadron and hadron-nucleus interactions

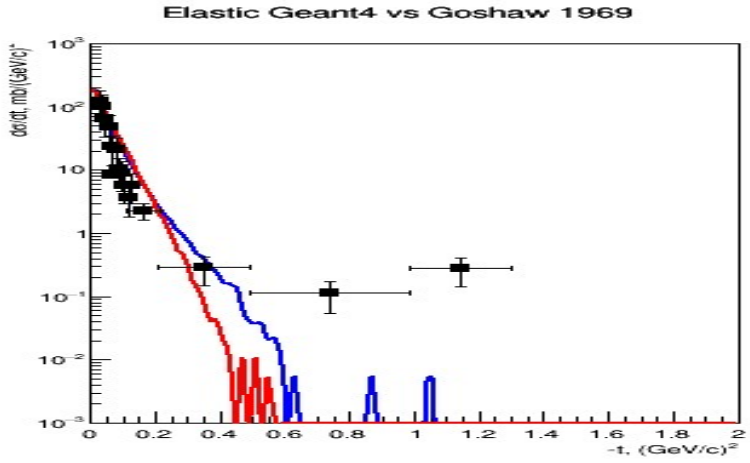
For d+d we can use only first 4 models, of which Elastic, ElasticLE and ElasticCHIPS give the same results.

d+d elastic scatterings
Results at Plab=1500 and 1750 MeV/c



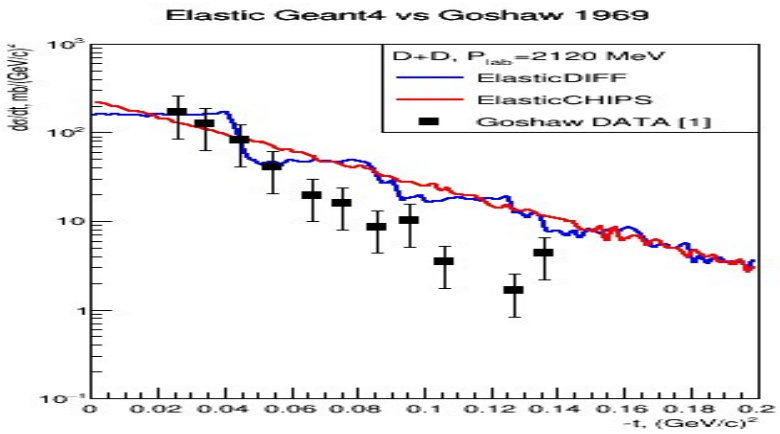


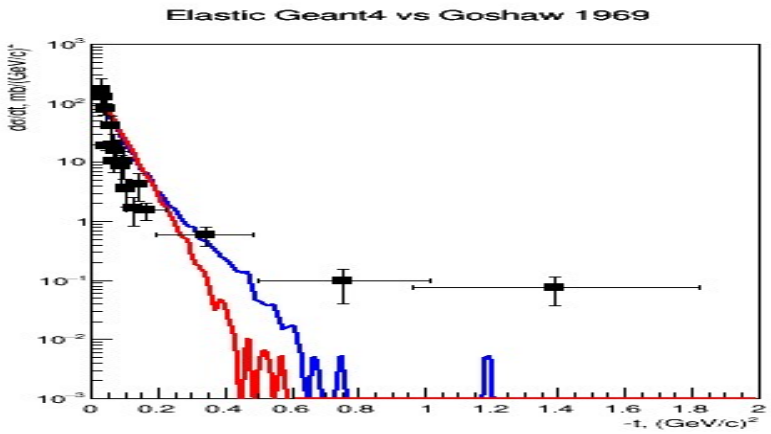




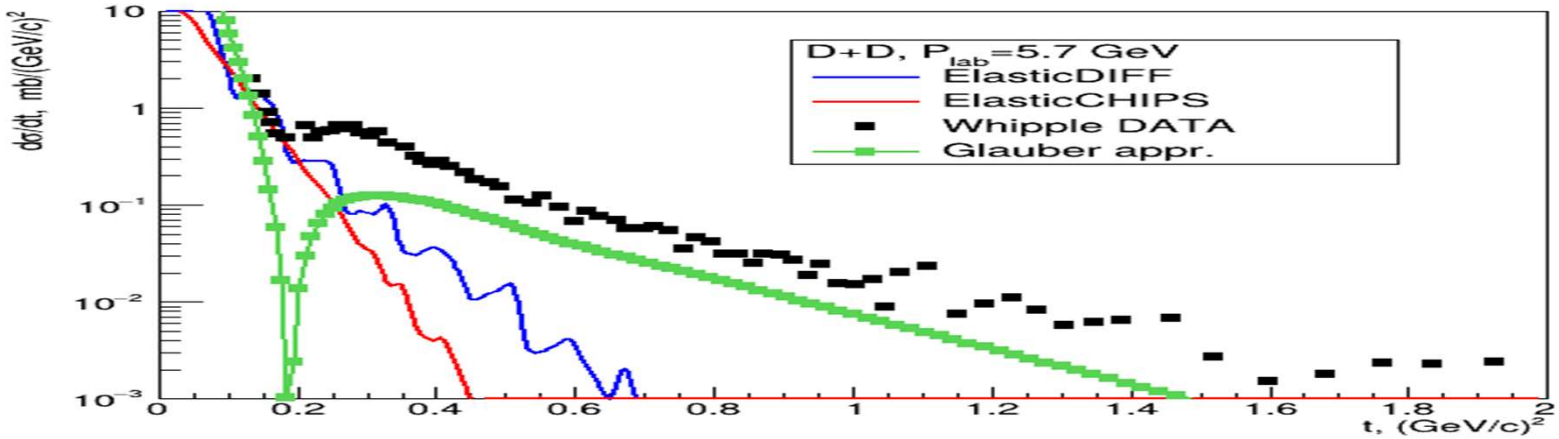
Geant4 elastic models allow to simulate small angle scatterings

d+d elastic scatterings
Results for Plab=2.12 GeV/c





[1]



E. T. Whipple et. al., Phys. Rev. Lett. 47, 774 – 14 September, 1981

An application of the Glauber theory gives more opportunities!

S.Yu. Shmakov, V.V. Uzhinskii et al., // **Comp. Phys. Commun**. V. 54, 123-125, 1989

Glauber theory

Differential cross sections of nucleon–nucleon interactions

$\frac{d\sigma}{d\Omega} = F(cos(\theta)) \propto \sum_{l=0}^{\infty} P_l(cos(\theta)) \left(1 - e^{i\ \delta_l}\right)$

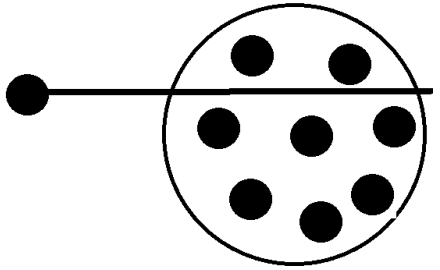
l=0 – s-wave, l=1 – P-wave and so on ...

At high energies: $\frac{d\sigma}{d\Omega} \propto \frac{d\sigma}{dt} \propto |A|^2\ e^{B\ t}$

$A = \frac{1}{2i\pi} \int \gamma(\vec{b})\ e^{i\ \vec{b}\cdot\ \vec{q}}\ d^2q, \qquad \gamma(\vec{b}) = 1 - e^{i\chi}$

Glauber's assumption: $\chi_{hA} = \sum_{i=1}^{A} \chi(\vec{b} - \vec{s}_i).$

$\Gamma_{hA}(\vec{b}) = 1 - e^{\sum \chi_i} = 1 - \prod_{i=1}^{A} [1 - \gamma(\vec{b} - \vec{s}_i)]$



Amplitude of elastic p+D scattering

$\Gamma_{pD}(\vec{b}) = \int \left[1 - \prod_{i=1}^{A} [1 - \gamma(\vec{b} - \vec{s}_i)]\right] |\psi(s)|^2\ d^2s$

$= \int \left[\gamma(\vec{b} - \vec{s}) + \gamma(\vec{b} + \vec{s}) - \gamma(\vec{b} - \vec{s}) \cdot \gamma(\vec{b} + \vec{s})\right] |\psi(s)|^2\ d^2s$

Glauber theory

In the simplest approximation $|\psi(s)|^2 = \frac{1}{\pi R_d^2} \cdot e^{-s^2/R_d^2}$
or it is a sum of Gaussians with varios radiuses as it is in the papers by Azhgirey et al.

L.S. Azhgirey, A. V. Tarasov, V.V. Uzhinsky et al., Nucl. Phys. A 3115 (1978) p.397

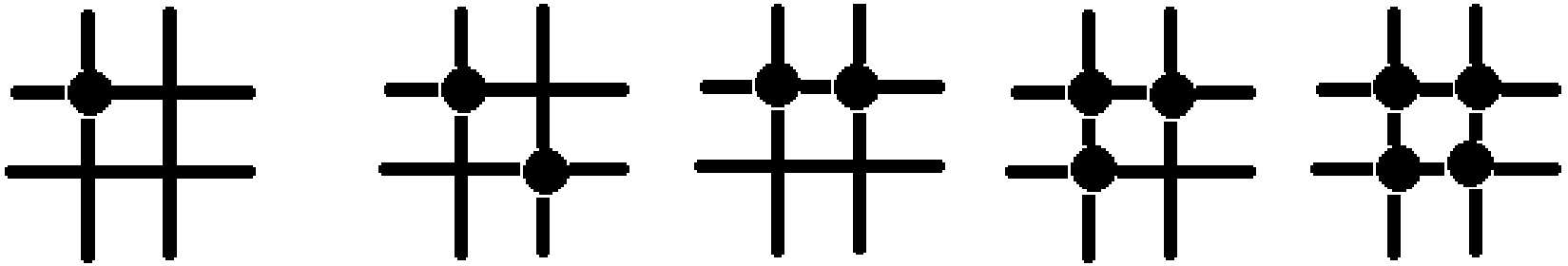
Now we can calculate the first and second term of the Glauber's serie for p+D scattering:

$$\int d^2b\, e^{i\,\vec{b}\cdot\vec{q}}\, \gamma(\vec{b}-\vec{s}) \cdot |\psi(s)|^2\, d^2s = \frac{\sigma_{nn}^{tot}(1-i\rho)}{2\cdot 2B\pi} \int b\, db\, J_0(q\, b) \cdot e^{-(\vec{b}-\vec{s})^2/2B} \frac{1}{\pi R_D^2} e^{-\vec{s}^2/R_D^2}$$

$$A(q) = \frac{iP_{lab}}{2\pi} \cdot \frac{\sigma_{nn}^{tot}(1-i\rho)}{2} \cdot e^{-q^2(B/2+R_D^2/4)}.$$

The third term is equal: $-\left[\frac{\sigma_{nn}^{tot}(1-i\rho)}{2}\right]^2 \frac{1}{4\pi(B+R_D^2)} e^{-B\ q^2/4}$

In the case of D+D there are many terms. All of them have different signs.



All of them are calculated.

Glauber theory

It is needed to determine for a calculation:

$\gamma(\vec{b})$ the NN scattering amplitude in the impact parameter representation

$$\gamma(\vec{b}) = \frac{\sigma_{NN}^{total}}{4B\pi}\, e^{-b^2/2B}$$

σ_{NN}^{total} – NN total cross section at a given energy;
B – slope parameter of elastic scattering cross section,
$\psi(s)$ – wave function of the deuteron.

Empirical parametrization of the nucleon-nucleon elastic scattering amplitude at high beam momenta for Glauber calculations and Monte Carlo simulations

V. Uzhinsky, A. Galoyan, Q. Hu, J. Ritman, H. Xu

Phys.Rev.C 94 (2016) 6, 064003 e-Print: 1603.04731 [hep-ph]

1 Standard parameterization of pp elastic scattering data

The amplitude of elastic proton-proton scattering must be symmetric when exchanging $t \leftrightarrow u$, where t and u are Mandelstam variables. Thus, we write the amplitude as $F(t) = f(t) + f(u)$ and thus an expression to fit the differential cross sections can be represented as:

$$d\sigma/dt = A^2\ (e^{Bt/2} + e^{Bu/2})^2. \qquad (4)$$

We analyzed and fitted 133 and 45 sets of experimental data on pp and np elastic scattering, respectively.

Fitting results for elastic np- interactions at Plab from 1.97 GeV/c up to 360GeV/c

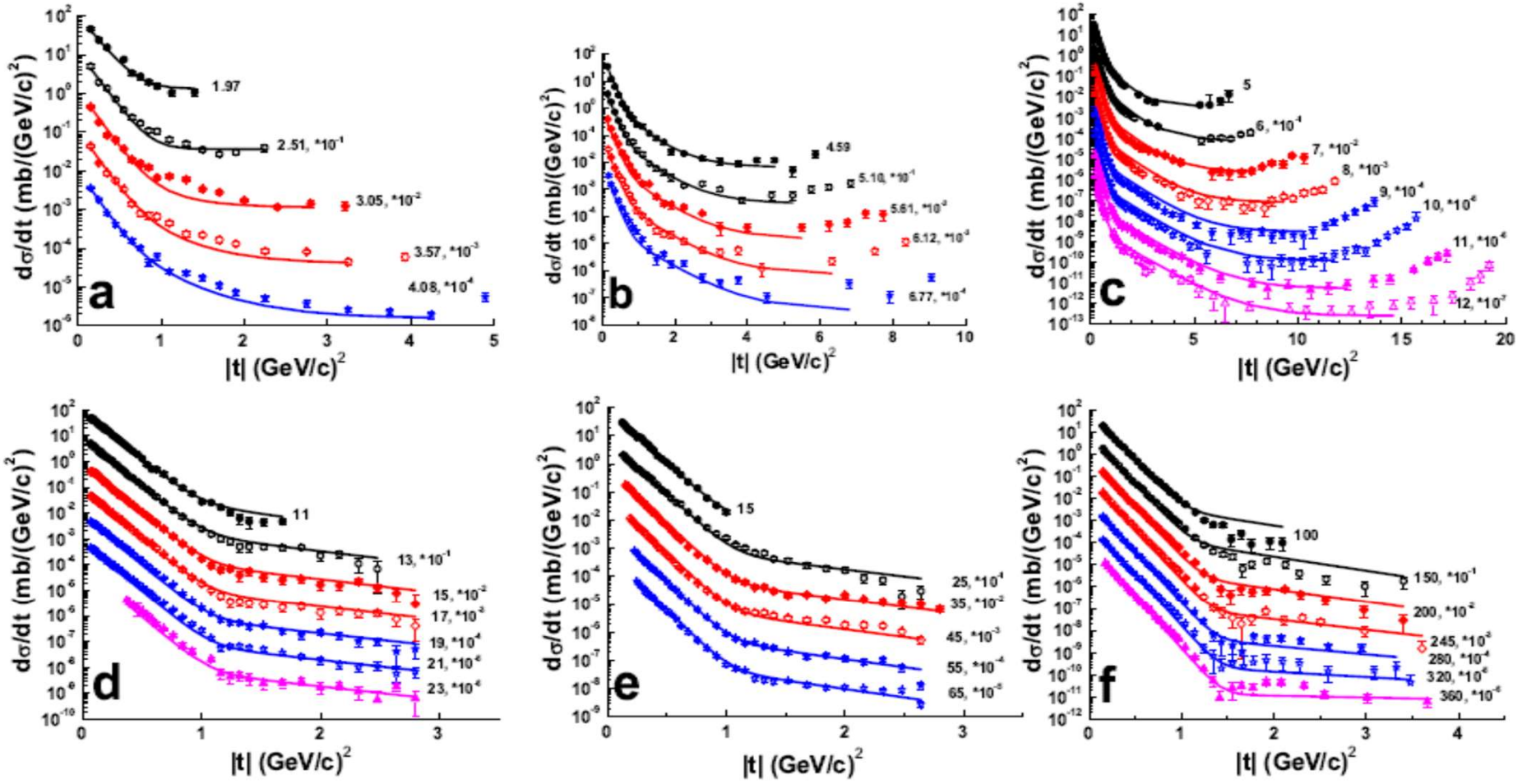


Figure 10: Description of *np* elastic differential cross sections by Eq. 27 with approximations 29 – 32. Points are experimental data [66, 67, 69, 70]. The experimental errors displayed are statistical only. Lines are calculation results.

133 and 45 sets of experimental data on pp and np elastic scattering, respectively, were analyzed and fitted and obtained approximations for σ_{tot}, σ_{el} , B, and ρ.

Approximations of σ_{tot}, σ_{el} , B, and ρ have been proposed as function of initial energy. They are used in calculations of light nuclei elastic scatterings in Glauber model and in our program for elastic PP-interactions.

$$B = 1.9\ \ln(s/s_0) + \frac{27}{\sqrt{s/s_0}} - \frac{47}{(s/s_0)} \quad [(\mathrm{GeV}/c)^{-2}].$$

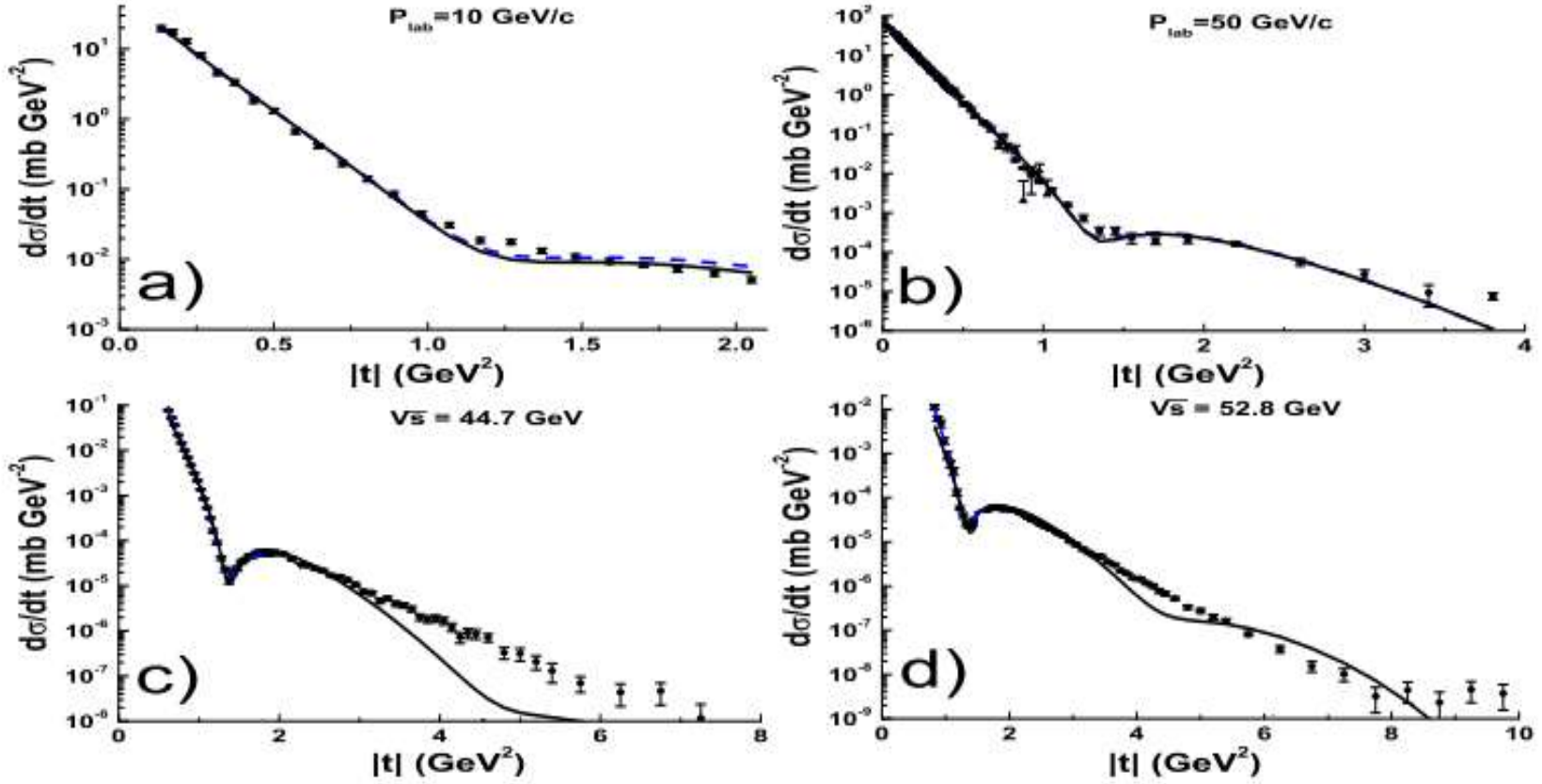
$$\rho_{pp} = \rho_{\mathrm{PDG}} + \frac{1.5}{(s/s_0)} - \frac{3}{(s/s_0)^2}, \quad s_0 = 1\ \mathrm{GeV}^2$$

$$\sigma^{\mathrm{tot}}_{pp} = \sigma^{\mathrm{tot}}_{\mathrm{PDG}} + \frac{7200}{(s/s_0)^{3.5}} - \frac{32}{(s/s_0 - 4)^2 + 0.45} \quad [\mathrm{mb}],$$

We created a code for simulation of elastic proton-proton interactions: PPelastic.C

The code is installed **in SPDRoot** and also in **Sampo** frameworks for **SPD** Experiment

Elastic PP- interactions



New code on C++ of Glauber approach for simulations of *dd* and light nuclei elastic scatterings

```
struct {
double q, bp[201],
prof[201]; } pr_;
```

```
struct {
complex <double> ca;
complex <double> ci;
double ga;} ampl_;
```

```
const int nstr=3, nstb=238;
```

Main Class of the Code : **class DDElastic**

Methods:

```
void Run(Int_t nmax, const char *fname3);
Int_t PREVIO(Int_t NA, Int_t NCA, …);
Int_t CONUCL(double X [nstr][nstb], Int_t N,...);
Double_t Gauss1(Double_t Bn, Double_t Bx ...);
Double_t Density(Double_t b);
Double_t Profile(Double_t b);
Double_t BesselJzero(Double_t z);
```

Class Members:

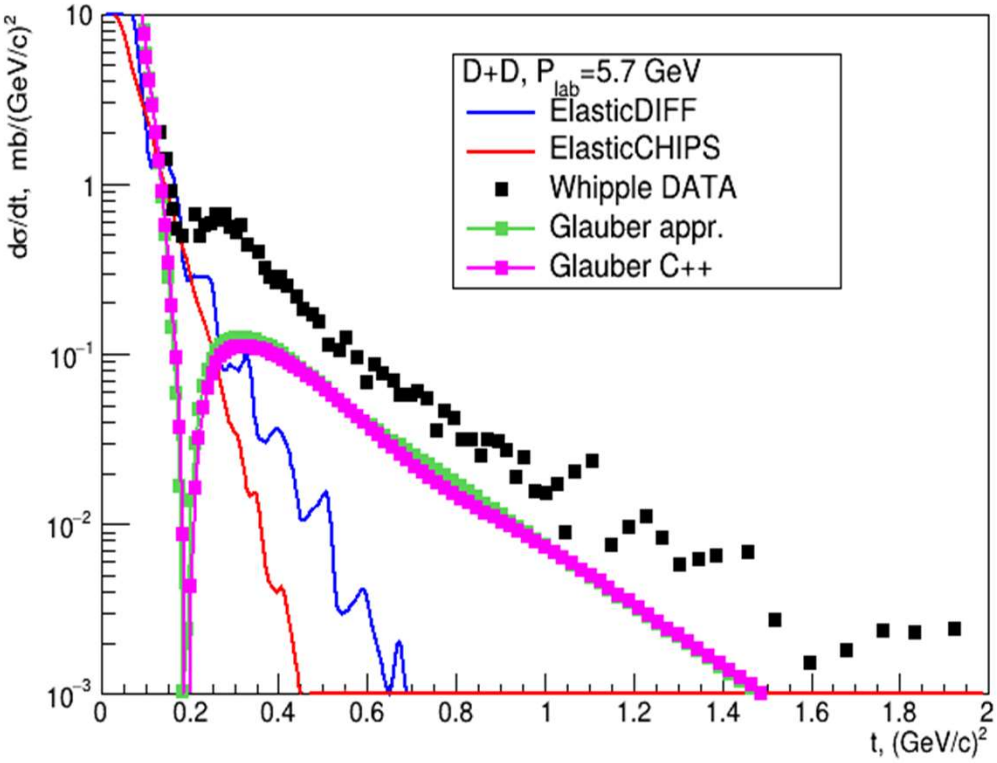
```
Double_t   Plab;
Double_t   Pcms;
Double_t   S;
Double_t   SqrtS;
Double_t   E;
Double_t   Mn;
Double_t   Ecms;
Double_t   Vcms;
```

At C++ code creation, problems were with operations with complex numbers and functions. The other problem was connected with choice of a random number generator. (TRandom, TRandom1, TRandom2…)

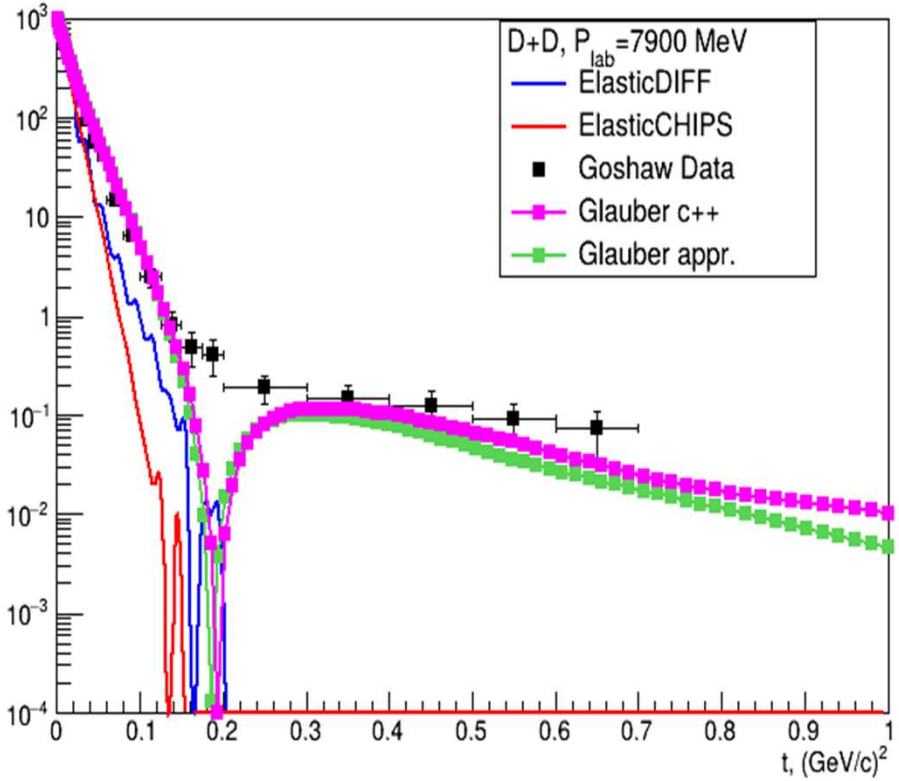
New code on C++ for simulations of *dd* (and light nucleus) elastic scatterings based on Glauber approach



Results for *dd*-elastic interactions at Plab = 5.7 and 7.9 GeV/c
(Elastic Geant4 models, fortran codes and new C++ code based on Glauber model)



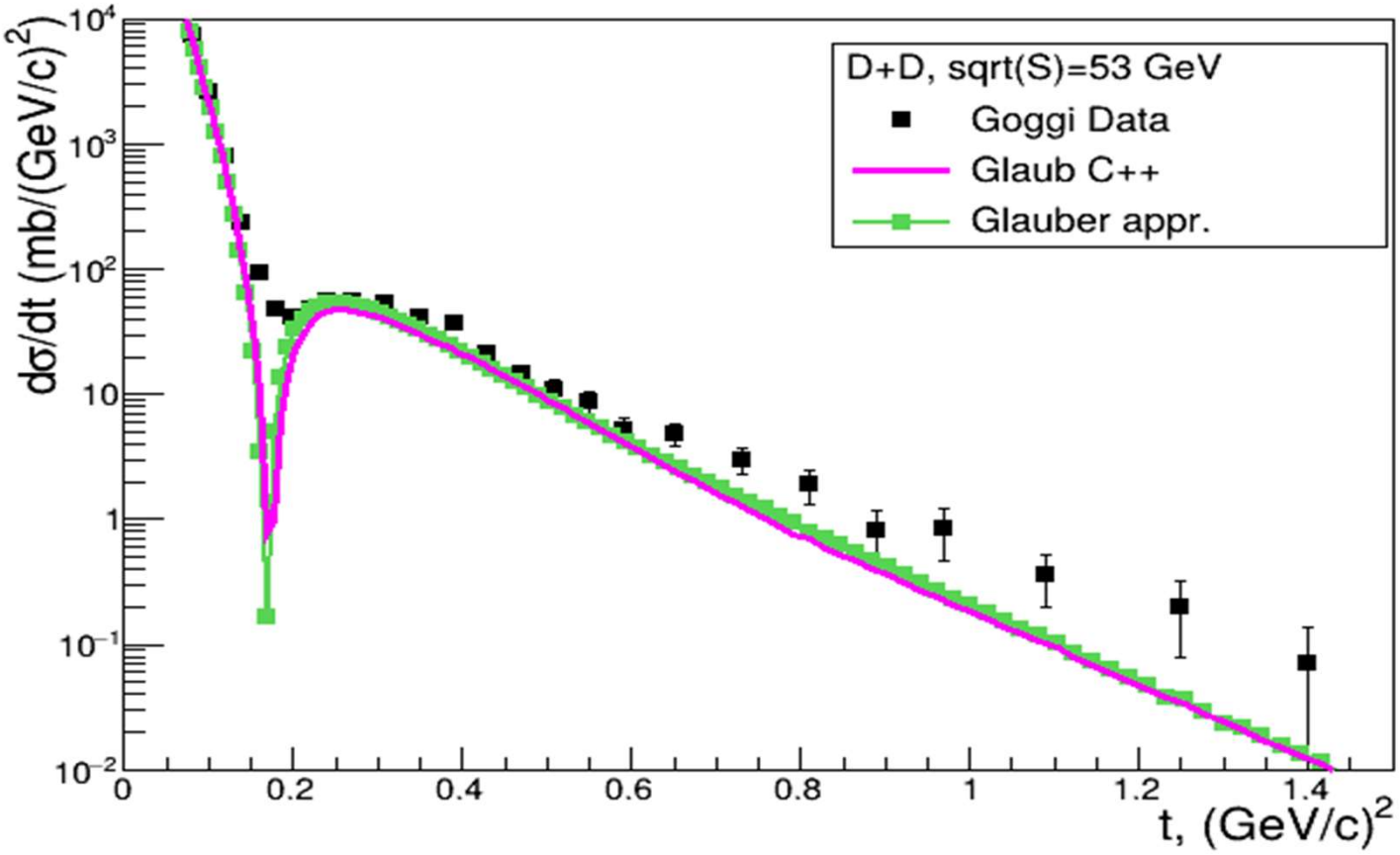
E. T. Whipple et. al., Phys. Rev. Lett. 47, 774,
14 Sept, 1981



T. Goshaw and P. J. Oddonef, Phys. Rev. Lett.,
V. 25, N. 4, P.249

An application of the new C++ code of Glauber theory gives more opportunities at higher energies!

Results for *dd*-elastic scatterings at $\sqrt{S_{dd}}$=53 GeV ($\sqrt{S_{nn}}$=26.5 GeV)
(fortran codes and new C++ code based on Glauber model)

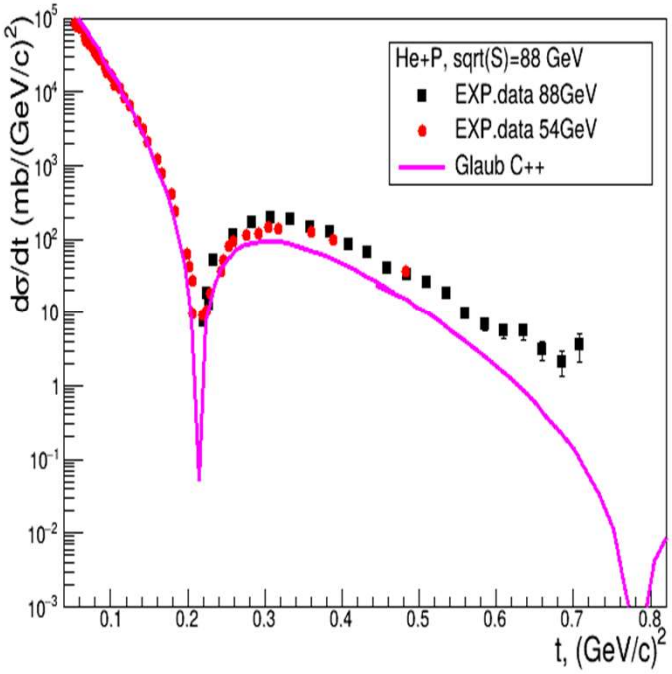


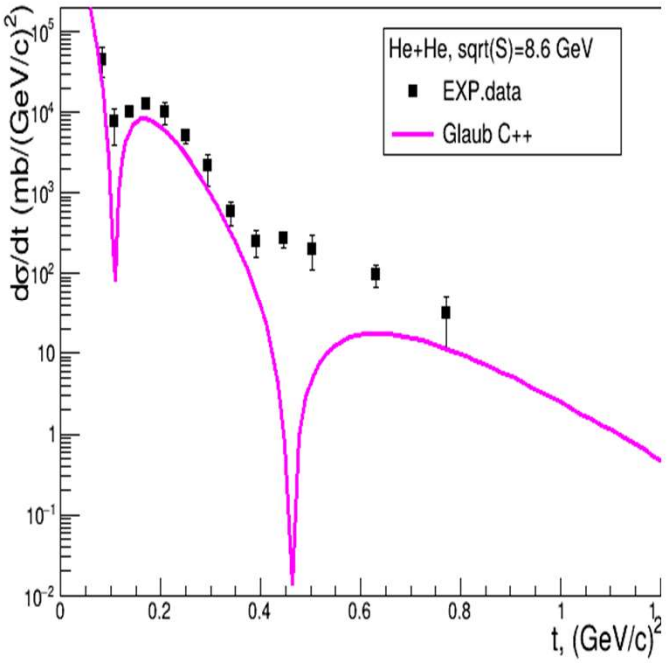
G. Goggi and G. Alberi, Nuclear Physics B149 (1979) 381-412

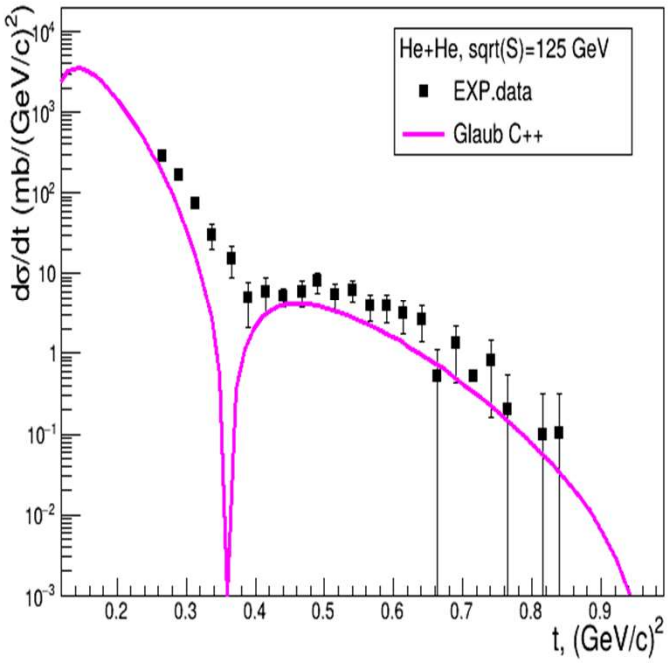
The new C++ code of Glauber theory can be used for other light nucleus elastic scattering simulations: p+d, *He3*+*He3*, α+α, …

Results for p+*He4* *at* $\sqrt{S_{Hep}}$ = 88 and 54 GeV, *He4*+*He4* - elastic scatterings at $\sqrt{S_{HeHe}}$= 8.6 GeV and 125 GeV (new C++ code based on Glauber model)

W. Bell et al., Measurement of αα and αp elastic scattering at the CERN ISR, //
Phys. Lett, V.117 B, 1982, P.131-134







The new C++ code of Glauber theory is applicable for light nucleus elastic scattering simulations: p+α, *He3*+*He3*, α+α, …

Conclusions

- **Geant4 FTF** is applicable for calculations of **inelastic deuteron-deuteron** interactions
- **Geant4** models for **elastic** interactions (DIFF, CHIPS) well describe elastic interactions at low transferred momentum and low energies
- But, they do not describe **elastic *dd-*** interactions at higher **t** correctly. This is due to the fact that the models available in Geant4 are not suitable for simulations of elastic processes with deuterons.
- To describe **elastic *dd-*** interactions we used **Glauber theoretical approach**. Some optimistic results are obtained for high energies ***dd***-interactions.
- New **code on C++** of Glauber approach for simulation of **elastic *dd-*** interactions is created. The new code can be also used for light nucleus elastic scattering simulations. It is operated under ROOT – software toolkit.
- Outlook:

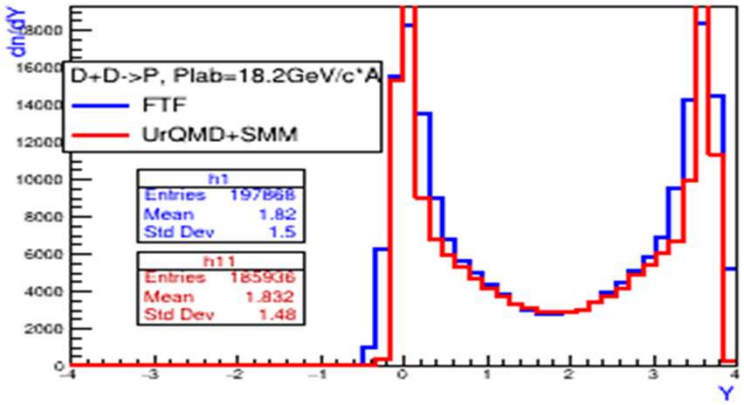
To develop the Glauber approach for more correct description of elastic d+d interactions at small energies.

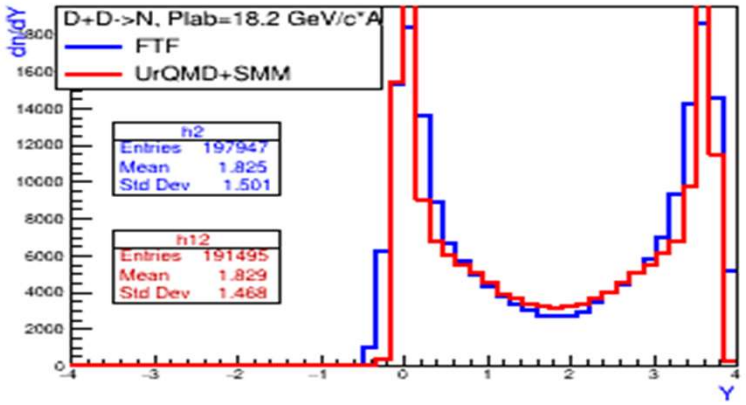
To implement in the new code creation of output ROOT-file with elastic events, similar to output ROOT-file of generator PPelastic.C in SpdRoot.

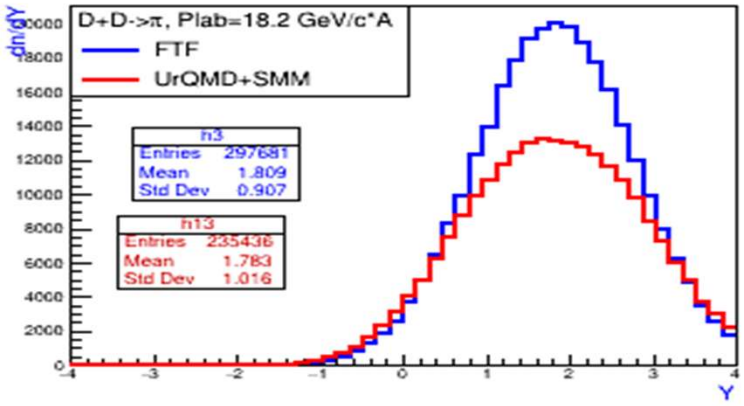
Simulations of deuteron - deuteron interactions in G4 FTF and UrQMD+SMM models

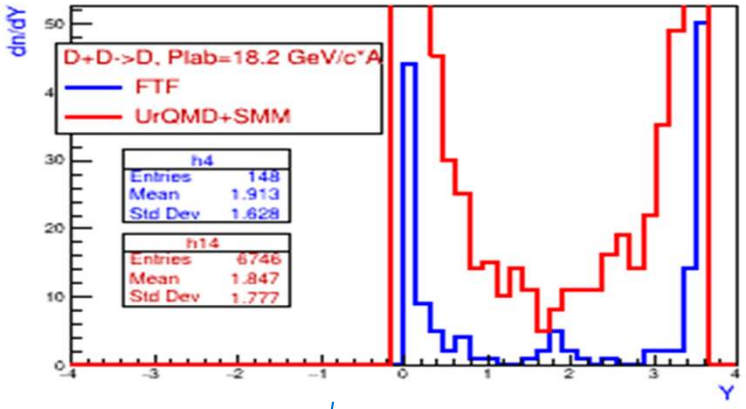
Rapidity distribution of particles produced in ***dd-*** interactions at Plab=18.2 GeV/c/N (100000 Events)

Calculations were done with Geant4 FTF (Geant4 11.4) and UrQMD (3.4)+clustering+SMM models







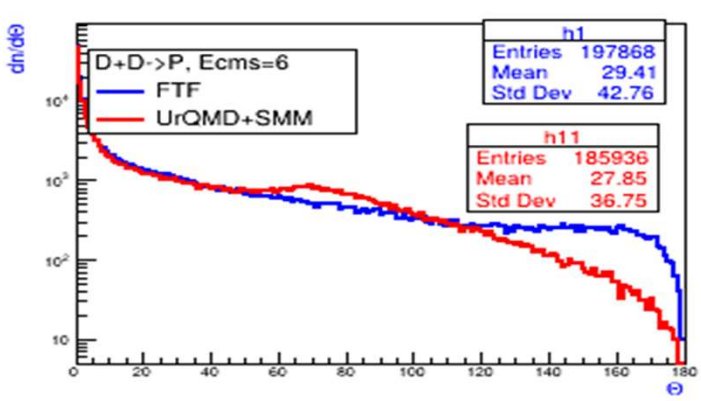


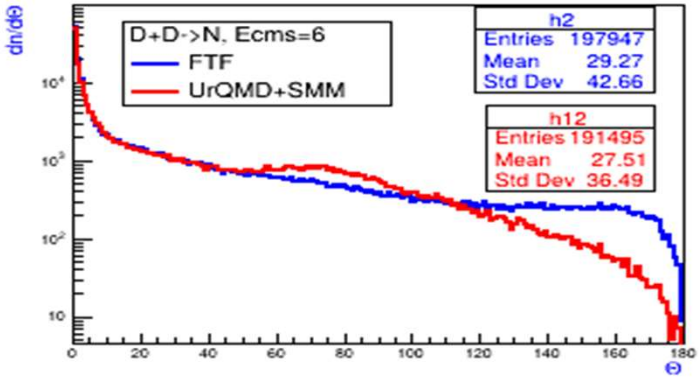
Initial Plab =18.2 GeV/c/N corresponds to $\sqrt{(Snn)}$= 6 GeV expected for dd-interactions SPD/NICA

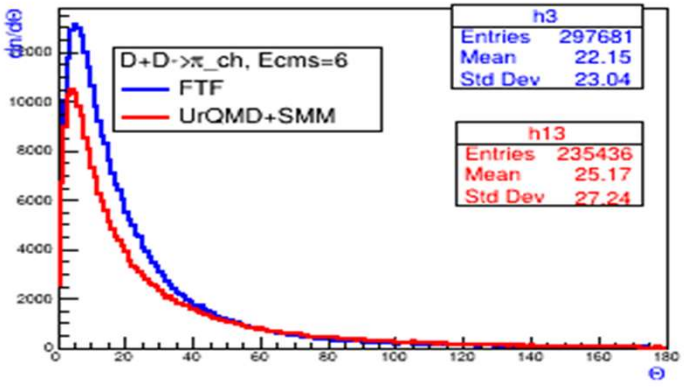
Simulation in SPDRoot using G4 FTF model interface

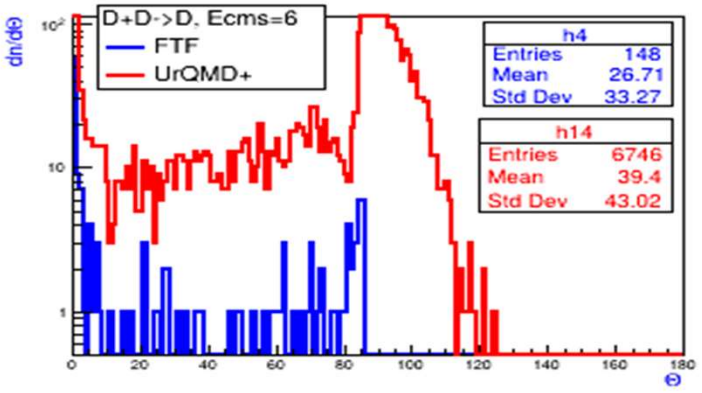
Polar angle distribution of particles produced in d+d interactions at Plab=18.2 GeV/c/N (100000 Events)

Calculations were made with Geant4 FTF (Geant4 11.3) and UrQMD (3.4)+SMM models



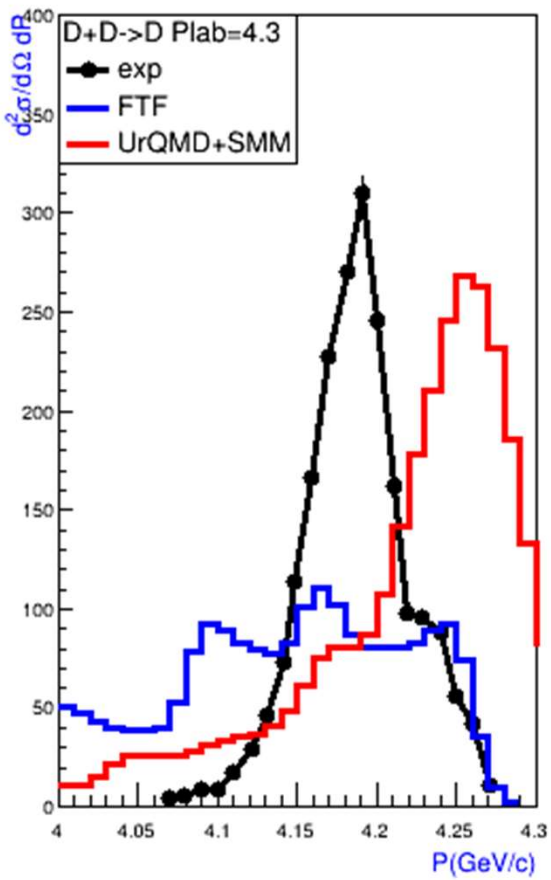


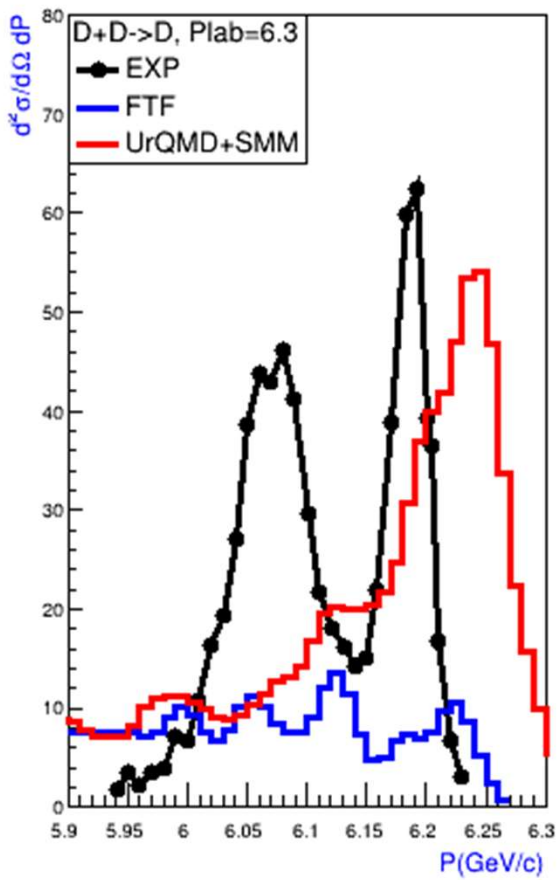


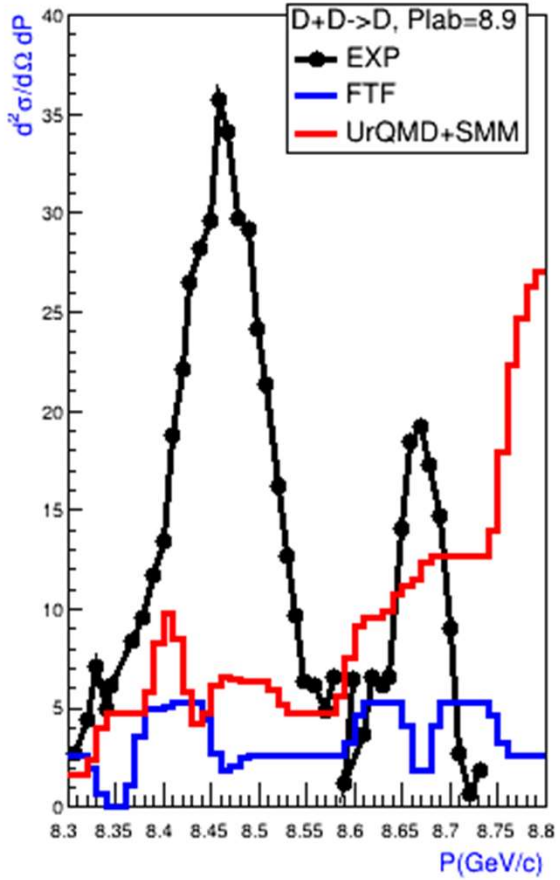


Plab 18.2 GeV/c corresponds to sqrt(Snn)= 6 GeV

Quasi-elastic d+d->d+X interactions at Plab=4.3, 6.3, 8.9 GeV/c, θ=105 mrad







Experimental data: L.S. Azhgirey… A.V. Tarasov, V.V. Uzhinsky et.al., Nuclear Physics A3115 (1978) 397-403.

It is not a regular way to simulate elastic and quasi-elastic interactions!

Parametrization of σ_{tot}, σ_{el} and ρ for PP

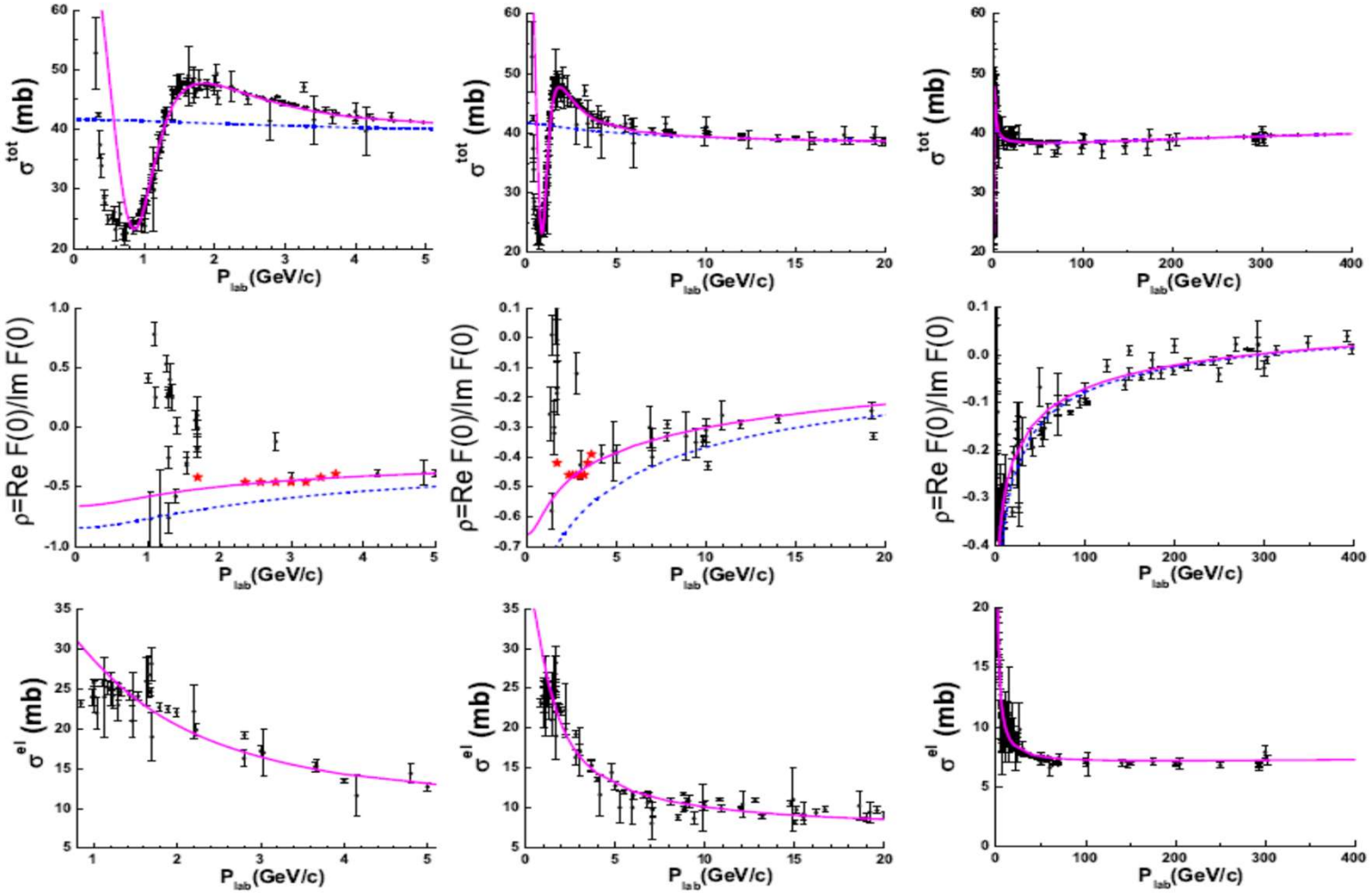


Figure 2: σ_{pp}^{tot}, σ_{pp}^{el} and ρ_{pp} as functions of P_{lab}. Points are data from PDG data-base [34]. Dashed lines are extrapolations of the PDG parameterizations. The solid lines are our approximations.