Cumulative processes in the region of large transverse momenta

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Институт физики высоких энергий имени А.А. Логунова,

Протвино, 2-5 июня 2026

Study of inclusive production cross-sections
of pions and protons in a new cumulative region:
central rapidities, large transverse momenta

$d+d \to \pi+X$

$d+d \to p+X$

flucton-flucton interaction
6q+6q

outside (and inside) p+p kinematics:

$p+p \to \pi+X$

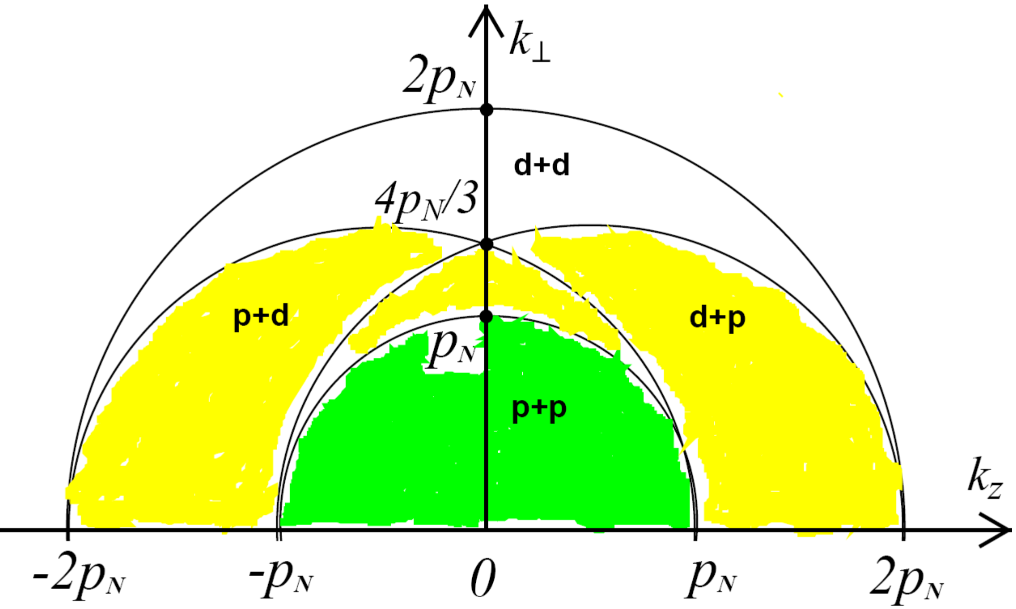
$p+p \to p+X$

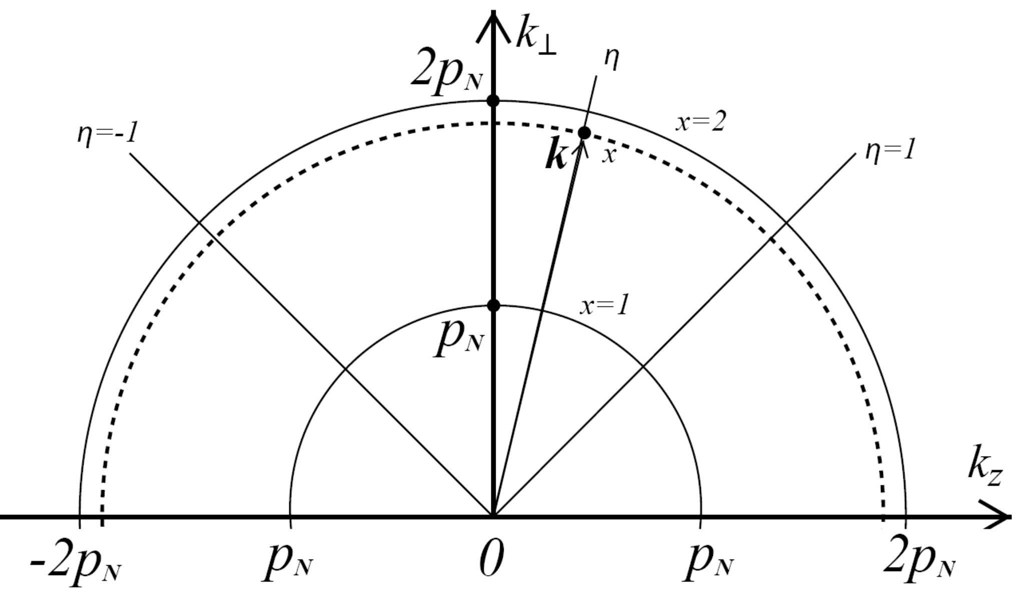
proton-proton interaction
3q+3q

This is possible only due to the moderate energy of the NICA collider (completely impossible at ultrahigh energies of the RHIC and LHC)

The mechanisms of pion and proton production are different!

Asymptotic form of the inclusive cross-section at mid-rapidities near kinematic thresholds





$p_N >> m_N$ $p_N \approx \sqrt{s}/2$

We expect
for the inclusive cross-section
of the production of particles with large transverse momenta at midrapidities:

$$\frac{k_0\, d^3\sigma}{d^3\mathbf{k}} = f(x,\eta) = C\, s^{-q_1}\, (f - x)^{q_2}\, F(\eta)$$

Quark Counting Rules for High pT Inclusive Production

two asymptotic parameters:
$s >> m^2$ and $(f\text{-}x) << 1$
at f=1 (pp) and f=2 (dd)

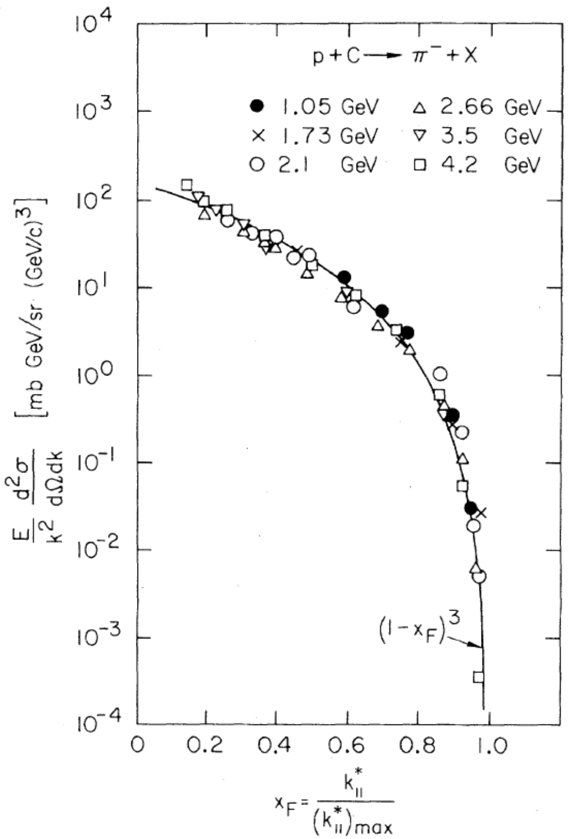
q_1, q_2 are different for production of pions and protons!

x - cumulative number
$\eta = -\ln \mathrm{tg}\, \frac{\theta^*}{2}$ - pseudorapidity

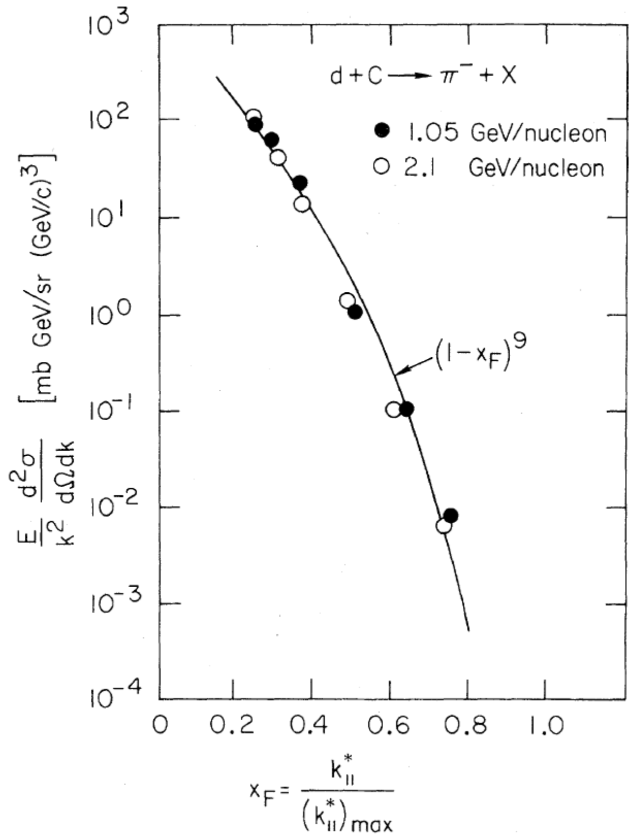
Schmidt I.A., Blankenbecler R. Phys.Rev. ***D15*** *(1977) 3321*

Threshold behaviour of ***inclusive cross sections*** (quark counting rules) at $|t| \ll s$.

The experimental points from J. Papp et al., Phys.Rev.Lett. 34, 601 (1975).



Δ^{2*2-1}



Δ^{2*5-1}

Scaling of cumulative inclusive cross section in the fragmentation region:

$$f_\pi(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C\, s^0\, (f - x)^{2p-1}\, \Phi_p\!\left(\frac{k_\perp}{m_q}\right)$$

M.A. Braun, V.V. Vechernin, Nucl.Phys.B 427,614(1994); Phys.Atom.Nucl 60,432(1997); ibid 63,1831(2000), V. Vechernin, AIP Conf.Proc.1701 (2016) 060020.

Quark counting rules for *elastic and quasi elastic* reactions with nuclei

Matveev V.A., Muradyan R.M., Tavkhelidze A.N. Lett. Nuovo Cimento 7 (1973) 719
Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

$s \to \infty, \quad t/s$ fixed

$(d\sigma/dt)_{\pi p \to \pi p} \sim s^{-8}, \ (d\sigma/dt)_{pp \to pp} \sim s^{-10}, \ (d\sigma/dt)_{\gamma p \to \pi p} \sim s^{-7}, \ (d\sigma/dt)_{\gamma p \to \gamma p} \sim s^{-6}$

$\sim s^{-n}$ $A+B \to C+D$ $n=n_A+n_B+n_C+n_D-2$ $n_p=3 \ \ n_\pi=2 \ \ n_\gamma=1$

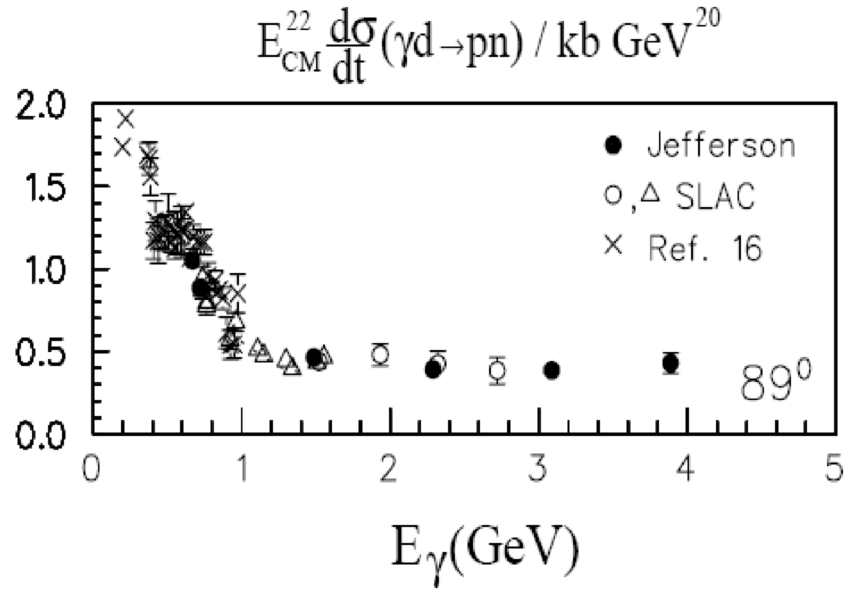
$$\frac{d\sigma}{dt}(A+B \to C+D) \to \frac{1}{t^{N-2}} f(t/s)$$

$N=n_A+n_B+n_C+n_D$

Yu.L. Dokshitzer, QCD Phenomenology, Lectures at the CERN–Dubna School, Pylos, August 2002

Early validity of QCR in the deuteron break-up by a photon, $\gamma + D \to p + n$

$$\frac{d\sigma}{dt} = \frac{f(\Theta)}{s^{K-2}}; \qquad \frac{t}{s} = \text{const}, \qquad K-2=1+6+3+3-2=11$$



For light nuclei:

Yu.N. Uzikov, Indication of Asymptotic Scaling in the Reactions $dd \to p^3H$, $dd \to n^3He$ and $pd \to pd$, JETP Letters 81 (2005) 303.

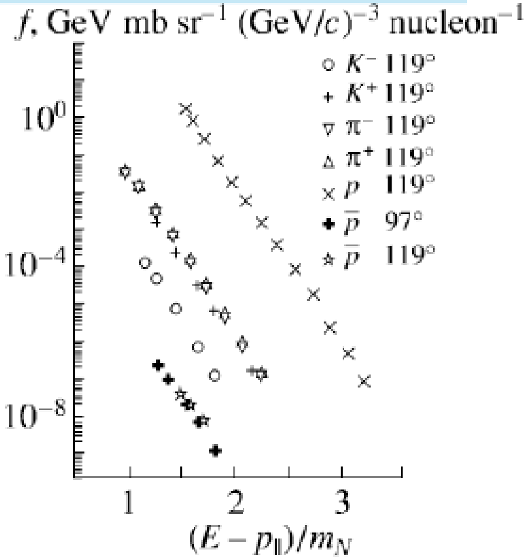
$\sim s^{-22}$ (6+6+3+9-2=22) and $\sim s^{-16}$ (3+6+3+6-2=16)

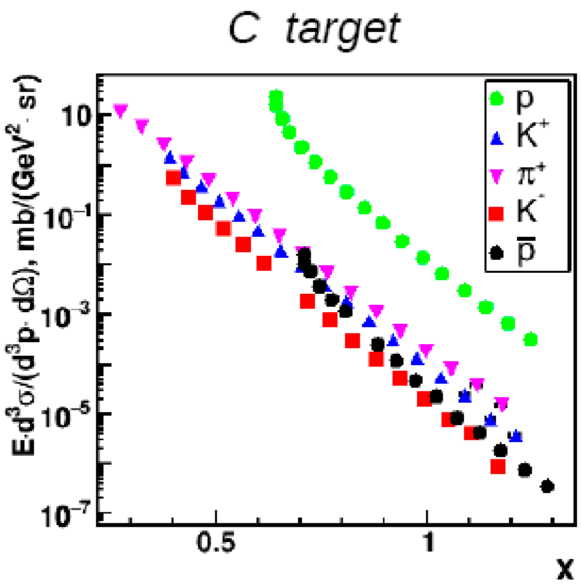
Study of flucton-flucton interaction in dd collisions at NICA SPD

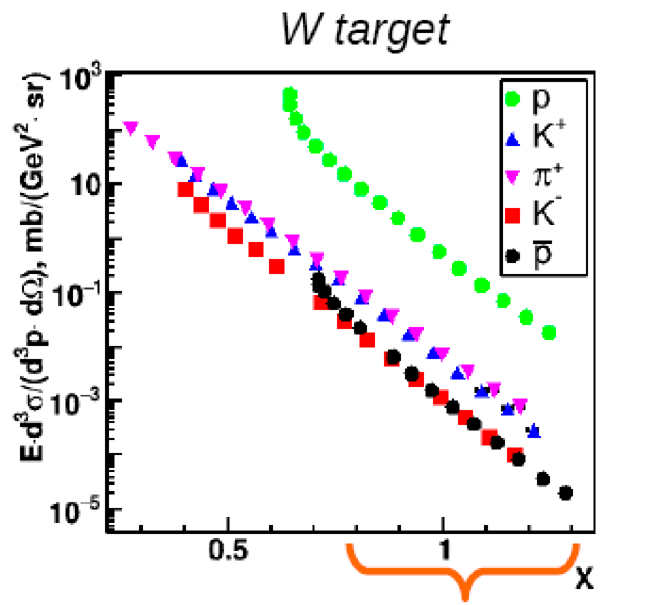
- **It can be studied only in new cumulative region of large transverse momenta in mid-rapidity region at NICA**
  (not in the traditional cumulative region of fragmentation of one of the nuclei).
- **There are no additional interactions in dd collision, compared with collisions of heavier nuclei, if both deuterons are in flucton configuration at the moment of collision.**
- **Higher frequency of dd collisions that can be recorded by the SPD, compared to the slower MPD (important for a registration of rare cumulative events).**
- **The studies in new cumulative region becomes possible due to the moderate energy of the NICA collider and is completely impossible at ultrahigh energies of the RHIC and LHC.**

What inspires us?

Г.А. Лексин, ЯФ, т. 65, № 11, 2002, стр. 2042 — 2051







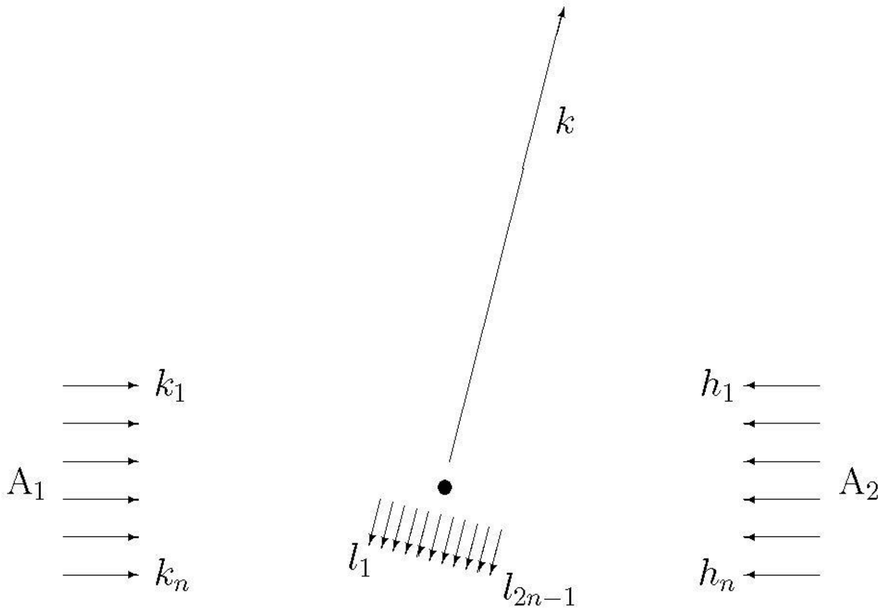
Large fraction of cumulative processes

$\sqrt{s_{NN}} = 9.8$ GeV

N. Antonov, V. Gapienko, G. Gapienko, M. Ilushin, A. Prudkoglyad, V. Romanovskiy, A. Semak, I. Solodovnikov, M. Ukhanov, V. Viktorov "High pt anti-proton and meson production in cumulative pA reaction at 50 GeV/c"
(National Research Center Kurchatov Institute - Institute for High Energy Physics, Protvino)
LXX International Conference "NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies", St Petersburg, October 11-17, 2020.

The mechanism of inclusive pion production in the new cumulative region of central rapidities and large transverse momenta due to flucton-flucton interaction.

Kinematics



d+d → π+X at quark level
($A_1 = A_2 = A = 2$, $n = 6$)

$p_N = P_A/A \qquad p_N \gg m_N$

Initial state:

$k_i \sim P_A/n = p_N/3$
$h_i \sim -P_A/n = -p_N/3$

$n = 3A$

Final state:

$k \sim P_A = A\, p_N = n\, p_N/3$

$l_i \sim -P_A/(2n-1) = -\frac{n}{3(2n-1)} p_N$

First small parameter: $\frac{m_N}{p_N} = \frac{2m_N}{\sqrt{s_{NN}}} \ll 1$

Second small parameter: $1 - \frac{k}{k_{max}} \ll 1$

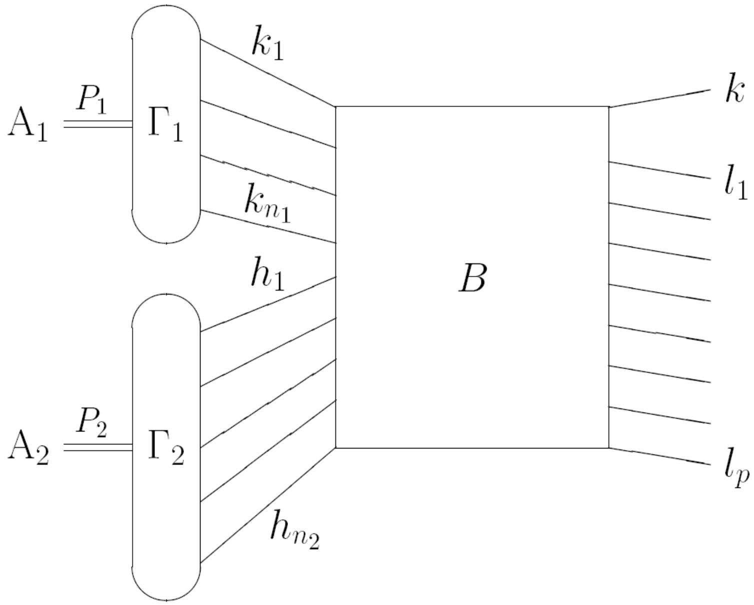
$k \to k_{max}$ => $M_X{}^2 = (\sum_{i=1}^{2n-1} l_i)^2 \to M_{Xmin}{}^2$

$$l_1 = l_2 = \ldots = l_{2n-1} = -\frac{k_{max}}{2n-1}$$

Amplitude of inclusive pion production in the region of central rapidities and large transverse momenta

V.Vechernin, S.Yurchenko,
Int. J. Mod. Phys. E 33, 2441022 (2024)
S.Yurchenko, V.Vechernin,
Phys. Atom. Nucl. 88, 349 (2025)

$T =$



$$p = n_1 + n_2 - 1$$

$$r_B \sim \frac{1}{\sqrt{s}}$$

$$I(\mathbf{k}) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{1}{J} \int |T|^2\, d\tau_p\,,$$

$$J = 2A_1A_2\sqrt{s(s-4m_N^2)}$$

$$d\tau_p \equiv (2\pi)^4 \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2l_{i0}(2\pi)^3}$$

$$s = s_{NN}$$

$$I(\mathbf{k}) = \frac{1}{J} |T|^2\, \tau_p$$

$$\tau_p = (2\pi)^4 \int \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2l_{i0}(2\pi)^3}$$

Used approximation

$$l^2 - m^2 + \mathrm{i}\epsilon = [l_0 + E(\mathbf{l})][l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon] \approx 2E(\mathbf{l})[l_0 - E(\mathbf{l}) + \mathrm{i}\epsilon]$$

Integration over zero components of momenta

It leads to Heitler's ("old fashioned") perturbation theory:

1) **All particles are considered on mass shell**
2) **Inclusion of "energy denominators" between interactions**
3) **Permutation of interactions in time where possible**
4) **Additional factor 1/[2*E*(l)] from each propagator**

$$\frac{1}{[\sum_i E(\mathbf{l}_i) - E_{init} - \mathrm{i}\epsilon]}$$

Weakly coupled systems

Third small parameter: $\alpha = \sqrt{\frac{\varepsilon}{m}} \ll 1$

$$M = n(m - \varepsilon)$$

$$\left\langle \frac{\mathbf{q}^{*2}}{2m} \right\rangle \simeq \varepsilon$$

$$|\mathbf{q}^*| \simeq \sqrt{m\varepsilon} = m\,\alpha$$

- in the rest frame

$$\gamma = \frac{E_p}{m}$$

$$|\mathbf{q}_\perp| \simeq \sqrt{m\varepsilon} = m\,\alpha$$

$$|q_z| \simeq \gamma |q_z^*| \simeq \gamma \sqrt{m\varepsilon} = \frac{E_p}{m}\sqrt{m\varepsilon} = E_p\,\alpha$$

$$E(\mathbf{p}+\mathbf{q}) \equiv \sqrt{(\mathbf{p}+\mathbf{q})^2 + m^2}$$

$$E(\mathbf{p}+\mathbf{q}) = E_p \left\{ 1 + \frac{(\mathbf{qp})}{E_p^2} + \frac{1}{2E_p^2}\left[\mathbf{q}^2 - \frac{(\mathbf{qp})^2}{E_p^2} \right] + O(\alpha^3) \right\}$$

$$E_p \equiv E(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}\ ,\ \ E_k \equiv E(\mathbf{k})$$

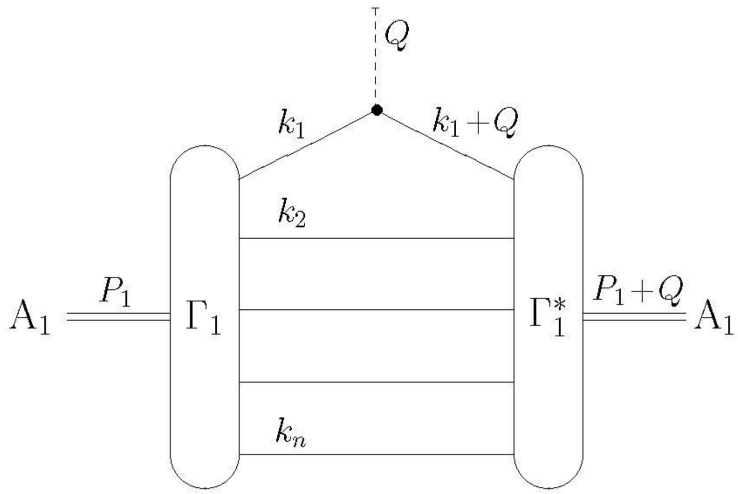
It is **not assumed** here that **|p|>>*m* !**

Valid both in the rest frame of the nucleus (p=0**) and**
in the frame in which the nucleus moves with relativistic momentum *n*p or *n*k
(|p|>>*m*, |k|>>*m*)

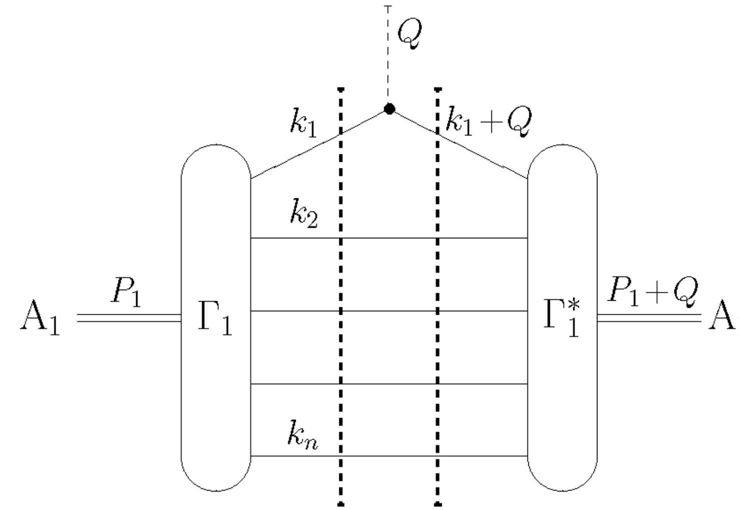
$$\frac{(\mathbf{qp})}{E_p^2} = \frac{q_z|\mathbf{p}|}{E_p^2} \simeq \frac{E_p\,\alpha\,|\mathbf{p}|}{E_p^2} = \frac{|\mathbf{p}|}{E_p}\alpha$$

Connection of vertices Γ_i with wave functions. Normalization.

$F(Q^2) =$



$=$



$$Q^2 \to 0 \quad \Rightarrow \quad F(Q^2) \to 1$$

$$\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)}) = \frac{\Gamma_{n\mathbf{p}}(\mathbf{q}^{(i)})}{\sqrt{n}2E_p\left[\sum_{i=1}^{n} E(\mathbf{p}+\mathbf{q}^{(i)}) - E_{init} - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

Connection of vertices Γ_i with wave functions. Normalization.

In the rest frame of the nucleus (p=0):

$$\varphi_0(\mathbf{q}^{*(i)}) = \frac{\Gamma_0(\mathbf{q}^{*(i)})}{\sqrt{n}2m\left[\sum_{i=1}^{n}\frac{\mathbf{q}^{*(i)2}}{2m} - n\varepsilon - i\epsilon\right]} = \frac{\Gamma_0(\mathbf{q}^{*(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}\mathbf{q}^{*(i)2} - n2m\varepsilon - i\epsilon\right]}$$

$$\int |\varphi_0(\mathbf{q}^{*(i)})|^2 \prod_{i=1}^{n-1}\frac{d^3\mathbf{q}^{*(i)}}{2m(2\pi)^3} = 1$$

In the frame in which the nucleus moves with relativistic momentum *n*p :

$$\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)}) = \frac{\Gamma_{n\mathbf{p}}(\mathbf{q}^{(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2 - n2m\varepsilon - i\epsilon\right]}$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1}\frac{d^3\mathbf{q}^{(i)}}{2m\gamma(2\pi)^3} = 1$$

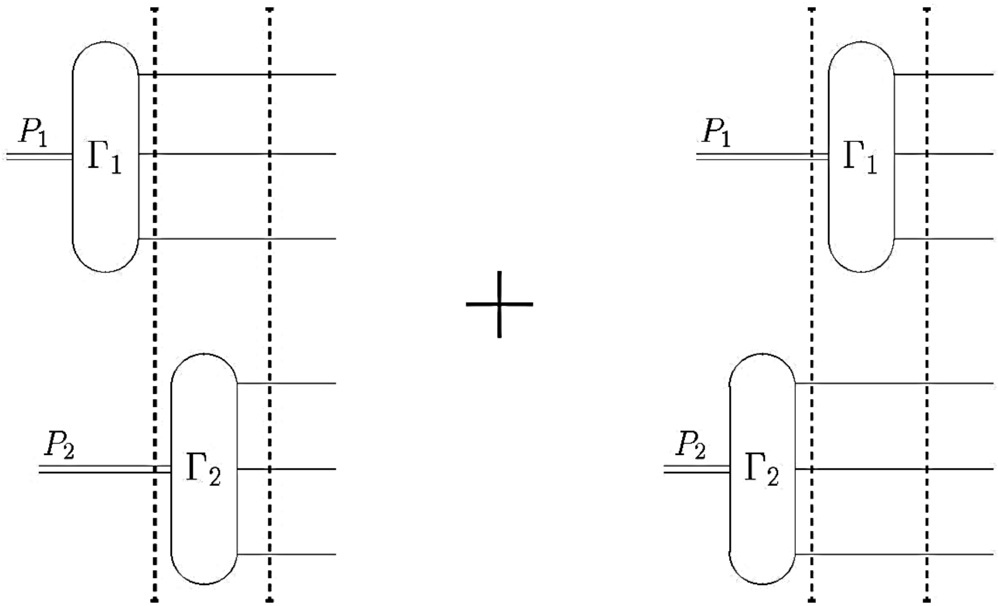
$$\gamma = \frac{E_p}{m}$$

$$\mathbf{q}_\perp^{(i)} = \mathbf{q}_\perp^{*(i)} \simeq \sqrt{m\varepsilon} = m\,\alpha$$

$$q_z^{(i)} = \gamma q_z^{*(i)} \simeq \gamma\sqrt{m\varepsilon} = E_p\,\alpha$$

$$\alpha = \sqrt{\frac{\varepsilon}{m}}$$

Permutations of Γ_i vertices



$$\varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)}) \equiv \frac{\Gamma_1}{[E_1 - E_1^{init}]2E_p\sqrt{3}} \; ,$$

$$\varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)}) \equiv \frac{\Gamma_2}{[E_2 - E_2^{init}]2E_p\sqrt{3}}$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

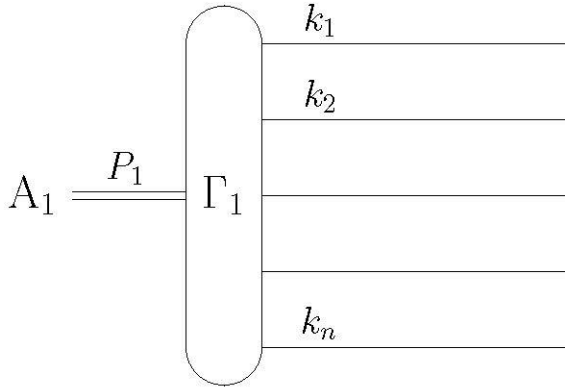
$$\left\{ \frac{1}{[E_1 - E_1^{init} - \mathrm{i}\epsilon]} + \frac{1}{[E_2 - E_2^{init} - \mathrm{i}\epsilon]} \right\} \frac{1}{[E_1 + E_2 - E_1^{init} + E_2^{init} - \mathrm{i}\epsilon]} =$$

$$= \frac{1}{[E_1 - E_1^{init}][E_2 - E_2^{init}]}$$

$$E_1 \equiv \sum_{i=1}^{3} E(\mathbf{p} + \mathbf{q}_1^{(i)}) \; , \quad E_2 \equiv \sum_{i=1}^{3} E(-\mathbf{p} + \mathbf{q}_2^{(i)}) = \sum_{i=1}^{3} E(\mathbf{p} - \mathbf{q}_2^{(i)})$$

Contribution of the vertices Γ_i in the case of pion production

$J_{n1} =$



$$\mathbf{k}_i = \mathbf{p} + \mathbf{q}_1^{(i)} \qquad \sum_{i=1}^{3} \mathbf{q}_1^{(i)} = 0$$

$$J_{n_1} = \int \varphi_{n\mathbf{p}}(\mathbf{q}^{(i)}) \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3}$$

$$J_{n_1} \sim \psi_1(\mathbf{r}_i - \mathbf{r}_j = 0)$$

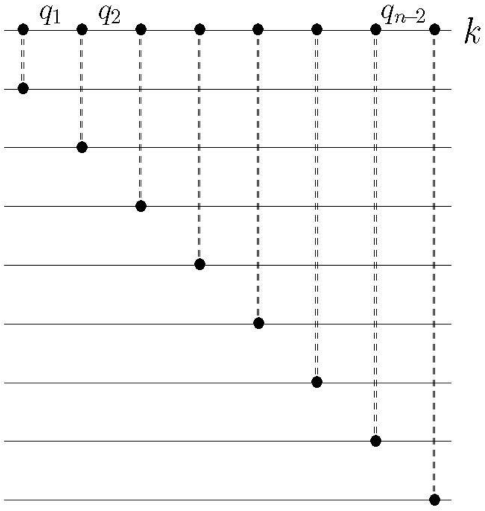
$$J_{n_1} = \frac{C_1}{m^{(n_1-1)/2} R_1^{3(n_1-1)/2}}$$

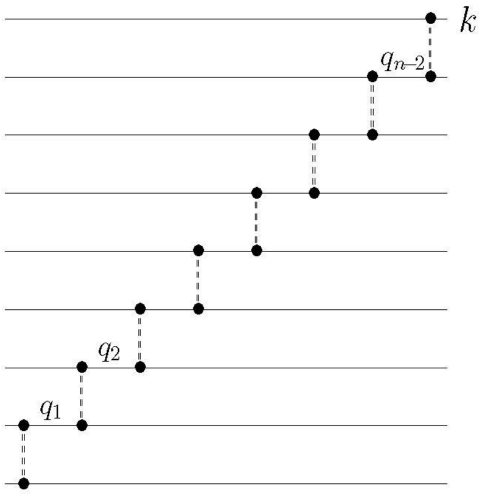
m - mass of the constituent quark
R_1- size of the system (R_1= R_A or R_N)
C_1- dimensionless constant, independent of the dimensional parameters of the model

$$|J_n|^2 \sim |\psi(\mathbf{r}_i - \mathbf{r}_j = 0)|^2 \sim \frac{1}{V^{n-1}} \sim \frac{1}{R^{3(n-1)/2}}$$

for a wave function with one dimensional parameter

Block of hard exchanges (B)





S.J. Brodsky, B.T. Chertok, Phys.Rev. D 14 (1976) 3003

$$B = \frac{C_B}{s^{n-2}} = \frac{C_B}{s^{n_1+n_2-2}}$$

$$n = n_1 + n_2$$

$$p = n - 1$$

$$r_B \sim \frac{1}{\sqrt{s}}$$

$$B(k_i, h_i; \mathbf{k}, \mathbf{l}_i) \approx B(P_1/n_1, P_2/n_2; \mathbf{k}_{max}, -\mathbf{k}_{max}/p)$$

$$T = J_{n_1} J_{n_2} B.$$

$$T = \frac{C_1\, C_2\, C_B}{m^{(n-2)/2} R_1^{3(n_1-1)/2} R_2^{3(n_2-1)/2} s^{n-2}} = \frac{C_1\, C_2\, C_B}{m^{(n-2)/2} R^{3(n-2)/2} s^{n-2}}$$

$$I(\mathbf{k}) = \frac{1}{J} |T|^2\, \tau_p$$

$$\tau_p = (2\pi)^4 \int \delta^4(P_1 + P_2 - k - \sum_{i=1}^{p} l_i) \prod_{i=1}^{p} \frac{d^{(3)}\mathbf{l}_i}{2 l_{i0} (2\pi)^3}$$

After calculating the phase volume and the relation with the cumulative number

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)\, m]^2}\ .$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p - \frac{5}{2}}}{(m^2R^3)^{p-1} s^{(p+3)/2}}$$

$p = p(A)$

two (!)
small parameters:
$m/\sqrt{s} \ll 1$
$A - x \ll 1$

A=2 (p=11) d+d → π+X $I_{dd\to\pi}(x) \sim s^{-7}\,(2-x)^{14}$

A=1 (p=5) p+p → π+X $I_{pp\to\pi}(x) \sim s^{-4}\,(1-x)^{5}$

$$r_B \sim \frac{1}{\sqrt{s}}$$

Changes may occur at moderate NICA energy:

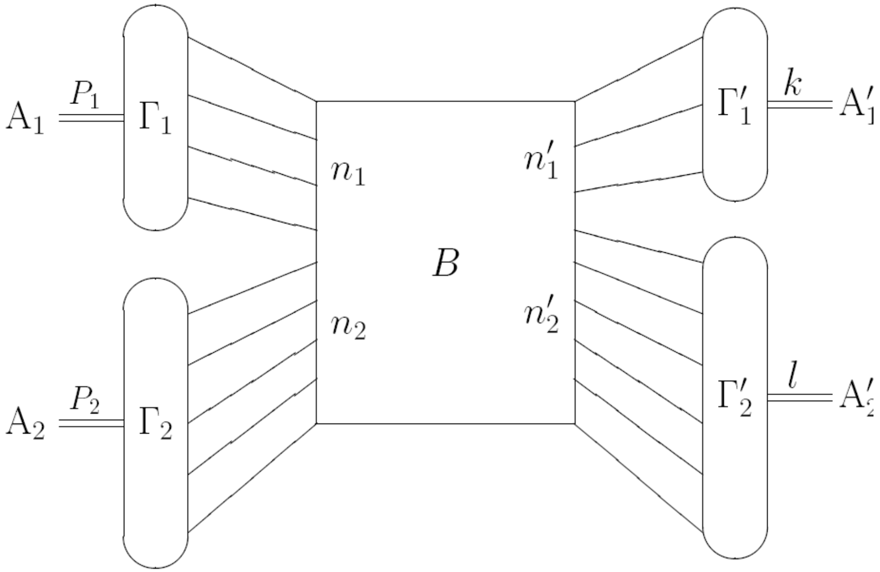
$M_X \sim (A-x)\sqrt{s}$

so at rather small NICA energies: partonic phase volume => hadronic phase volume
only **d+d** → **π+NNNN,** neglecting **d+d** → **π+ πNNNN** and so on.

Comparison with Quark Counting Rules for Quasi-Elastic Processes

one
small parameters:
$m/\sqrt{s} \ll 1$

check up of the approach



$$\frac{d\sigma}{dt} \sim \frac{1}{s^{n_1+n_2+n'_1+n'_2-2}}$$

Matveev V.A., Muradyan R.M., Tavkhelidze A.N., Lett. Nuovo Cimento 7 (1973) 719
Brodsky S., Farrar G., Phys.Rev.Lett. 31 (1973) 1153
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003
Uzikov Yu. N., JETP Lett. 81 (2005) 303

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\, s^2_{A_1 A_2}} |T_{2\to2}|^2 = \frac{1}{16\pi\, A_1^2 A_2^2 s^2} |T_{2\to2}|^2$$

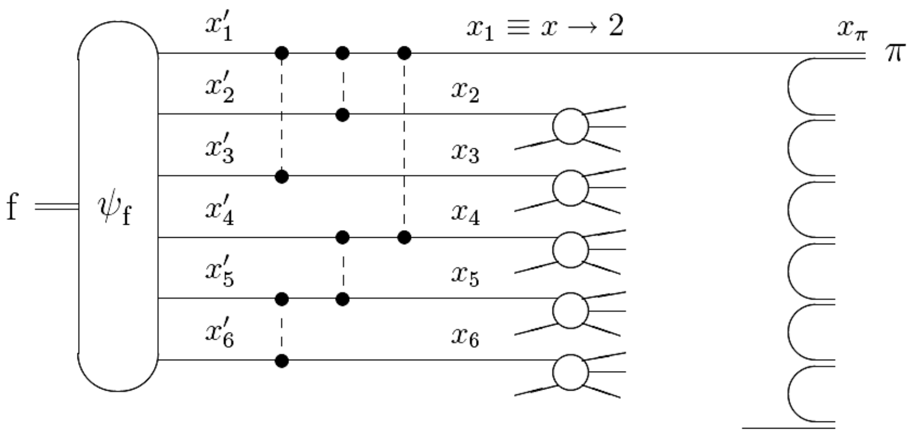
$$T_{2\to2} = J_{n_1}\, J_{n_2}\, B\, J_{n'_1}\, J_{n'_2}$$

$$\frac{d\sigma}{dt} = \frac{C'}{s^{2n-2}\, m^{2n-4} R_1^{3(n_1-1)} R_2^{3(n_2-1)} R_1'^{3(n'_1-1)} R_2'^{3(n'_2-1)}}$$

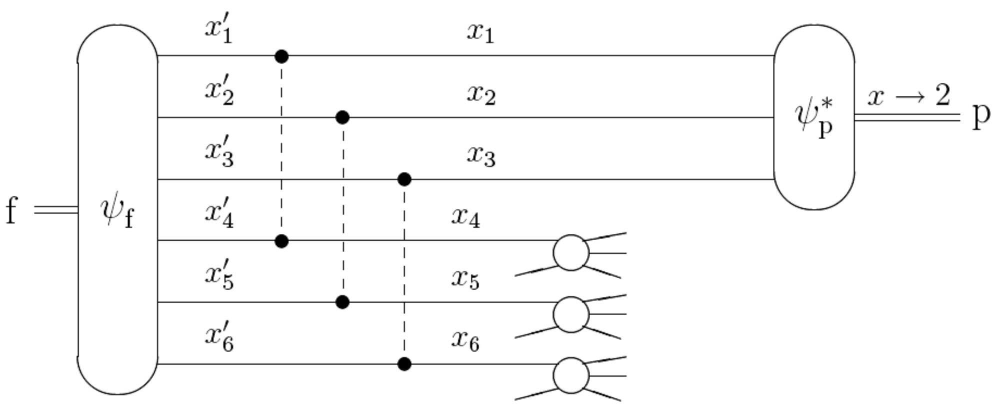
$n = n_1 + n_2$
$= n'_1 + n'_2$

The mechanism of proton production
at central rapidities and large transverse momenta due to flucton-flucton interaction.

Coherent Quark Coalescence and Production of Cumulative Protons **in Fragmentation Region**



- the cumulative pion production
by hadronization of one fast quark
M.A. Braun, V.V. Vechernin,
*Nucl.Phys.**B 427**, 614 (1994);*
*Phys.Atom.Nucl. **60**, 432 (1997);*
***63**, 1831 (2000)*



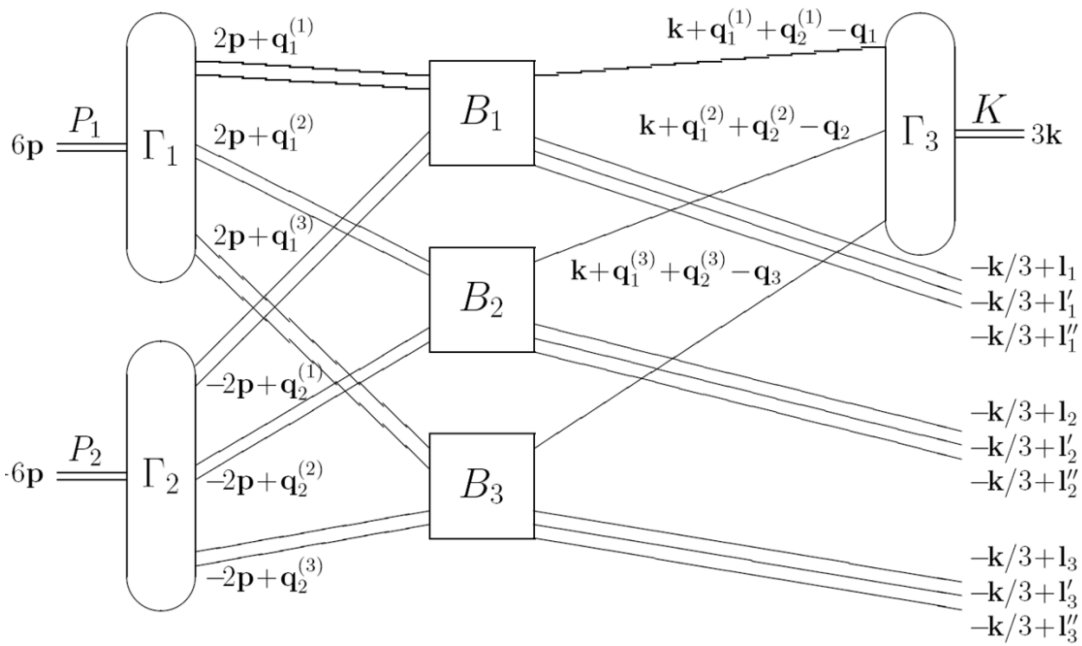
- the cumulative proton production
by **coherent** quark coalescence mechanism:
M.A. Braun, V.V. Vechernin,
*Nucl.Phys.**B 92**, 156 (2001);*
*Theor.Math.Phys **139**, 766 (2004);*
V.Vechernin,
AIP Conf.Proc.1701 (2016) 060020.

The last diagram **recalls** the few nucleon **short-range correlations** in a nucleus
L.L. Frankfurt, M.I. Strikmann, Phys. Rep. 76, 215 (1981); ibid 160, 235 (1988).
But instead of using the relativistic generalization of non-relativistic NN wave function
the microscopic analysis of the flucton fragmentation process near cumulative thresholds on the base of the intrinsic diagrams of QCD in light-cone gauge
*Brodsky S.J., Hoyer P., Mueller A., Tang W.-K., Nucl. Phys. **B369** (1992) 519.*
was developed and applied.

Amplitude of cumulative proton production in d+d → p+X at

$$x \to 2, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}, \quad \theta^* \sim 90^\circ$$

Mechanism of Coherent Coalescence of Quark also dominates the proton production in the new cumulative region of large transverse momenta. This leads to Modified Quark Counting Rules (due to production from few points) and changes the dependence of inclusive cross-section on two asymptotic parameters $s \gg m^2$ and $(x_{max} - x) \ll 1$.



$$r_N \sim r_f \gg r_B \sim \frac{1}{\sqrt{s}}$$

So, the asymptotic behaviour of cross-sections depends on details of flucton-flucton interaction!

V. Vechernin, S. Yurchenko, Phys. Part. Nucl. 57 (5) (2026) [in press].

$$\mathbf{q}_1^{(i)} = \mathbf{k}_1^{(i)} + \mathbf{k}_1'^{(i)}$$
$$\overline{\mathbf{q}}_1^{(i)} = (\mathbf{k}_1^{(i)} - \mathbf{k}_1'^{(i)})/2$$

$$\sum_{i=1}^{3} \mathbf{q}_i = \sum_{i=1}^{3} [\mathbf{l}_i + \mathbf{l}_i' + \mathbf{l}_i''] = 0$$

Amplitude of proton production in d+d → p+X at

$$x \to 2, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}, \quad \theta^* \sim 90^\circ$$

$$T(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i) = J(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i)\prod_{j=1}^{3} B_j(2\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{l}_i,\mathbf{l}'_i,\mathbf{l}''_i) = \int \prod_{i=1}^{2} \frac{d^3\mathbf{q}_1^{(i)}}{2E_p(2\pi)^3}\frac{d^3\mathbf{q}_2^{(i)}}{2E_p(2\pi)^3}\times$$

$$\times\left[\int \varphi_{6\mathbf{p}}(\mathbf{q}_1^{(i)},\overline{\mathbf{q}}_1^{(i)})\prod_{i=1}^{3}\frac{d^3\overline{\mathbf{q}}_1^{(i)}}{2E_p(2\pi)^3}\right]\left[\int \varphi_{-6\mathbf{p}}(\mathbf{q}_2^{(i)},\overline{\mathbf{q}}_2^{(i)})\prod_{i=1}^{3}\frac{d^3\overline{\mathbf{q}}_2^{(i)}}{2E_p(2\pi)^3}\right]$$

$$\times D(a_1,a_2,a_3)\,\varphi^*_{3\mathbf{k}}(\mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-\mathbf{q}_i)$$

$$D(a_1,a_2,a_3) = -2\pi^2E_p^2[\delta(a_1)\delta(a_2)+\delta(a_2)\delta(a_3)+\delta(a_1)\delta(a_3)]$$

$$a_i \equiv (2\mathbf{p},\ \mathbf{q}_1^{(i)}-\mathbf{q}_2^{(i)}) - (\mathbf{k},\ \mathbf{q}_1^{(i)}+\mathbf{q}_2^{(i)}-2\mathbf{q}_i) \qquad \mathbf{q}_i = \mathbf{l}_i+\mathbf{l}'_i+\mathbf{l}''_i$$

$$\mathbf{q}_1^{(i)} = \mathbf{k}_1^{(i)}+\mathbf{k}_1'^{(i)}$$

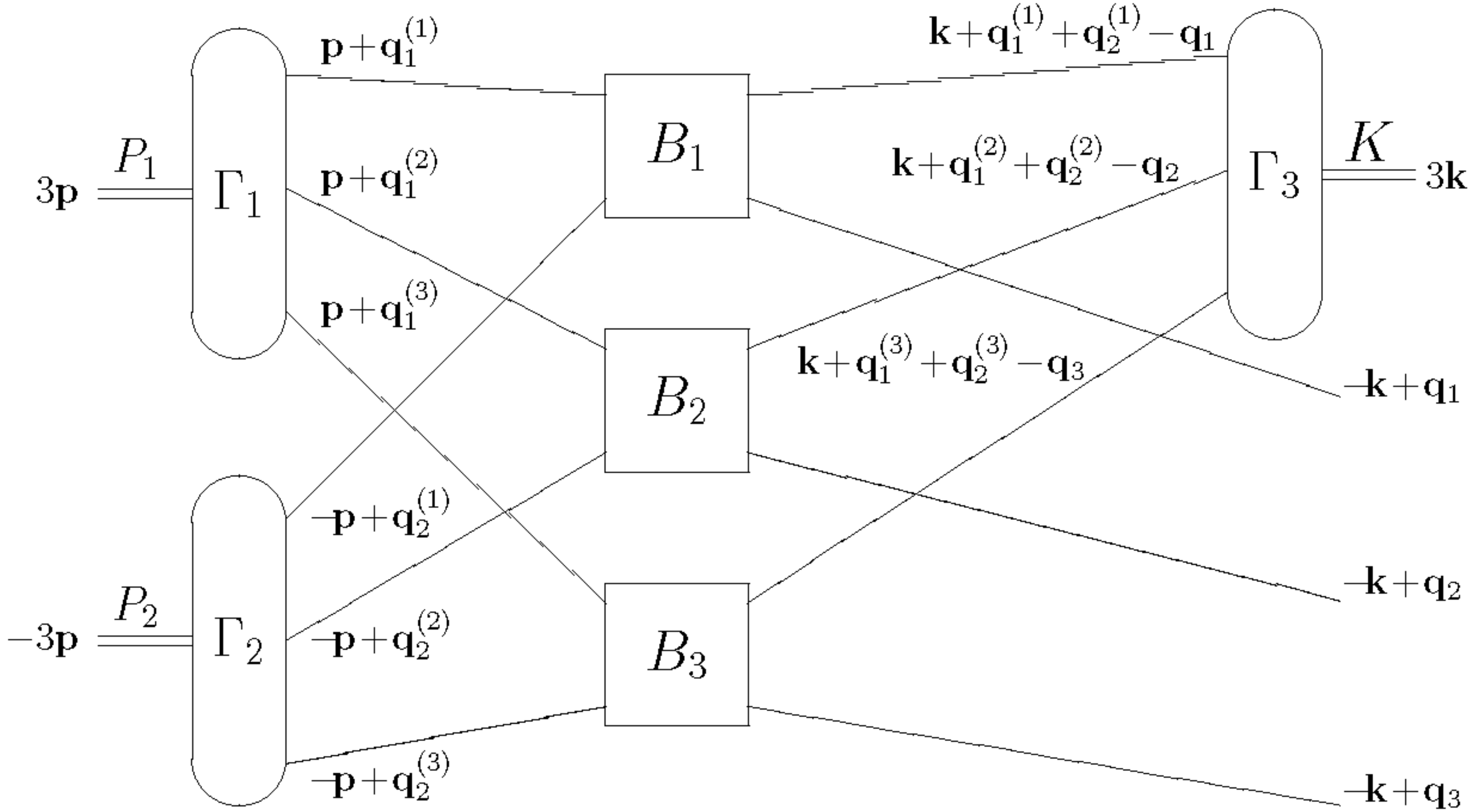
$$\overline{\mathbf{q}}_1^{(i)} = (\mathbf{k}_1^{(i)}-\mathbf{k}_1'^{(i)})/2$$

$$\sum_{i=1}^{3}\mathbf{q}_i = \sum_{i=1}^{3}[\mathbf{l}_i+\mathbf{l}'_i+\mathbf{l}''_i] = 0$$

Quark Coherent Coalescence technic for p+p → p+X in 3q-picture of a nucleon (n=3) at

$$x \to 1, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}/2, \quad \theta^* \sim 90^\circ$$

2/3 < x < 1



$$\sum_{i=1}^{3} \mathbf{q}_1^{(i)} = 0$$

$$\sum_{i=1}^{3} \mathbf{q}_2^{(i)} = 0$$

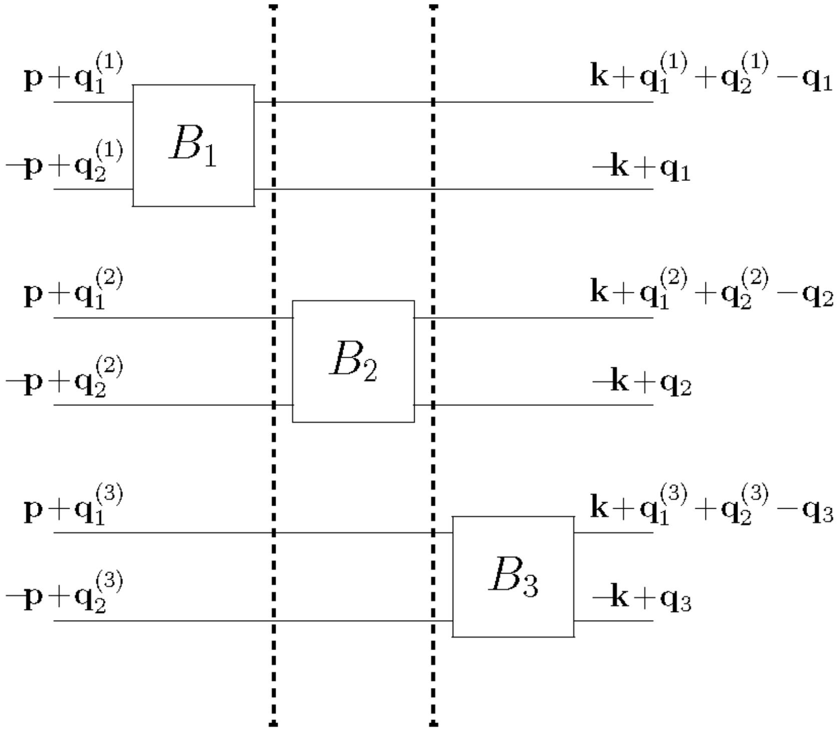
$$\sum_{i=1}^{3} \mathbf{q}_i = 0$$

Now there will be no proportionality to

$$J_{n_1} \sim \psi_1(\mathbf{r}_i - \mathbf{r}_j = 0)$$

$$I_{\mathrm{pp}\to\mathrm{p}}(x) \sim (1-x)^2$$

Permutations of B blocks (n=3)



$$\frac{E_p^2}{[-a_1 - i\epsilon][a_3 - i\epsilon]}$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)} - \mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - 2\mathbf{q}_i)$$

$$\sum_{i=1}^{3} a_i = 0$$

$$\frac{1}{b - i\epsilon} = \frac{1}{b} + i\pi\delta(b)$$

$$\frac{-E_p^2}{[a_1 + i\epsilon][a_3 - i\epsilon]} + \frac{-E_p^2}{[a_3 + i\epsilon][a_1 - i\epsilon]} = -\frac{2E_p^2}{a_1 a_3} - 2\pi^2 E_p^2 \delta(a_1)\delta(a_3)$$

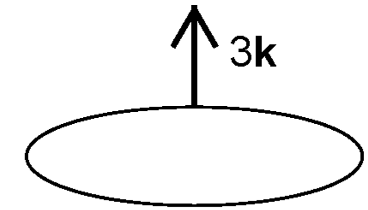
$$\frac{1}{a_1 a_3} + \frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} = \frac{a_1 + a_2 + a_3}{a_1 a_2 a_3} = 0$$

$$D(a_1, a_2, a_3) = -2\pi^2 E_p^2 [\delta(a_1)\delta(a_2) + \delta(a_2)\delta(a_3) + \delta(a_1)\delta(a_3)]$$

Inclusive cross section of proton prod. in p+p → p+X (n=3)

$$x \to 1, \quad \sqrt{s} \gg m, \quad |\mathbf{k}_\perp| \sim \sqrt{s}/2, \quad \theta^* \sim 90^\circ$$

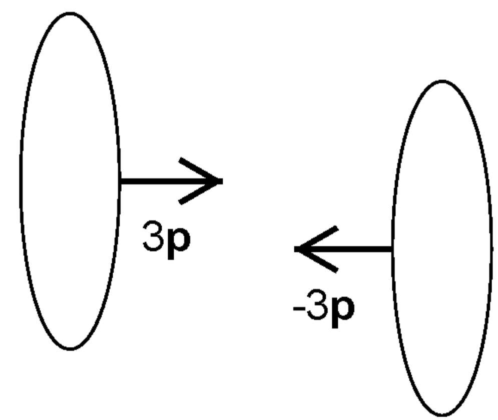
$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}} \int |T(\mathbf{p};\mathbf{k},\mathbf{q}_i)|^2\, d\tau_n$$



$$T(\mathbf{p};\mathbf{k},\mathbf{q}_i) = J(\mathbf{p};\mathbf{k},\mathbf{q}_i) \prod_{j=1}^{3} B_j(\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_i) = \int \prod_{i=1}^{2} \frac{d^3\mathbf{q}_1^{(i)}}{2E_p(2\pi)^3} \frac{d^3\mathbf{q}_2^{(i)}}{2E_p(2\pi)^3} \times$$

$$\times \varphi_{3\mathbf{p}}(\mathbf{q}_1^{(i)}) \varphi_{-3\mathbf{p}}(\mathbf{q}_2^{(i)}) D(a_1,a_2,a_3) \varphi^*_{3\mathbf{k}}(\mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - \mathbf{q}_i)$$

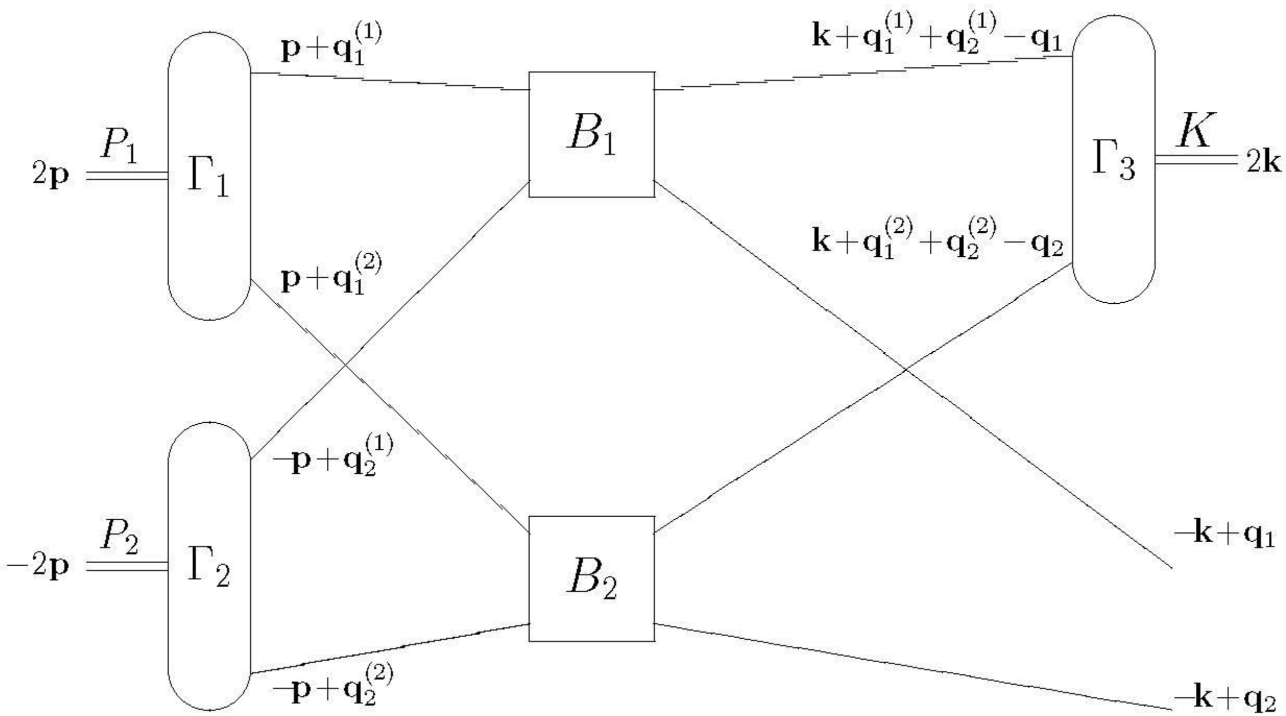


$$D(a_1,a_2,a_3) = -2\pi^2 E_p^2 [\delta(a_1)\delta(a_2) + \delta(a_2)\delta(a_3) + \delta(a_1)\delta(a_3)]$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)} - \mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - 2\mathbf{q}_i)$$

$$\int |\varphi_{n\mathbf{p}}(\mathbf{q}^{(i)})|^2 \prod_{i=1}^{n-1} \frac{d^3\mathbf{q}^{(i)}}{2E_p(2\pi)^3} = 1$$

Coherent Coalescence (n=2)



can be used to describe:

1) **d+d → d+p+n**
at the nucleon level

Braun M.A., Vechernin V.V., Yad.Fiz. 36 (1982) 614; 44 (1986) 784; 47 (1988) 1452; J. Phys. G 16 (1990) 1615.

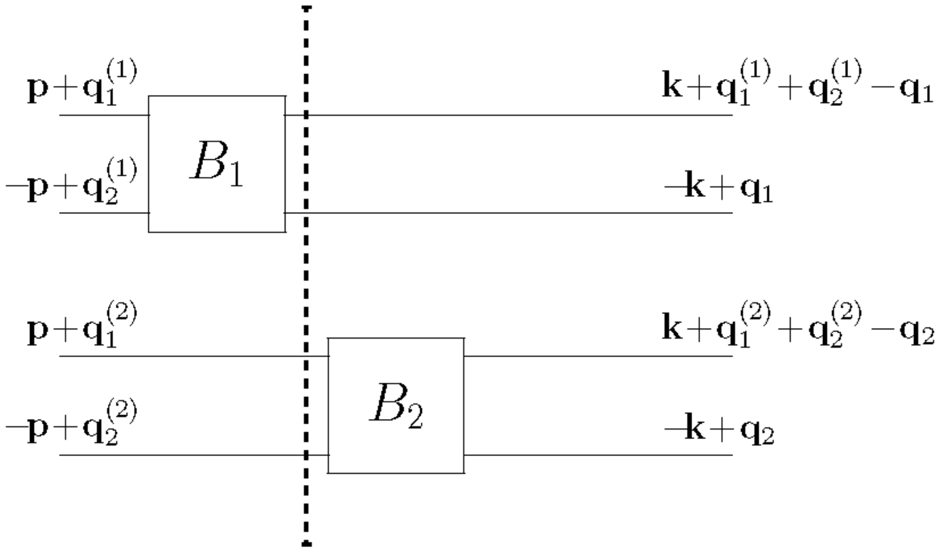
p + A → d,Tr,He + X

2) **p+p → p+X**
in the quark-diquark picture of the nucleon

$$\mathbf{q}_1^{(1)}+\mathbf{q}_1^{(2)}=0\ ,\ \ \mathbf{q}_2^{(1)}+\mathbf{q}_2^{(2)}=0\ ,\ \ \mathbf{q}_1+\mathbf{q}_2=0$$

$$r_B \sim \frac{1}{\sqrt{s}}$$

Permutations of B blocks (n=2)



$$\frac{E_p}{[-a_1 - \mathrm{i}\epsilon]} + \frac{E_p}{[-a_2 - \mathrm{i}\epsilon]}$$

$$a_i \equiv (\mathbf{p},\, \mathbf{q}_1^{(i)} - \mathbf{q}_2^{(i)}) - (\mathbf{k},\, \mathbf{q}_1^{(i)} + \mathbf{q}_2^{(i)} - 2\mathbf{q}_i)$$

$$a_1 + a_2 = 0$$

$$\frac{-E_p}{[a_1 + \mathrm{i}\epsilon]} + \frac{-E_p}{[a_2 + \mathrm{i}\epsilon]} = i\pi E_p[\delta(a_1) + \delta(a_2)]$$

$$\frac{1}{b - \mathrm{i}\epsilon} = \frac{1}{b} + i\pi\delta(b)$$

$$D(a_1, a_2) = i\pi E_p[\delta(a_1) + \delta(a_2)] = 2\pi i E_p\,\delta(a_1)$$

Inclusive cross section (n=2)

$$(2\pi)^3\, 2K_0 \frac{d^3\sigma}{d^3\mathbf{K}} = \frac{1}{J_{in}} \int |T(\mathbf{p};\mathbf{k},\mathbf{q}_1)|^2\, d\tau_2$$

$$r_B \sim \frac{1}{\sqrt{s}}$$

$$T(\mathbf{p};\mathbf{k},\mathbf{q}_1) = J(\mathbf{p};\mathbf{k},\mathbf{q}_1) \prod_{j=1}^{2} B_j(\mathbf{p};\mathbf{k})$$

$$J(\mathbf{p};\mathbf{k},\mathbf{q}_1) = \int \frac{d^3\mathbf{q}_1^{(1)}}{2E_p(2\pi)^3} \frac{d^3\mathbf{q}_2^{(1)}}{2E_p(2\pi)^3} \times$$

$$\times 2\pi i E_p\, \delta(a_1)\, \varphi_{2\mathbf{p}}(\mathbf{q}_1^{(1)}) \varphi_{-2\mathbf{p}}(\mathbf{q}_2^{(1)}) \varphi^*_{2\mathbf{k}}(\mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)} - \mathbf{q}_1)$$

$$\int |\varphi_{2\mathbf{p}}(\mathbf{q})|^2 \frac{d^3\mathbf{q}}{2E_p(2\pi)^3} = 1$$

$$\int |\varphi_{2\mathbf{k}}(\mathbf{q})|^2 \frac{d^3\mathbf{q}}{2E_k(2\pi)^3} = 1$$

$$a_1 \equiv (\mathbf{p},\, \mathbf{q}_1^{(1)} - \mathbf{q}_2^{(1)}) - (\mathbf{k},\, \mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)} - 2\mathbf{q}_1)$$

$$\gamma = \frac{E_p}{m}$$

$$I(x) = C_B^2(\theta) \frac{(1-x)^{0.5}}{R\, s^{2.5}}\, F(\theta)$$

$$F(\theta) = \frac{1}{\frac{12}{\gamma^2} + \sin^2\theta \left(1 - \frac{1}{\gamma^2}\right)\left(2 - \frac{3}{\gamma^2}\right)}$$

Angular dependence of the Coalescence **(n=2)**

$$F(\theta) = \frac{1}{\frac{12}{\gamma^2} + \sin^2\theta(1 - \frac{1}{\gamma^2})(2 - \frac{3}{\gamma^2})}$$

$$\gamma = 1 \;\Rightarrow\; F(\theta) = \frac{1}{12} = const$$

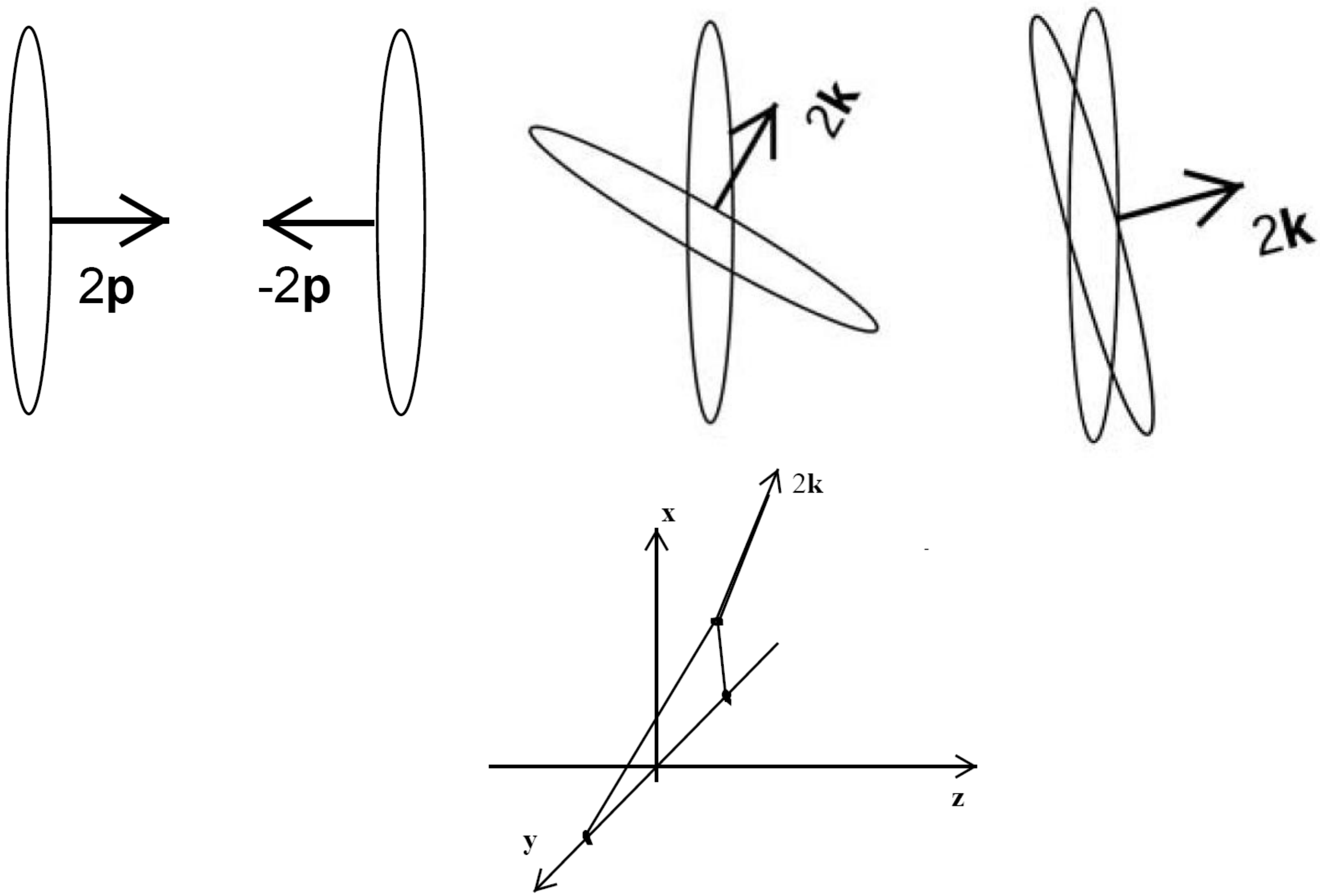
$$\gamma \gg 1 \;\Rightarrow\; F(\theta) = \frac{\gamma^2}{2(6 + \gamma^2 \sin^2\theta)} = \frac{\gamma^2 \mathrm{ch}^2\eta}{2(6\,\mathrm{ch}^2\eta + \gamma^2)}$$

$\eta = \ln \mathrm{tg}(\theta/2)$ - pseudorapidity

$$\sin\theta = 1/\mathrm{ch}\,\eta$$

Increase in the coalescence coefficient for small angles in the relativistic case

Physical interpretation in the center of mass system



Incorporating diquarks

V.T. Kim, Diquarks and Dynamics of Large P(T) Baryon Production, Mod.Phys.Lett.A 3 (1988) 909.

p/π^+ - ratio explanation, using that the diquark distribution function is harder: $(1\text{-}x)^1$ vs $(1\text{-}x)^3$ for quarks [$(1\text{-}x)^{2p-1}$].

Yu.L. Dokshitzer, QCD Phenomenology,
Lectures at the CERN–Dubna School, Pylos, August 2002

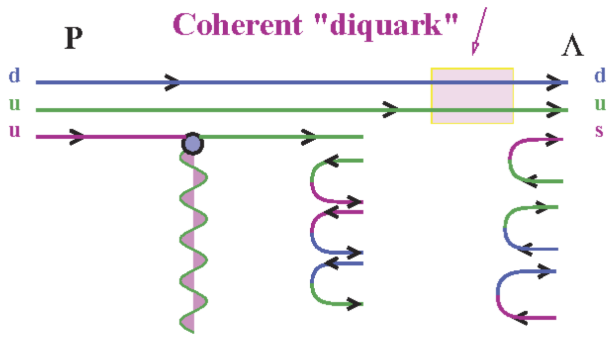


Fig. 4a: Gluon exchange produces a leading baryon.

M.A. Braun, V.V. Vechernin, Nuclear Structure Functions and Particle Production in the Cumulative Region in the Parton Model, Nucl.Phys. B427 (1994) 614

Can string junction carries the baryon number?

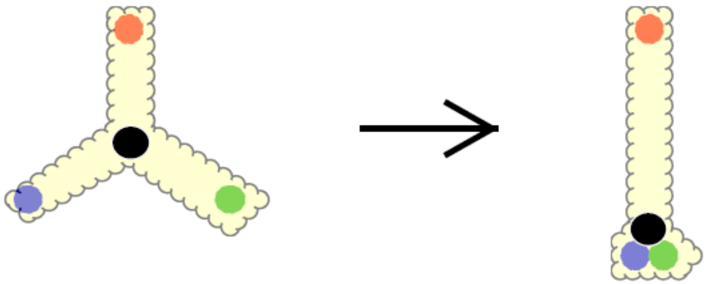
L. Montanet, G. C. Rossi, and G. Veneziano, “Baryonium Physics,”
Phys. Rept. 63, 149–222 (1980).

D. Kharzeev, “Can gluons trace baryon number?”
Phys.Lett. B 378, 238–246 (1996), arXiv:nucl-th/9602027.
Can be verified experimentally by studing of
baryon stopping in central pp and AA collisions.

Yu.M. Shabelski,
String Junction and Diffusion of Baryon Charge in Multiparticle Production Processes,
arXiv: 0705.0947 [hep-ph], (2007).
F. Bopp, Yu.M. Shabelski,
String junction effects for forward and central baryon production in hadron-nucleus collisions
Eur.Phys.J.A 28 (2006) 237-243

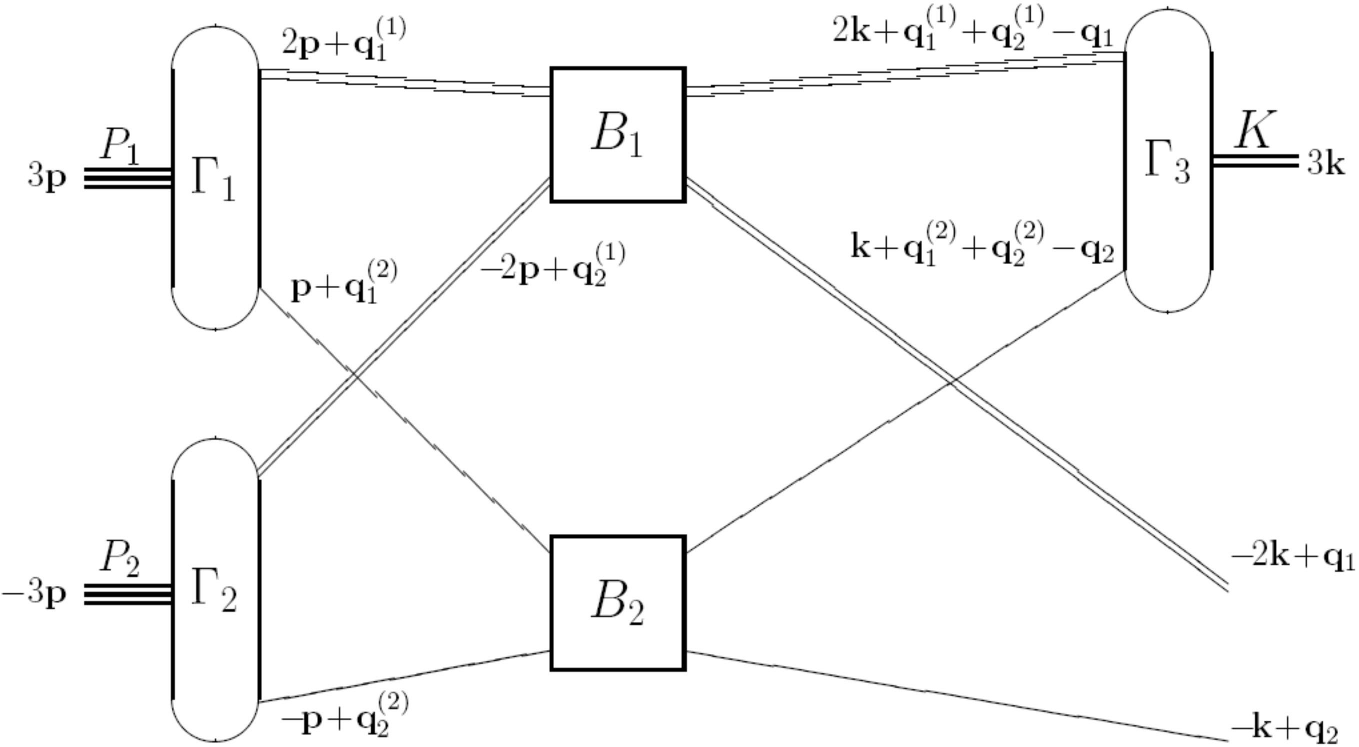
G.Pihan, A.Monnai, B.Schenke, Chun Shen,
Unveiling baryon charge carriers through charge stopping in isobar collisions
arXiv:2405.19439v1 [nucl-th] (2024).

Connection with diquarks:
Now B=1 corresponds to diquark



At $x \to 1\ ,\quad |\mathbf{k}| \to |\mathbf{p}|$ **in the region** $2/3 < x < 1$

the mechanism of coherent coalescence of diquark and quark dominates over diquark fragmentation into proton.



Coherent coalescence of diquark and quark

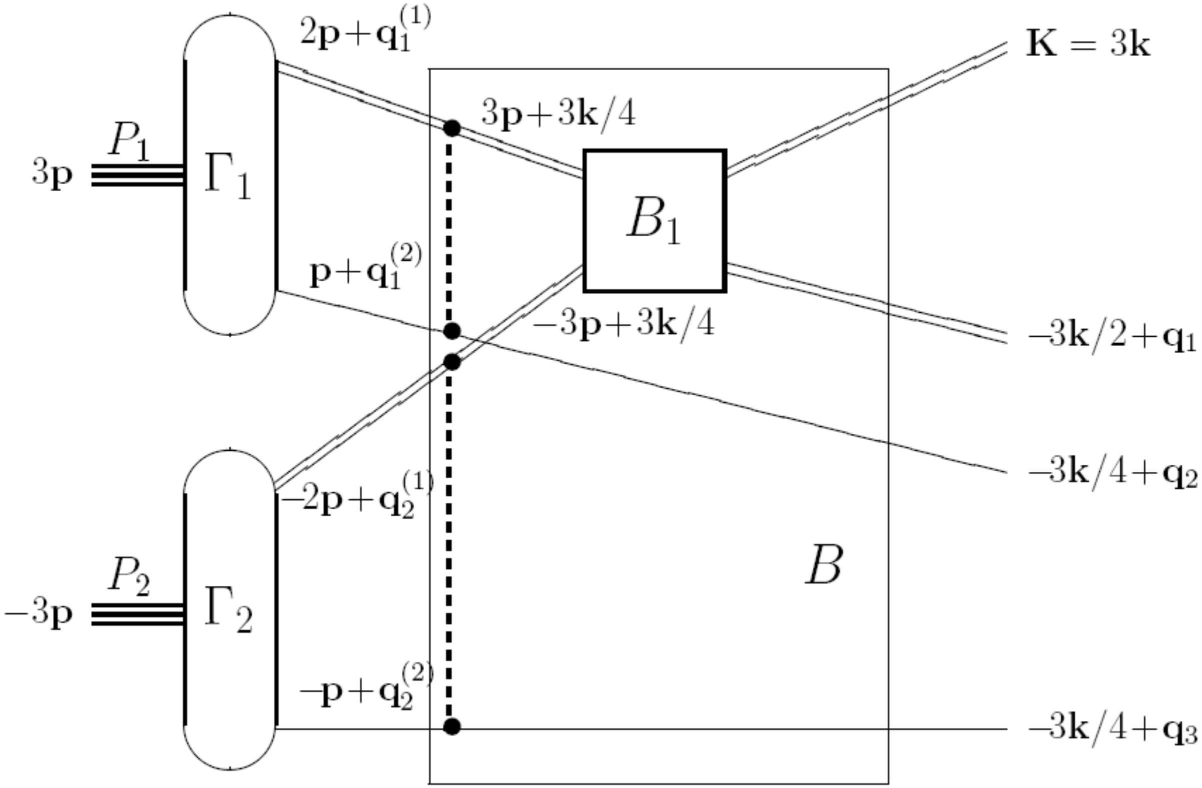
$$r_B \sim \frac{1}{\sqrt{s}}$$

$$I_{pp\to p}(x) \sim \frac{(1-x)^{0.5}}{s^{2.5}}$$

$$\mathbf{q}_1^{(1)}+\mathbf{q}_1^{(2)}=0\ ,\quad \mathbf{q}_2^{(1)}+\mathbf{q}_2^{(2)}=0\ ,\quad \mathbf{q}_1+\mathbf{q}_2=0$$

At $x \to 1\ ,\quad |\mathbf{k}| \to |\mathbf{p}|$ in the region 2/3 < x < 1

the mechanism of **coherent coalescence of diquark and quark** dominates over **diquark fragmentation** into proton.



Diquark fragmentation into proton.

$$r_B \sim \frac{1}{\sqrt{s}}$$

$$I_{pp\to p}(x) \sim \frac{(1-x)^2}{s^3}$$

$$\mathbf{q}_1^{(1)} + \mathbf{q}_1^{(2)} = 0\ ,\quad \mathbf{q}_2^{(1)} + \mathbf{q}_2^{(2)} = 0\ ,\quad \mathbf{q}_1 + \mathbf{q}_2 = 0$$

Asymptotic behaviour of inclusive cross sections in pp at 2/3 < x < 1

n=2, q+(qq):

Coalescence of diquark and quark:

$$I_{pp\to p}(x) \sim \frac{(1-x)^{0.5}}{s^{2.5}}$$

Diquark fragmentation:

$$I_{pp\to p}(x) \sim \frac{(1-x)^{2}}{s^{3}}$$

n=3, q+q+q:

Coalescence:

$$I_{pp\to p}(x) \sim \frac{(1-x)^{2}}{s^{?}}$$

Quark fragmentation:

$$I_{\mathrm{pp}\to\pi}(x) = C'' \frac{1}{(m^2R^3)^4} \frac{(1-x)^5}{s^4}$$

Characteristic features of the amplitude in mechanism of Coherent Quark Coalescence

- **no proportionality to the wave function value at zero**
- **smaller number of hard exchanges compared to fragmentation mechanism**
- **the absence of the need for all quarks to gather in an ever-smaller region as energy increases**
- **the coherence - the convolution of wave functions (not probabilities!)**
- **MC simulations cannot be applied (just like for Gribov screening)**
- **usual factorization assumption is invalid**
- **calculation of Feynman graphs automatically leads to the correct space-time picture of the process**

(see papers on **Coherent Coalescence Mechanism** at nucleon level:
Braun M.A., Vechernin V.V., Yad.Fiz. 36 (1982) 614; 44 (1986) 784.
Braun M.A., Vechernin V.V., Production of fast fragments in high-energy hadron collisions with nuclei. J. Phys. G 16 (1990) 1615-1626.)

$$r_N \sim r_f \gg r_B \sim \frac{1}{\sqrt{s}}$$

Summary

1. The Quark Counting Rules for cross sections of inclusive production of particles with large transverse momentum are formulated.

2. Mechanism of Coherent Quark Coalescence leads to the increase of proton production in considered region at $x \to 1$, compared to the mechanism of fragmentation of one quark (diquark) into proton, which was used for description of pion production, due to fewer number of hard exchanges and the absence of the need for all quarks to gather in an ever-smaller region as energy increases.

3. The last leads to further modifications of Quark Counting Rules.

4. Taking into account the stretching of the wave function of the resulting relativistic particle in momentum space is essential. In this case the probability of Quark Coalescence occurs lower for angles about 90^o. The physical interpretation of this phenomenon is presented.

MC simulations of the particle production with high pT in pp collisions

in the region 2/3 < x < 1

(to analyse new QCR in pp at NICA SPD)

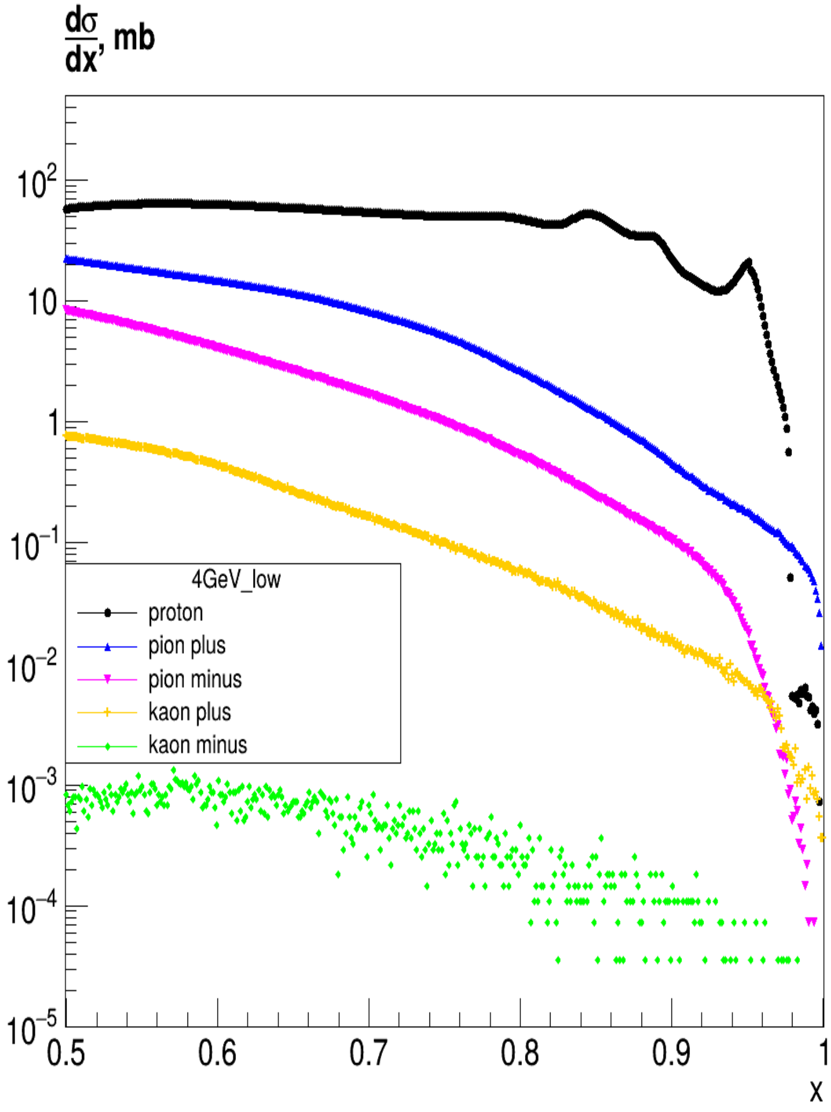
S. Yurchenko, Phys. Part. Nucl. (2026) [in press].

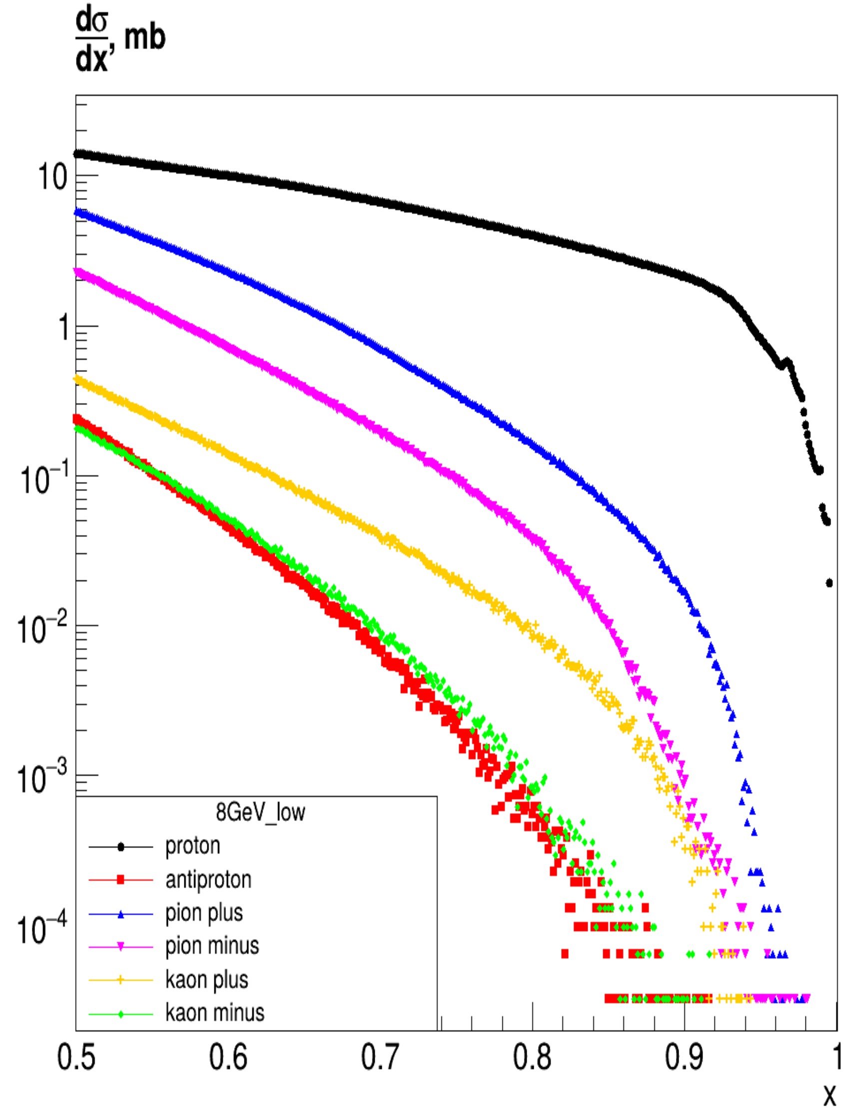
Simulations with Pythia8

- LowEnergyQCD: all = on. $\mathrm{N_{ev}} = 10^9$
- HeavyIons: mode = 1. $\mathrm{N_{ev}} = 5 \cdot 10^8$
- Pseudorapidity interval: $-2 < \eta < 2\ (15^\circ < \theta < 165^\circ)$
- Particle types: $p, \bar{p}, \pi^+, \pi^-, K^+, K^-$

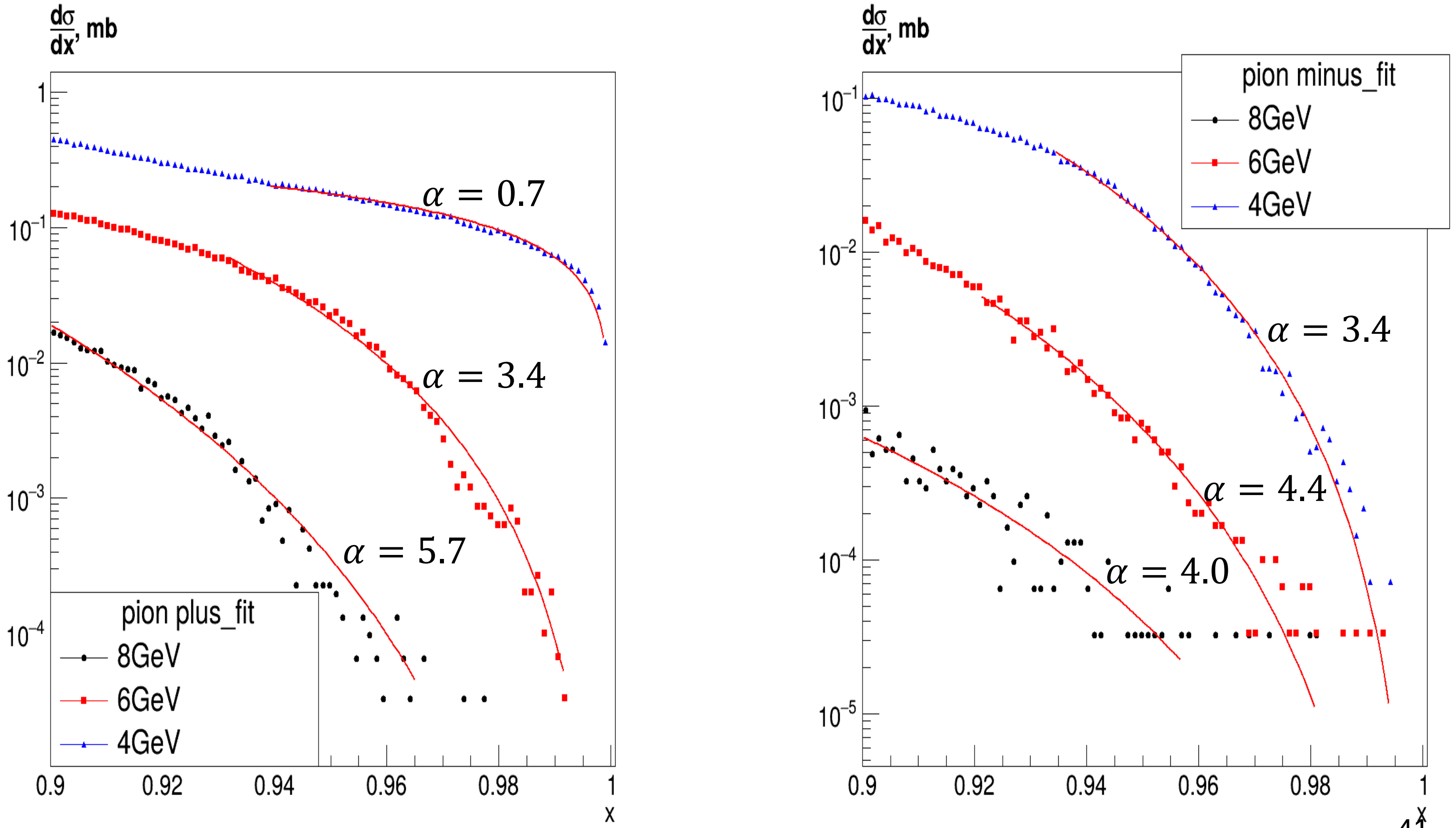
No propagation through detector material were done!

All particles, 4 and 8 GeV, pp





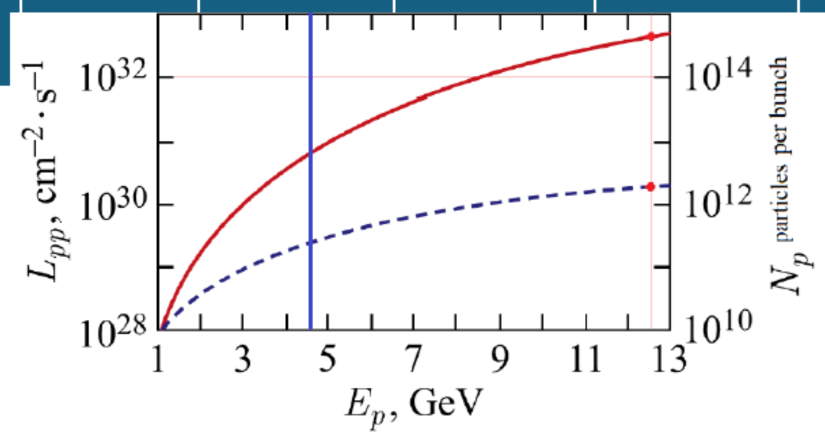
Fits of π^+ and π^- with $C \cdot (1-x)^{\alpha}$



Yields of particles: pp with $0.95 < x < 1$

$\sqrt{S_{NN}} = 4\ GeV$ | $\sqrt{S_{NN}} = 6\ GeV$ | $\sqrt{S_{NN}} = 8\ GeV$

Particle type	Per sec	Per day	Per month	Per sec	Per day	Per month	Per sec	Per day	Per month
p	$8.6 \cdot 10^2$	$7.4 \cdot 10^7$	$2.2 \cdot 10^9$	$2.6 \cdot 10^3$	$2.3 \cdot 10^8$	$6.8 \cdot 10^9$	$1.5 \cdot 10^3$	$1.3 \cdot 10^8$	$4 \cdot 10^9$
π^+	$2.1 \cdot 10^1$	$1.8 \cdot 10^6$	$5.4 \cdot 10^7$	$5.8 \cdot 10^0$	$5 \cdot 10^5$	$1.5 \cdot 10^7$	$1.3 \cdot 10^{-1}$	$1.1 \cdot 10^4$	$3.3 \cdot 10^5$
π^-	$8.1 \cdot 10^{-1}$	$5.4 \cdot 10^4$	$2.1 \cdot 10^6$	$1.6 \cdot 10^{-1}$	$1.4 \cdot 10^4$	$4.1 \cdot 10^5$	$4.4 \cdot 10^{-2}$	$3.8 \cdot 10^3$	$1.2 \cdot 10^5$
Number of collisions	$1.7 \cdot 10^5$	$1.5 \cdot 10^{10}$	$4.4 \cdot 10^{11}$	$8.8 \cdot 10^5$	$7.6 \cdot 10^{10}$	$2.3 \cdot 10^{12}$	$3.1 \cdot 10^6$	$2.7 \cdot 10^{11}$	$8.2 \cdot 10^{12}$



Backup slides

Connection of vertices Γ_i with wave functions. Light-cone variables.

Light-cone partonic wave function.

S.J. Brodsky, P. Hoyer, A. Mueller, W.-K. Tang, Nucl.Phys. B369 (1992) 519;
M.A. Braun, V.V. Vechernin, Nucl.Phys. B427 (1994) 614.

$$\varphi_{lc}(x_i, \mathbf{q}_\perp^{(i)}) = \frac{\Gamma_{lc}(x_i, \mathbf{q}_\perp^{(i)})}{\sqrt{Ax_0}\left[\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} - \mathrm{i}\epsilon\right]}$$

$$\int |\varphi_{lc}(x_i, \mathbf{q}_\perp^{(i)})|^2 \prod_{i=1}^{n-1} \frac{dx_i\, d^2\mathbf{q}_\perp^{(i)}}{2x_i(2\pi)^3} = 1$$

$$x_i = \frac{k_+^{(i)}}{p_+}$$

$$\sum_{i=1}^{n} \mathbf{q}_\perp^{(i)} = 0$$

$$\sum_{i=1}^{n} x_i = A$$

$$x_0 \equiv A/n$$

$$A = 1 \quad n = 2$$

$$\frac{m_q^2 + \mathbf{q}_\perp^2}{x_1} + \frac{m_q^2 + (-\mathbf{q}_\perp)^2}{x_2} = \frac{m_q^2 + \mathbf{q}_\perp^2}{x(1-x)}$$

Connection of vertices Γ_i with wave functions.

Light-cone variables.

Light-cone partonic wave function.

$$M_A^2/A = (nm_q - n\varepsilon_q)^2/A \approx \frac{n}{x_0}(m_q^2 - 2m_q\varepsilon_q)$$

$$x_i = \frac{k_+^{(i)}}{p_{N+}} \approx \frac{k_z^{(i)}}{p_{Nz}} = \frac{p_{Nz}A/n + q_z^{(i)}}{p_{Nz}} = x_0 + \frac{q_z^{(i)}}{p_{Nz}} = x_0 + \frac{q_z^{(i)}}{p_{Nz}}$$

$$\gamma = \frac{E_q}{m_q} \approx \frac{p_{Nz}A/n}{m_q} = \frac{p_{Nz}x_0}{m_q}$$

$$\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} = \sum_{i=1}^{n} \frac{\mathbf{q}_\perp^{(i)2}}{x_0} + \frac{m_q^2}{x_0} \sum_{i=1}^{n} \left[1 - \frac{q_z^{(i)}}{x_0 p_{Nz}} + \frac{q_z^{(i)2}}{(x_0 p_{Nz})^2}\right] - \frac{n}{x_0}(m_q^2 - 2m_q\varepsilon_q)$$

$$\sum_{i=1}^{n} \frac{m_q^2 + \mathbf{q}_\perp^{(i)2}}{x_i} - \frac{M_A^2}{A} = \frac{1}{x_0} \sum_{i=1}^{n} \left[\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2\right] - \frac{2nm_q\varepsilon_q}{x_0}$$

$$\varphi_{lc}(q_z^{(i)}, \mathbf{q}_\perp^{(i)}) = \frac{\Gamma_{lc}(q_z^{(i)}, \mathbf{q}_\perp^{(i)})}{\sqrt{n}\left[\sum_{i=1}^{n}[\mathbf{q}_\perp^{(i)2} + (q_z^{(i)}/\gamma)^2] - 2nm_q\varepsilon_q - i\epsilon\right]}$$

Behavior of block B at large transferred momenta

Brodsky S., Farrar G. Phys.Rev.Lett. 31 (1973) 1153; Phys.Rev. D11 (1975) 1309
Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003; Phys.Rev.Lett. 37 (1976) 269

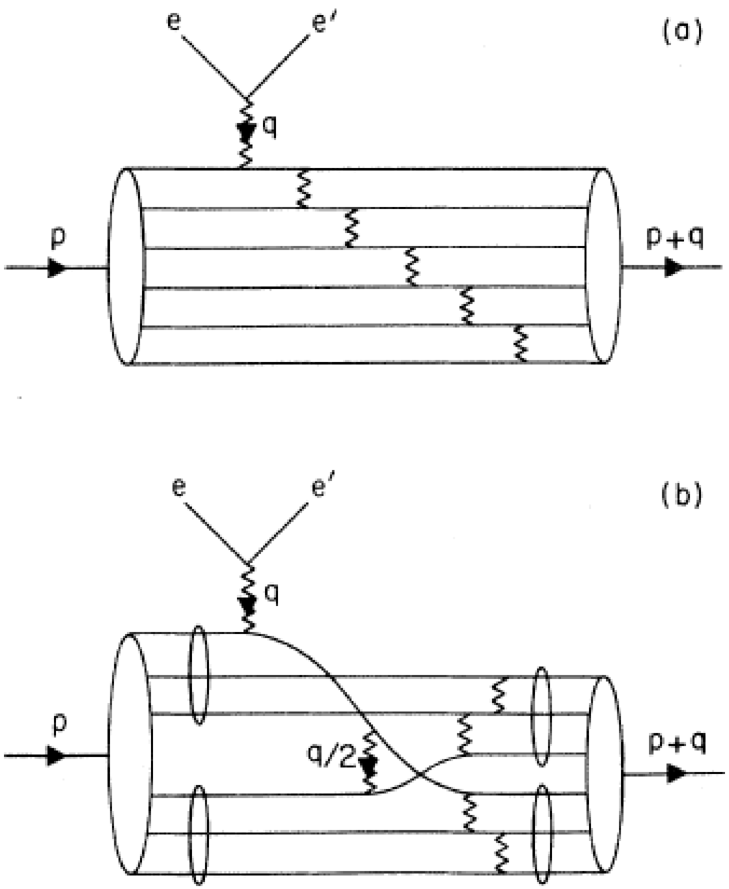


FIG. 2. Two possible quark-constituent views of e–D elastic scattering are (a) the democratic chain (cascade) model and (b) the quark-interchange model.

$$\psi_n(0) \equiv \int \prod_{j=1}^{n-1} d^3\vec{k}_j \psi(\vec{k}_j)$$

Hence, e interacts with d, when d is in the flucton configuration.

$$F_n(\vec{q}^2) \sim \left[\frac{2m}{\vec{q}^2} V(\vec{q}^2)\right]^{n-1} \psi_n^2(0)$$

In the case of quantum electrodynamics, and in fact any renormalizable theory, we have effectively (modulo powers of $\log q^2$ from finite orders in perturbation theory)

$$V(q^2) \sim \frac{e^2}{q^2}\left[1 + O\left(\frac{q^2}{m^2}\right)\right],$$

i.e., $V(q^2)$ becomes constant in the relativistic domain and

for large q^2 the gluon propagator is always compensated by its couplings to the quark currents

$$F_n(q^2) \sim \left(\frac{1}{q^2}\right)^{n-1}$$

The results of formfactor calculations

Brodsky S., Chertok B.T., Phys.Rev. D14 (1976) 3003

$$F_n(q^2) \sim \left(\frac{1}{q^2}\right)^{n-1}$$

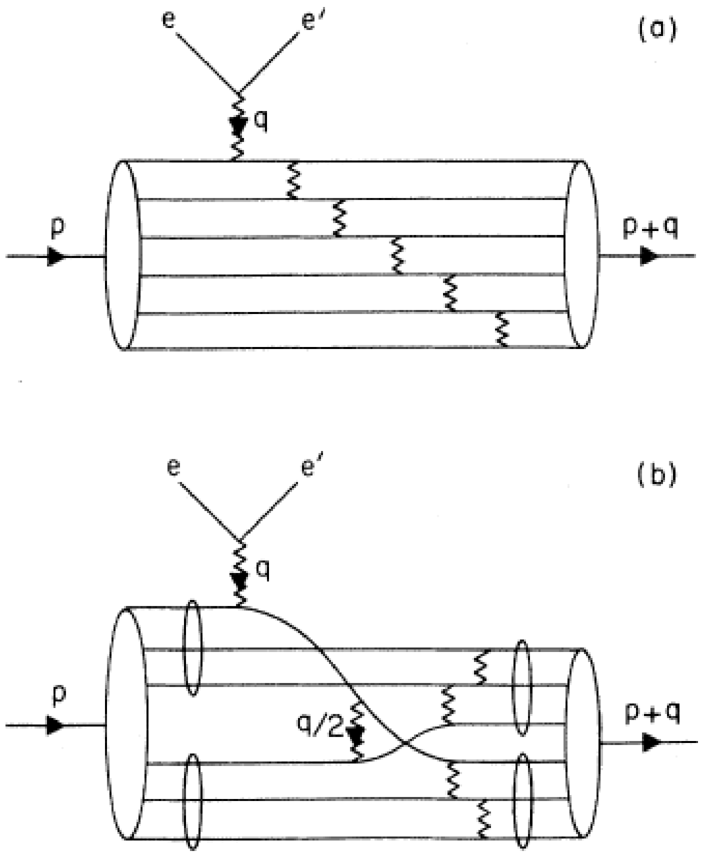


FIG. 2. Two possible quark-constituent views of e–D elastic scattering are (a) the democratic chain (cascade) model and (b) the quark-interchange model.

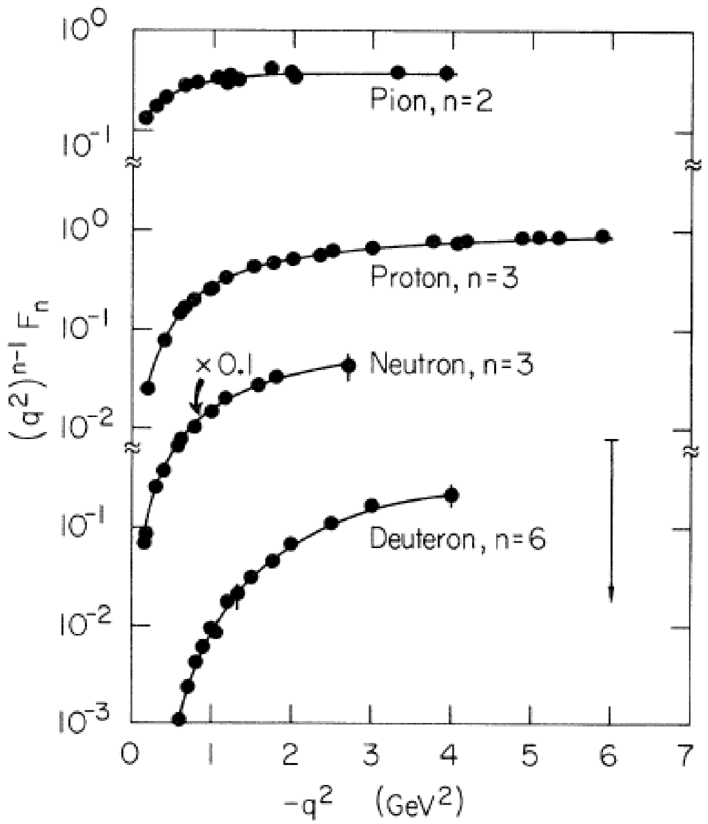
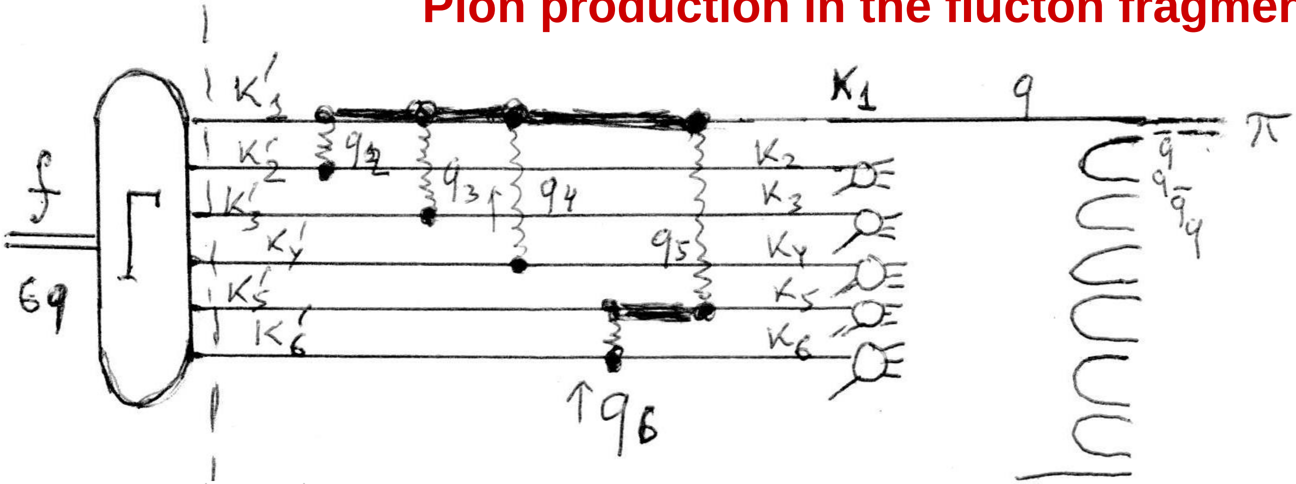


FIG. 1. Elastic electromagnetic form factors of hadrons for large spacelike q^2 in terms of the dimensional-scaling quark model. The curves simply connect the data points. (The neutron data have been multiplied by 0.1.)

Pion production in the flucton fragmentation region



f – number of nucleons which formed flucton

n - number of quarks in flucton

p=n-1 - number of "donors", stopped quarks

$\Gamma = \Gamma(k'_{+i}, k'_{\perp i})$ **then after integration over all** k'_{-i} **we get:**

$\Gamma(k'_{+i}, k'_{\perp i}) \to \Psi(k'_{+i}, k'_{\perp i})$ – light cone parton wave function of flucton

In all rest parts of the diagram we can put: $k'_{+i} = \dfrac{f\,p_+}{n} = \dfrac{f}{n} p_+ = \dfrac{1}{3} p_+$

Then we get:

$$\int \Psi(k'_{-i}, k'_{\perp i})\ \delta(\sum_{i=1}^{n} k'_{+i} - f\,p_+)\ \delta^2(\sum_{i=1}^{n} k'_{\perp i}) \prod_{i=1}^{n} \frac{dk'_{+i}}{2k'_{+i}} d^2k'_{\perp i} \sim \overline{\Psi}_{cms}(\{r_i - r_j = 0\})$$

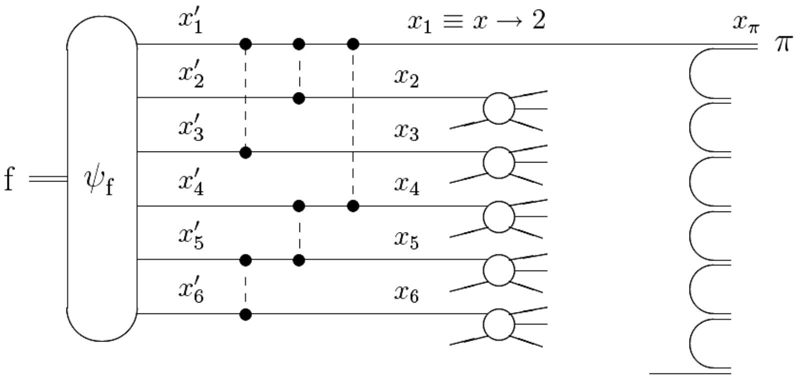
Contribution of (n-1) "Gluon" exchanges and (n-2) internal quark propagators limits to constant, when at $x_1 \Rightarrow f$ **all** $x_2, \ldots, x_n \Rightarrow 0$

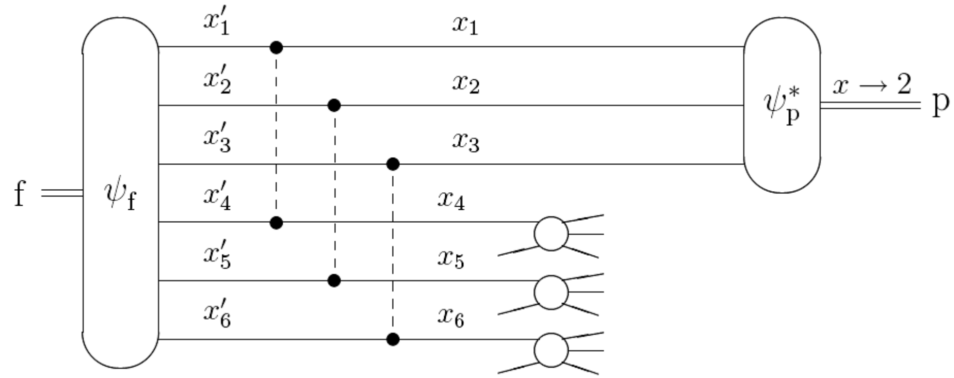
The main contribution comes from intermediate quark propagators, which defined the longitudial and transverse momentum dependence.

Scaling of cumulative inclusive cross section in the flucton fragmentation region:

$$f_\pi(x, k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C\, s^0\, (f - x)^{2p-1}\, \Phi_p\left(\frac{k_\perp}{m_q}\right)$$

Comparison of the mechanisms of pion and proton production in dd collisions in traditional cumulative region of fragmentation of one of the colliding nuclei





$$f_\pi(x,k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C_\pi (2-x)^9\, \Phi_5\left(\frac{k_\perp}{m_q}\right) / \Phi_5(0)$$

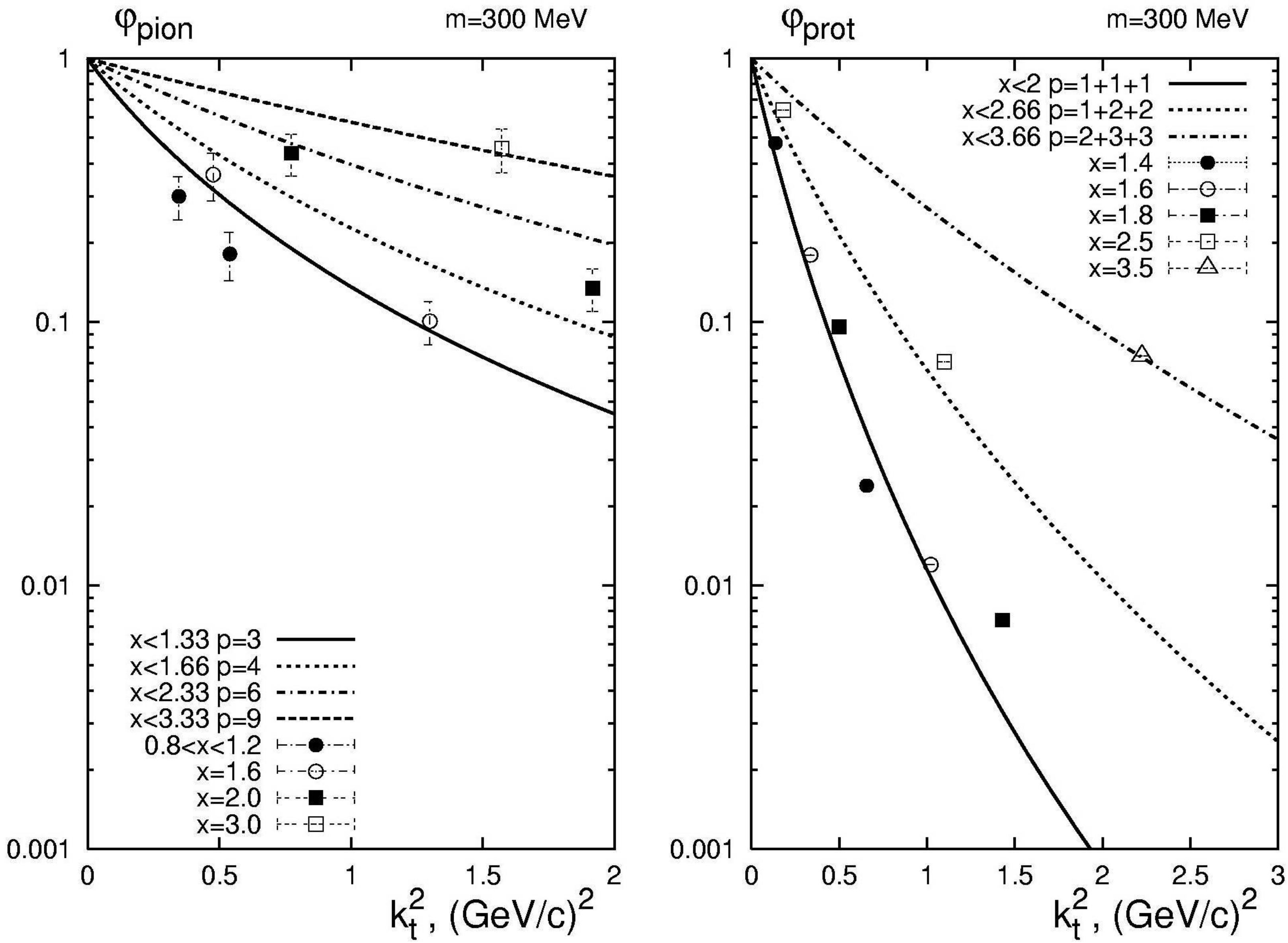
$$f_{\rm p}(x,k_\perp) \equiv \frac{k_0\, d^3\sigma_{\rm p}}{d^3\mathbf{k}} = C_{\rm p} (2-x)^5\, \Phi_1^3\left(\frac{k_\perp}{3m_q}\right) / \Phi_1^3(0)$$

$$\Phi_p(t) = 2\pi \int_0^\infty dz\, z J_0(tz)[zK_1(z)]^p$$

$$\Phi_1(t) = \frac{4\pi}{(t^2+1)^2}$$

Energy scaling of cumulative inclusive cross section in the flucton fragmentation region:

$$f_\pi(x,k_\perp) \equiv \frac{k_0\, d^3\sigma_\pi}{d^3\mathbf{k}} = C\, s^0\, (f-x)^{2p-1}\, \Phi_p\left(\frac{k_\perp}{m_q}\right)$$



V.Vechernin,
AIP Conference Proceedings
1701 (2016) 060020.

S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***46****, 871 (1987)*
S.V. Boyarinov et al., Physics of Atomic Nuclei ***57****, 1379 (1994)*
S.V. Boyarinov et al., Sov.J.Nucl.Phys. ***55****, 917 (1992)*

Calculation of Phase Volume

$$\tau_p = (2\pi)^{4-3p} \int \prod_{i=1}^{p} \frac{d^3\mathbf{l}'_i}{2l_{i0}} \delta^{(3)}\left(\sum_{i=1}^{p} \mathbf{l}'_i\right) \times$$
$$\times \delta\left(\sum_{i=1}^{p}\left[\sqrt{(\mathbf{k}/p+\mathbf{l}'_i)^2+m^2} - \sqrt{(\mathbf{k}/p)^2+m^2}\right] - \Delta\right)$$

$$\mathbf{l}'_i = -\mathbf{k}/p + \mathbf{l}_i$$

$$l_{i0} = \sqrt{(\mathbf{k}/p+\mathbf{l}'_i)^2+m^2}$$

$$p = n_1 + n_2 - 1$$

$$\Delta = A\sqrt{s} - \sqrt{k^2+m^2} - \sqrt{k^2+p^2m^2}$$

$k \to k_{max}$ => $\Delta \to 0$

$$\sqrt{k_{max}^2+m^2} + \sqrt{k_{max}^2+(p\,m)^2} = A\sqrt{s}$$

$$\tau_p = \frac{1}{2^p m^{p-1} p^{\frac{3}{2}}} \frac{\left(\frac{E_p}{2\pi}\Delta\right)^{\frac{3}{2}p-\frac{5}{2}}}{\left(\frac{3}{2}p-\frac{5}{2}\right)!}$$

$$E_p \equiv \sqrt{k^2/p^2+m^2}$$

Relation with Cumulative Number

$$x\sqrt{s} = \sqrt{k^2 + m^2} + \sqrt{k^2 + [p(x)\, m]^2}\ .$$

$$p(x) = n_1 + n_2 - 1 = 3A_1 + 3A_2 - 1 = 6A - 1 = 6x - 1.$$

$$\Delta = (A - x)[\sqrt{s} + O(1/\sqrt{s})]$$

$$\tau_p = \frac{1}{2^{4p-5} p^{3p/2-1} m^{p-1}} \frac{\left[\frac{A}{\pi} s(A-x)\right]^{\frac{3}{2}p - \frac{5}{2}}}{\left(\frac{3}{2}p - \frac{5}{2}\right)!}$$

$$p = p(A)$$

$$I(x) \equiv (2\pi)^3\, 2k_0 \frac{d^3\sigma}{d^3\mathbf{k}} = \frac{C(A-x)^{\frac{3}{2}p - \frac{5}{2}}}{(m^2 R^3)^{p-1} s^{(p+3)/2}}$$

two (!)
small parameters:
$m/\sqrt{s} \ll 1$
$A - x \ll 1$

Historical note

Coherent Coalescence Mechanism at nucleon level:

Braun M.A., Vechernin V.V., Yad.Fiz. 47 (1988) 1452; J. Phys. G 16 (1990) 1615.

p+A → d, Tr, ^{3}He +X - **in central and fragmentation regions**

- using the Feynman diagram technique
- taking into account the stretching of the wave function of the resulting fragment in momentum space

$$\kappa_2(\mathbf{k}) \sim \int d^2\mathbf{b}\, dz_1\, dz_2\ \rho_{n_1}(\mathbf{b}, z_1)\, \rho_{n_2}(\mathbf{b}, z_2)\, |\psi(y^*)|^2.$$

$$k \equiv |\mathbf{k}|$$

$$y^* \equiv |\mathbf{y}^*| = \frac{k_-}{m}|z_1 - z_2|$$

$$k_- = \sqrt{k^2 + m^2} - k\cos\theta$$

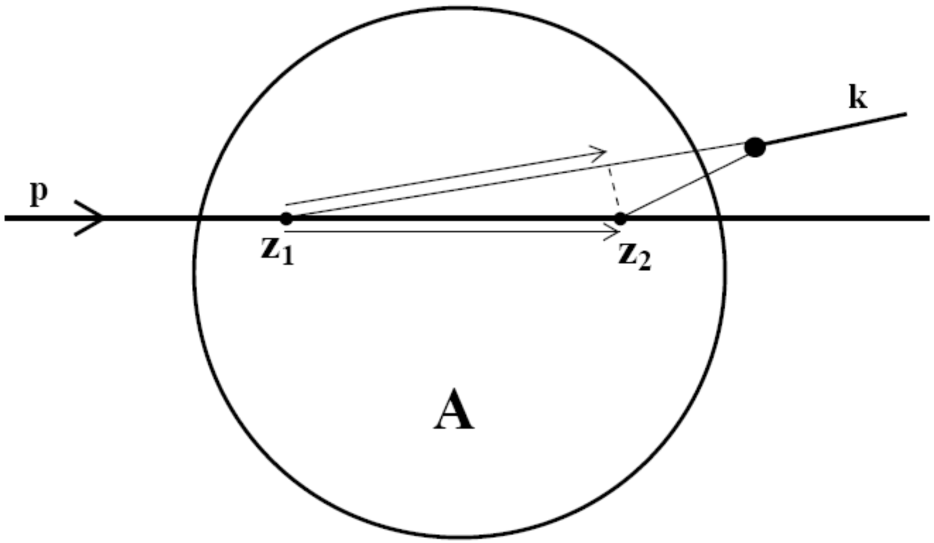
$$\kappa_2(\mathbf{k}) \sim \frac{1}{k_-}$$

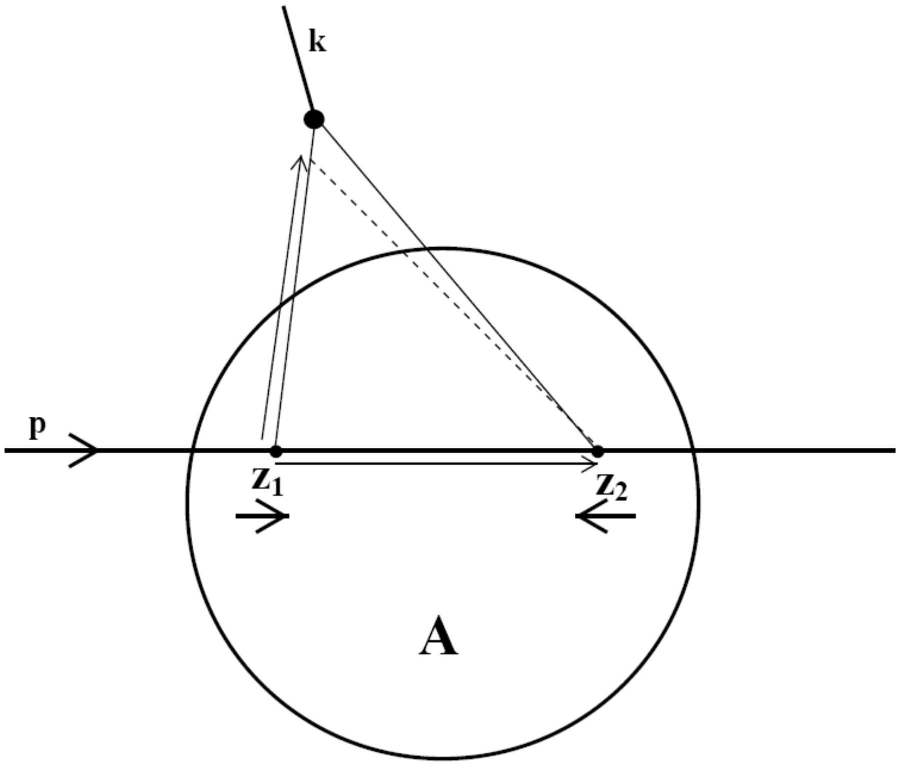
$$\theta \simeq 0^\circ \quad k_- \to 0$$
$$\theta \simeq 90^\circ \quad k_- \simeq |\mathbf{k}|$$
$$\theta \simeq 180^\circ \quad k_- \simeq 2|\mathbf{k}|$$

- increase of the Coalescence Coefficient

$$z_1 \longrightarrow z_2$$

Physical interpretation in the laboratory frame





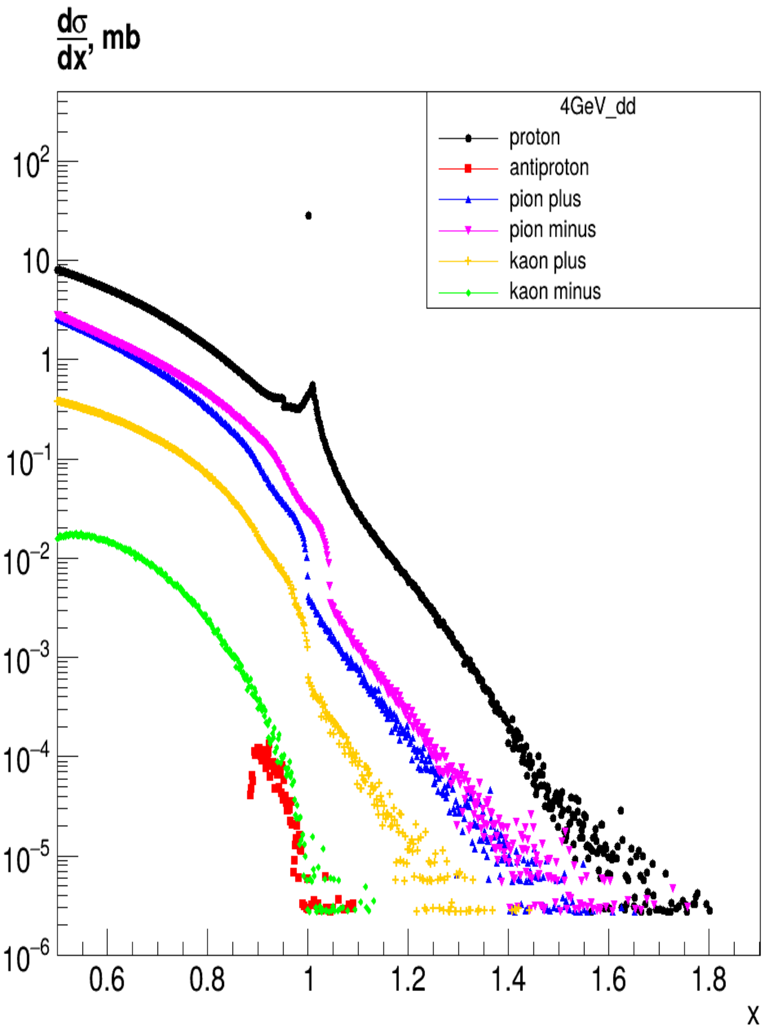
Braun M.A., Vechernin V.V., Yad.Fiz. 47 (1988) 1452; J. Phys. G 16 (1990) 1615.

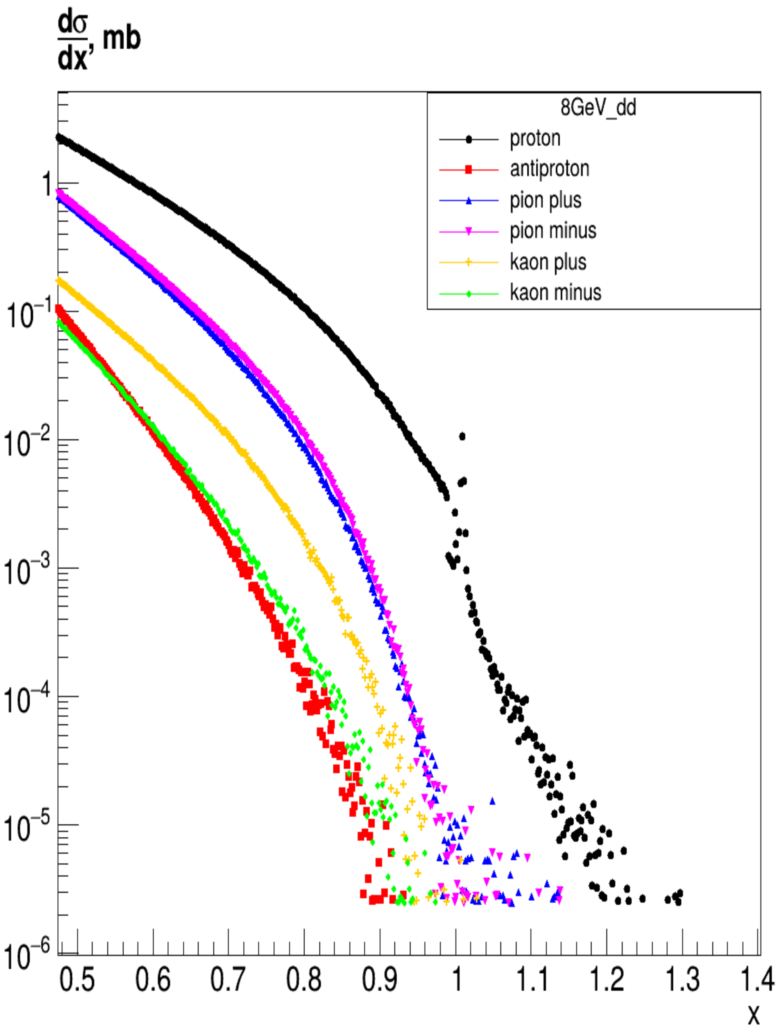
Gavrilov V.B., Kornienko N.L., Leksin G.A., Semenov S.V., Sov. J. Nucl. Phys. 41 (1985) 540;

Preprint ITEP-69 Moscow, 1985.

MC simulations of the particle production in cumulative region in dd collisions
(to study interaction of multiquark fluctons)

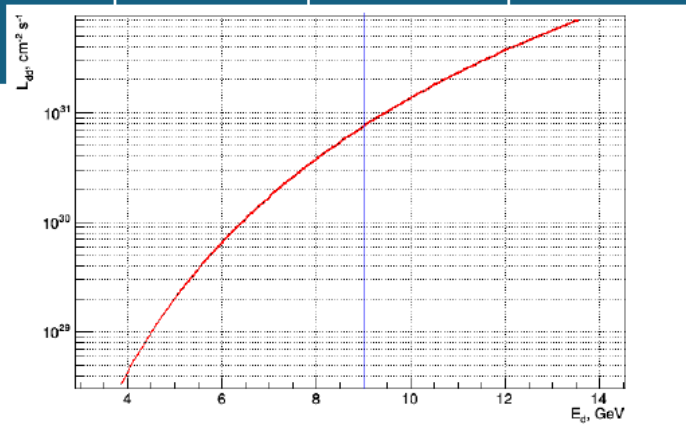
All particles in dd at 4 and 8 GeV



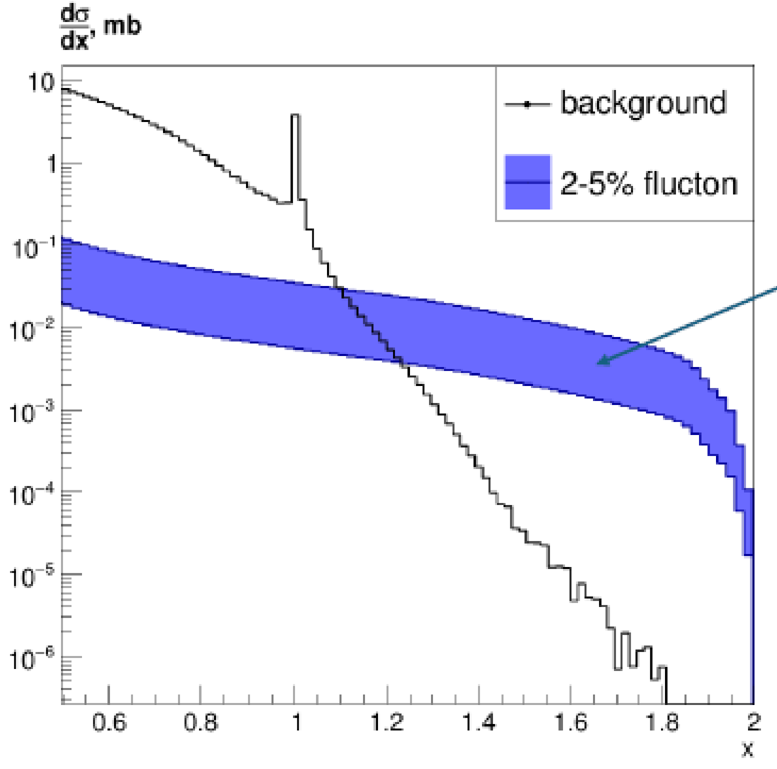


Yields of particles: dd with $x > 1$

	$\sqrt{S_{NN}} = 4\ GeV$			$\sqrt{S_{NN}} = 6\ GeV$			$\sqrt{S_{NN}} = 8\ GeV$		
Particle type	**Per sec**	**Per day**	**Per month**	**Per sec**	**Per day**	**Per month**	**Per sec**	**Per day**	**Per month**
p	$2.9 \cdot 10^{0}$	$2.5 \cdot 10^{5}$	$7.5 \cdot 10^{6}$	$1.2 \cdot 10^{0}$	$1 \cdot 10^{5}$	$3.1 \cdot 10^{6}$	$2.9 \cdot 10^{-1}$	$2.5 \cdot 10^{4}$	$7.5 \cdot 10^{5}$
π^{+}	$9.1 \cdot 10^{-3}$	$7.9 \cdot 10^{2}$	$2.4 \cdot 10^{4}$	$1.8 \cdot 10^{-3}$	$1.5 \cdot 10^{2}$	$4.6 \cdot 10^{3}$	$1.2 \cdot 10^{-3}$	$1.1 \cdot 10^{2}$	$3.2 \cdot 10^{3}$
π^{-}	$4.1 \cdot 10^{-2}$	$3.6 \cdot 10^{3}$	$1.1 \cdot 10^{5}$	$2.5 \cdot 10^{-3}$	$2.2 \cdot 10^{2}$	$6.5 \cdot 10^{3}$	$7 \cdot 10^{-4}$	$6 \cdot 10^{1}$	$1.8 \cdot 10^{3}$
Number of collisions	$5 \cdot 10^{3}$	$4.3 \cdot 10^{8}$	$1.3 \cdot 10^{10}$	$7.5 \cdot 10^{4}$	$6.5 \cdot 10^{9}$	$2 \cdot 10^{11}$	$4.2 \cdot 10^{5}$	$3.6 \cdot 10^{10}$	$1.1 \cdot 10^{12}$



Protons in dd collisions at $\sqrt{s_{NN}} = 4$ GeV



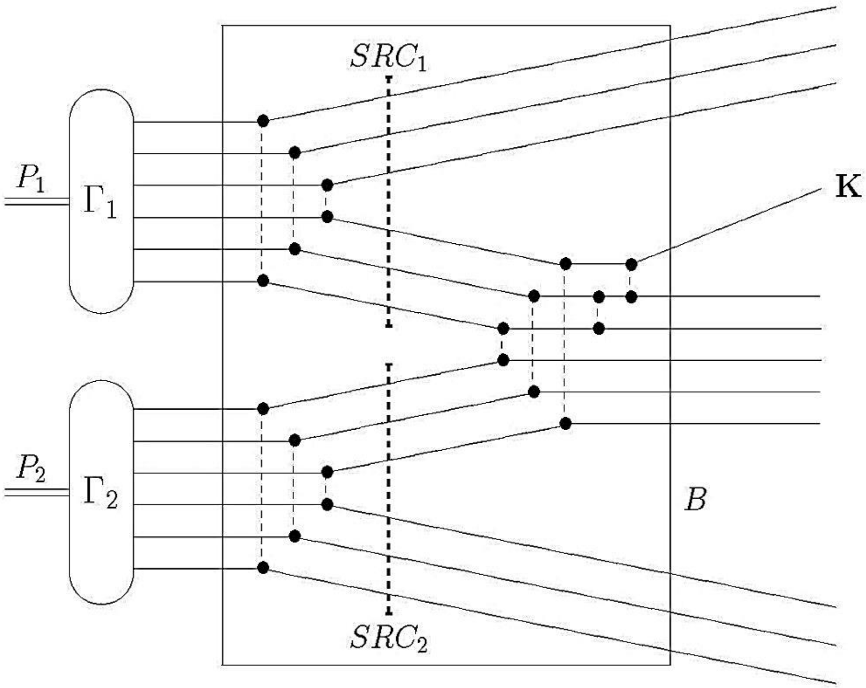
Flucton-flucton contribution is modeled as pp collisions at doubled energy $\sqrt{s_{NN}} = 8$ GeV

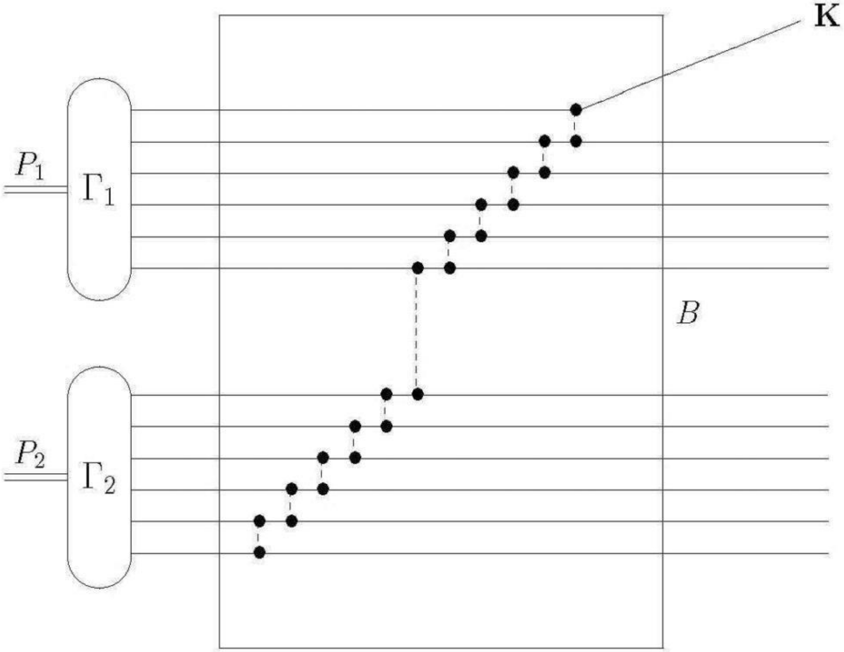
$(0.02)^2 = 4\ 10^{-4}$

$(0.05)^2 = 2.5\ 10^{-3}$

Correspondence between the SRC approach and the diagrammatic picture of multiquark fluctons

For the production of pions





$$r_N \sim r_f \gg r_B \sim \frac{1}{\sqrt{s}}$$

$\rightarrow$ QCR

V.Vechernin, S.Yurchenko,
Int. J. Mod. Phys. E 33, 2441022 (2024)
S.Yurchenko, V.Vechernin,
Phys. Atom. Nucl. 88, 349 (2025)

Contribution of Fermi motion does not decrease with increasing initial collision energy

Weakly coupled systems:

$$M = n(m - \varepsilon)$$

Small parameter:

$$\alpha = \sqrt{\frac{\varepsilon}{m}} \simeq \frac{1}{10} \triangleright \frac{1}{20}$$

$$Q_\perp^{*(i)} = Q_\perp^{(i)} \simeq \sqrt{m\varepsilon} = m\,\alpha$$

$$Q_z^{*(i)} = \gamma\, Q_z^{(i)} \simeq \gamma\,\sqrt{m\varepsilon} = E_p\,\alpha$$

$$\gamma = \frac{E_p^*}{m}$$

$$E_p^* = \sqrt{\mathbf{p}^{*2} + m^2}$$

$$\mathbf{P}^{*(i)} = \mathbf{p}^* + \mathbf{Q}^{*(i)}$$

$$E_p = 7000\ GeV \qquad Q_\perp^{*(i)} = Q_\perp^{(i)} \simeq 50 \triangleright 90\ MeV$$

$$Q_z^{*(i)} \simeq 350 \triangleright 700\ GeV\ \ (!)$$

The compression of the wave function of the nucleus in coordinate space corresponds to its expansion in momentum space.