Leptonic angular coefficients in J/ψ production within Soft Gluon Resummation approach

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Particle Physics at Intermediate and High Energies, IHEP, Protvino
June 4, 2026

Outline

- Factorisation framework:
  - TMD factorisation, Soft Gluon Resummation approach, Boer-Mulders PDFs
  - InEW, matching of TMD and fixed-order (CPM) calculations
- Hadronisation model: NRQCD
- Polarised J/ψ production & leptonic angular coefficients within NRQCD
- Results of calculations:
  - Unpolarised J/ψ production with up-to-date PDFs
  - Leptonic angular coefficients, PHENIX & HERA-B data
  - Predictions for SPD NICA

Introduction

- J/ψ production as a tool to study gluon PDF in proton
  - In the small-p_T region, TMD PM and TMD PDFs
  - In the large-p_T region, CPM and Collinear PDFs
- Hadronisation approaches: NRQCD and ICEM
- V. Saleev and K. Shilyaev, «Production of S-wave charmonia in the Soft Gluon Resummation approach using the NRQCD», Mod. Phys. Lett. A 40 (2025) 32, 2550145.
- V. Saleev and K. Shilyaev, «Prompt production of J/ψ in the soft gluon resummation approach using the ICEM», Mod. Phys. Lett. A [hep-ph: 2601.22817] (2026).
- V. Saleev and K. Shilyaev, «Small-p_T production of η_c mesons within the Soft Gluon Resummation approach», Phys. Atom. Nucl. 88 (2025) 2, 338-341.

Current status of studies

- Previous tasks:
  - leading contributions in TMD factorisation — unpolarised partons (PDFs) within NRQCD and ICEM for J/ψ and η_c production in LL-LO approximation
  - calculation of J/ψ polarisation, agreement with PHENIX data at small-p_T
  - estimation of contribution of gluon Boer-Mulders PDFs (BM PDFs), i.e. linearly polarised gluons within protons, for J/ψ and η_c production
- Current work:
  - J/ψ production in NLL-LO approximation, including gluon BM PDFs
  - impact of BM PDFs on the leptonic angular coefficients in J/ψ production

TMD factorisation and initial parton transverse momenta

- **Transverse Momentum Dependent (TMD) factorisation**: $q_T, k_T \ll \mu_F \sim M$
- TMD parton distribution functions $f(x, \boldsymbol{q_T}, \mu_F, \zeta) \Rightarrow$ two-scale **Collins-Soper** equations:

$$\begin{cases} \dfrac{\partial \ln \hat{f}(x, \boldsymbol{b_T}, \mu_F, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu_F) & \text{with CS kernel } \tilde{K}(b_T, \mu_F) \\ \dfrac{\partial \tilde{K}(b_T, \mu)}{\partial \ln \mu} = -\gamma_K[\alpha_s(\mu_F)] & \text{with anomalous dimension } \gamma_K[\alpha_s(\mu_F)] \end{cases}$$

- Partons' momenta decomposition:

$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu \tag{1}$$

  - preserving $\mathcal{O}(q_T/M)$ terms, neglecting $\mathcal{O}(q_T^2/M^2)$ terms and, therefore, assuming $y_{1,2} \to 0$:

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2}\right), \qquad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2}\right) \tag{2}$$

- Relevant $2 \to 1$ processes for production of charmonium state C:
  - gluon-gluon fusion $g + g \to C$ and quark-antiquark annihilation $q + \bar{q} \to C$

TMD factorisation and TMD PDFs

- General formula of TMD factorisation [TMD Handbook, arXiv:2304.03302]:

$$d\sigma = \frac{(2\pi)^4}{2s}\frac{d^3p}{(2\pi)^3\,2p^0}\int dx_1\,dx_2\,d\boldsymbol{q}_{1T}\,d\boldsymbol{q}_{2T}\,\delta^{(4)}(q_1+q_2-p)\,\Phi_g^{\mu\nu}(x_1,\mathbf{q}_{1T})\Phi_g^{\rho\sigma}(x_2,\mathbf{q}_{2T})\overline{\mathcal{M}_{\mu\rho}\mathcal{M}^*_{\nu\sigma}} \tag{3}$$

with $\Phi_g^{\mu\nu}(x,\mathbf{q}_T)$ being the correlator of gluon field strengths in leading approximation w.r.t. kinematic power corrections:

$$\Phi_g^{\mu\nu}(x,\mathbf{q}_T) = -\frac{1}{2x}\left[g_T^{\mu\nu}f_1^g(x,\mathbf{q}_T) - \left(\frac{q_T^\mu q_T^\nu}{M_h^2} + g_T^{\mu\nu}\frac{\mathbf{q}_T^2}{2M_h^2}\right)h_1^{\perp g}(x,\mathbf{q}_T)\right] \tag{4}$$

- Differential cross section after simplification:

$$\frac{d\sigma}{dy dp_T} = \frac{2\pi^2 p_T}{s^2}\int d\boldsymbol{q}_{1T}\,d\boldsymbol{q}_{2T}\,\delta^{(2)}(\boldsymbol{q}_{1T}+\boldsymbol{q}_{2T}-\boldsymbol{p}_T)\Phi_g^{\mu\nu}\Phi_g^{\rho\sigma}\overline{\mathcal{M}_{\mu\rho}\mathcal{M}^*_{\nu\sigma}} \tag{5}$$

- PDF convolutions over transverse momenta:

$$\mathcal{C}[wff] = \int d\boldsymbol{q}_{1T}\,d\boldsymbol{q}_{2T}\,\delta^{(2)}(\boldsymbol{q}_{1T}+\boldsymbol{q}_{2T}-\boldsymbol{p}_T)\,w(\boldsymbol{q}_{1T},\boldsymbol{q}_{2T},\boldsymbol{p}_T)\,f(x_1,\boldsymbol{q}_{1T})\,f(x_2,\boldsymbol{q}_{2T}) \tag{6}$$

  - the form of $w(\boldsymbol{q}_{1T},\boldsymbol{q}_{2T},\boldsymbol{p}_T)$ is defined by PDF weight in $\Phi_g^{\mu\nu}(x,\mathbf{q}_T)$ and structure of $\mathcal{M}_{\mu\rho}$
  - different convolutions contribute variously to final angular distribution

Soft Gluon Resummation approach, perturbative evolution

- To implement **Collins-Soper** evolution, the transfer to impact parameter $\boldsymbol{b_T}$ space by 2D Fourier transform is done
- PDF convolutions in $\boldsymbol{b}_T$-space:

$$\mathcal{C}[wff] = \int d\boldsymbol{q}_{1T}\, d\boldsymbol{q}_{2T}\, \delta^{(2)}(\boldsymbol{q}_{1T} + \boldsymbol{q}_{2T} - \boldsymbol{p}_T)\, w(\boldsymbol{q}_{1T}, \boldsymbol{q}_{2T}, \boldsymbol{p}_T)\, f(x_1, \boldsymbol{q}_{1T}, \mu, \zeta)\, f(x_2, \boldsymbol{q}_{2T}, \mu, \zeta) = \\ = \frac{1}{2\pi} \int db_T\, b_T\, J_n(p_T b_T) \hat{f}(x_1, \boldsymbol{b}_T, \mu, \zeta) \hat{f}(x_2, \boldsymbol{b}_T, \mu, \zeta) \tag{7}$$

- Soft and collinear gluon resummation leads to **perturbative** Sudakov factor $S_P(\mu, \mu_b, b_T)$ [J. Collins, D. Soper (1981)]:

$$\hat{f}(x_1, \boldsymbol{b}_T, \mu, \zeta)\, \hat{f}(x_2, \boldsymbol{b}_T, \mu, \zeta) = e^{-S_P(\mu, \mu_b, b_T)} \hat{f}(x_1, \boldsymbol{b}_T, \mu_b, \mu_b^2) \hat{f}(x_2, \boldsymbol{b}_T.\mu_b, \mu_b^2) \tag{8}$$

- Perturbative calculation of S_P terms:

$$S_P(\mu, \mu_b, b_T) = \int_{\mu_b^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[A(\mu') \ln \frac{\mu^2}{\mu'^2} + B(\mu') \right], \qquad A(\mu') = \sum_{n=1}^{\infty} A^{(n)} \left(\frac{\alpha_s(\mu')}{\pi} \right)^n, \qquad B(\mu') = \sum_{n=1}^{\infty} B^{(n)} \left(\frac{\alpha_s(\mu')}{\pi} \right)^n \tag{9}$$

  - $A^{(1)}$, $A^{(2)}$ and $B^{(1)}$ refer to NLL-LO approximation

Soft Gluon Resummation approach, nonperturbative content

- Soft gluon resummation formula for any PDF convolution:

$$\mathcal{C}[wff] = \frac{1}{2\pi}\int db_T\, b_T\, J_n(p_T b_T)\hat{f}(x_1,\boldsymbol{b}_T)\hat{f}(x_2,\boldsymbol{b}_T) =$$
$$= \frac{1}{2\pi}\int db_T\, b_T\, J_n(p_T b_T)\, e^{-S_P(\mu,\mu'_{b^*},b_T^*)} e^{-S_{NP}(b_T,\mu)}\hat{f}(x_1,\mu'_{b^*},b_T^*)\hat{f}(x_2,\mu'_{b^*},b_T^*) \quad (10)$$

  - scale prescription $\mu'_b = \mu b_0/(\mu b_T + b_0)$ and impact parameter cut-off $b_T^* = b_T/\sqrt{1+(b_T/b_{T,\mathbf{max}})^2}$

- **Nonperturbative** quark factor obtained in SIDIS data fitting [S. Aybat, T. Rogers (2011)]:

$$S_{NP}(b_T,\mu) = \left[g_1 \ln\frac{\mu}{2Q_{NP}} + g_2\left(1 + 2g_3 \ln\frac{10xx_0}{x_0+x}\right)\right] b_T^2 \quad (11)$$

  - Casimir-scaled by C_A/C_F for initial gluons, Casimir-scaling is preserved up to $\mathcal{O}(\alpha_s^3)$

- In LO, the perturbative tails of TMD PDFs are expressed with collinear PDF [P. Sun, B.-W. Xiao, F. Yuan (2011)]:

$$\hat{f}_1^g(x,\mu'_{b^*},b_T^*) = f(x,\mu'_{b^*}) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T\Lambda_{\mathsf{QCD}}) \quad (12)$$

$$\hat{h}_1^{\perp g}(x,\mu'_{b^*},b_T^*) = -\frac{\alpha_s(\mu'_{b^*})}{\pi}\int\frac{dx'}{x'}\left(\frac{x'}{x}-1\right)\left[C_A f^g(x,\mu'_{b^*}) + C_F\sum_{i=q,\bar{q}} f^i(x,\mu'_{b^*})\right] + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T\Lambda) \quad (13)$$

Matching of small-p_T and high-p_T regions within Inverse-Error Weighting Scheme

- Matched cross-section as a weighted sum of CPM and TMD terms
  [M. Echevarria, T. Kasemets, J.-P. Lansberg, C. Pisano, A. Signori (2018)]:

$$d\sigma = \mathcal{W}\, d\sigma^{\text{TMD}} + \mathcal{Z}\, d\sigma^{\text{CPM}} \tag{14}$$

  - $d\sigma$ approximates TMD-term at small-p_T and CPM-term at high-p_T
  - intermediate p_T-region is covered with sum of TMD- and CPM-terms weighted with their power corrections

- Normalised weights for each of the two terms:

$$\mathcal{W} = \frac{\Delta\mathcal{W}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}, \qquad \mathcal{Z} = \frac{\Delta\mathcal{Z}^{-2}}{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}$$

$$\Delta\mathcal{W} = \left(\frac{p_T}{Q}\right)^2 + \left(\frac{m}{Q}\right)^2, \qquad \Delta\mathcal{Z} = \left(\frac{m}{p_T}\right)^2 \left(1 + \ln^2 \frac{\sqrt{Q^2 + p_T^2}}{p_T}\right) \tag{15}$$

- Uncertainty due to the matching procedure:

$$\Delta d\sigma = \frac{d\sigma}{\sqrt{\Delta\mathcal{W}^{-2} + \Delta\mathcal{Z}^{-2}}} = \frac{\Delta\mathcal{W} \cdot \Delta\mathcal{Z}}{\sqrt{\Delta\mathcal{W}^2 + \Delta\mathcal{Z}^2}} d\sigma \tag{16}$$

Hadronisation model: NRQCD

- J/ψ wave function as a series w.r.t. relative constituent quarks velocity v (approximate scaling, $v^2 \approx 0.2$):

$$|J/\psi\rangle = \mathcal{O}(v^0)\,|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)\,|c\bar{c}[^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^1S_0^{(8)}]g\rangle + \dots$$

- Hard cross section factorisation:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_n d\hat{\sigma}(ab \to c\bar{c}[n]X)\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$$

- Long-distance matrix elements (LDME) $\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$: phenomenological potential models, **experimental data fitting**
- Amplitude of J/ψ production projected onto necessary spin state: $\mathcal{M}_{\mu\rho} = \mathcal{M}_{\mu\rho\alpha}\varepsilon^{\alpha}(S_z)$
- Unpolarised J/ψ production within TMD factorisation:

$$\frac{d\sigma}{dy dp_T} = \frac{2\pi^2 p_T}{s^2}\Bigg[\frac{5\pi^2\alpha_s^2}{12M}\langle\mathcal{O}^{J/\psi}[^1S_0^{(8)}]\rangle\left(\mathcal{C}[f_1^g f_1^g] - \mathcal{C}[w_2\, h_1^{\perp g} h_1^{\perp g}]\right) + \\ + \frac{5\pi^2\alpha_s^2}{M^3}\langle\mathcal{O}^{J/\psi}[^3P_0^{(8)}]\rangle\left(\mathcal{C}[f_1^g f_1^g] + \mathcal{C}[w_2\, h_1^{\perp g} h_1^{\perp g}]\right) + \\ + \frac{4\pi^2\alpha_s^2}{3M^3}\langle\mathcal{O}^{J/\psi}[^3P_2^{(8)}]\rangle\left(\mathcal{C}[f_1^g f_1^g]\right)\Bigg] \tag{17}$$

Leptonic angular coefficients, NRQCD

- Angular distribution of leptonic J/ψ-decay:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{3+\lambda}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\varphi+\frac{\nu}{2}\sin^2\theta\cos 2\varphi\right] \tag{18}$$

  - in a rest frame of J/ψ: Collins-Soper (CS), Gottfried-Jackson (GJ), Helicity (HX) frames

- Leptonic angular coefficients related to spin states of J/ψ:

$$\lambda = \frac{d\sigma - 3\,d\sigma_{00}}{d\sigma + d\sigma_{00}}, \qquad \mu = \frac{\sqrt{2}\,\mathrm{Re}\,d\sigma_{10}}{d\sigma + d\sigma_{00}}, \qquad \nu = \frac{2\,d\sigma_{1-1}}{d\sigma + d\sigma_{00}} \tag{19}$$

- Calculation of cross sections and interferences as projections onto necessary spin states

$$\frac{d\sigma_{ij}}{dy\,dp_T} = \frac{2\pi^2 p_T}{s^2}\int d\boldsymbol{q}_{1T}\,d\boldsymbol{q}_{2T}\,\delta^{(2)}(\boldsymbol{q}_{1T}+\boldsymbol{q}_{2T}-\boldsymbol{p}_T)\,\Phi_g^{\mu\nu}\,\Phi_g^{\rho\sigma}\,\overline{\mathcal{M}_{\mu\rho\alpha}\mathcal{M}^*_{\nu\sigma\beta}}\,\varepsilon^{\alpha}(S_z=i)\,\varepsilon^{*\beta}(S_z=j) \tag{20}$$

Leptonic angular coefficients, NRQCD

- Angular distribution of leptonic J/ψ-decay:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{3+\lambda}\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\varphi+\frac{\nu}{2}\sin^2\theta\cos 2\varphi\right] \tag{21}$$

- Expressions for terms in J/ψ angular coefficients, i.e longitudinally polarised J/ψ, ν (double spin-flip), μ (spin-flip):
  - longitudinally polarised J/ψ cross section

$$\frac{d\sigma_{00}}{dy dp_T} = \frac{2\pi^2 p_T}{s^2}\Bigg[\frac{1}{3}\cdot\frac{5\pi^2\alpha_s^2}{12M}\langle\mathcal{O}^{J/\psi}[{}^1S_0^{(8)}]\rangle\left(\mathcal{C}[f_1^g f_1^g]-\mathcal{C}[w_2\, h_1^{\perp g} h_1^{\perp g}]\right)+ \tag{22}$$

$$+\frac{1}{3}\cdot\frac{5\pi^2\alpha_s^2}{M^3}\langle\mathcal{O}^{J/\psi}[{}^3P_0^{(8)}]\rangle\left(\mathcal{C}[f_1^g f_1^g]+\mathcal{C}[w_2\, h_1^{\perp g} h_1^{\perp g}]\right)\Bigg]$$

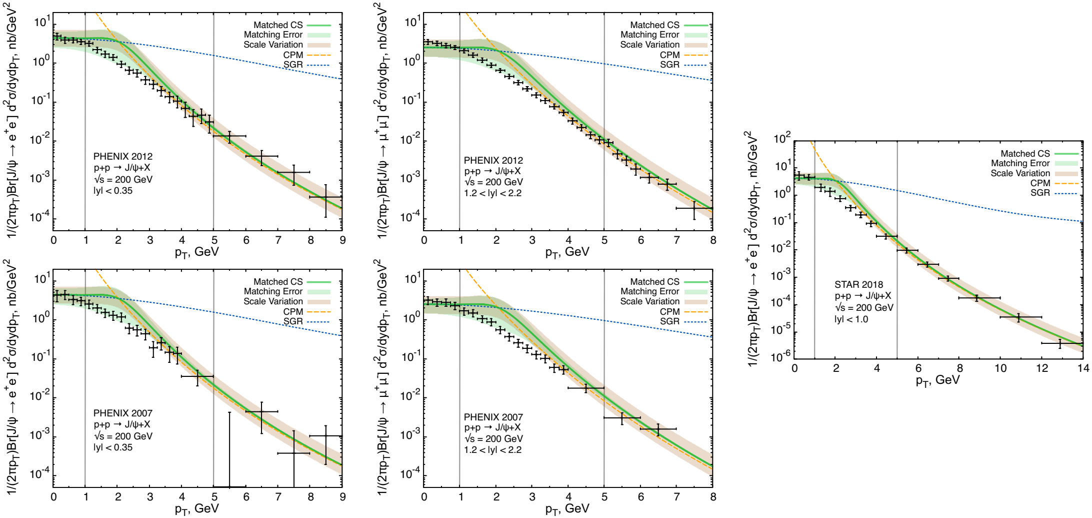
  - double spin-flip term which contributes to ν coefficient

$$\frac{d\sigma_{1-1}}{dy dp_T} = \frac{2\pi^2 p_T}{s^2}\left[-\frac{10\pi^2\alpha_s^2}{3M^3}\langle\mathcal{O}^{J/\psi}[{}^3P_0^{(8)}]\rangle\left(\mathcal{C}[w_{31} h_1^{\perp g} f_1^g]+\mathcal{C}[w_{32} f_1^g h_1^{\perp g}]\right)\right] \tag{23}$$

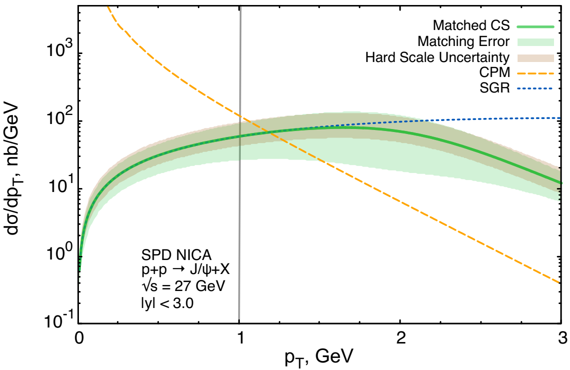
  - spin-flip term which contributes to μ coefficient

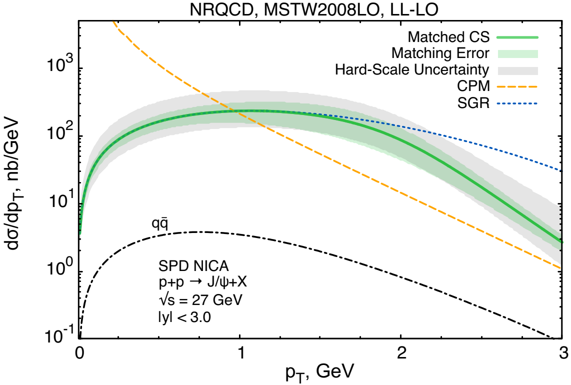
$$\frac{d\sigma_{10}}{dy dp_T} = 0 \tag{24}$$

Unpolarised J/ψ production at $\sqrt{s} = 200$ GeV



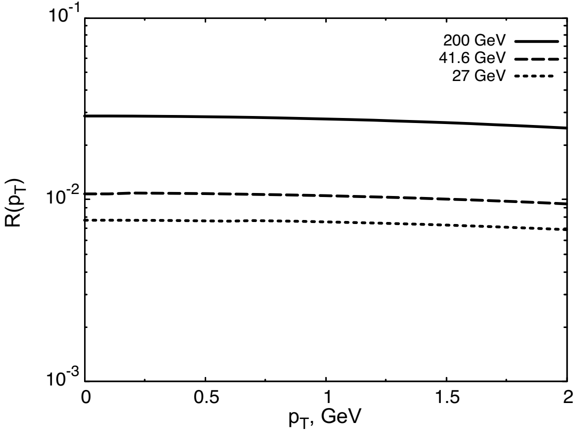
Prediction for unpolarised J/ψ production at SPD NICA



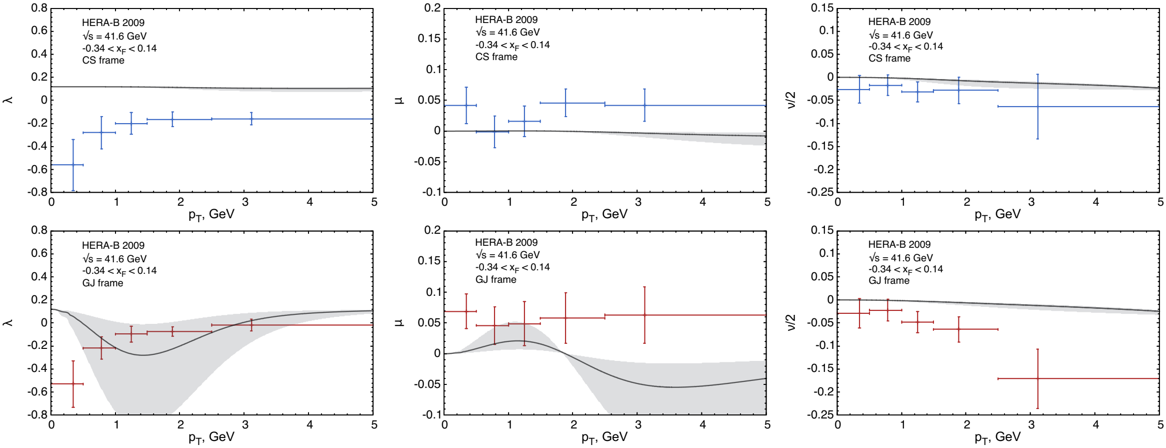


Estimation of Boer-Mulders contribution

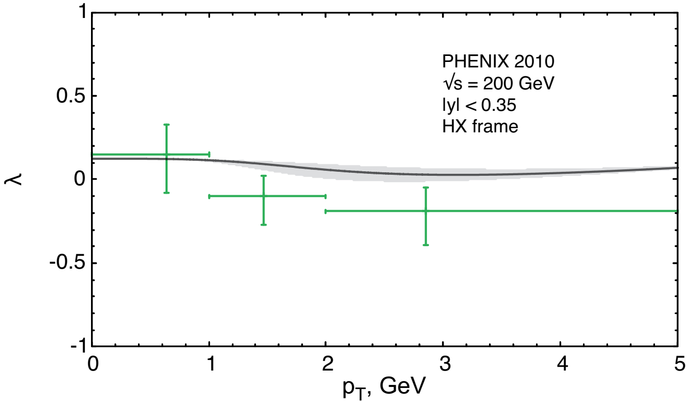
$$R(p_T) = \frac{\mathcal{C}[w_2\, h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$$



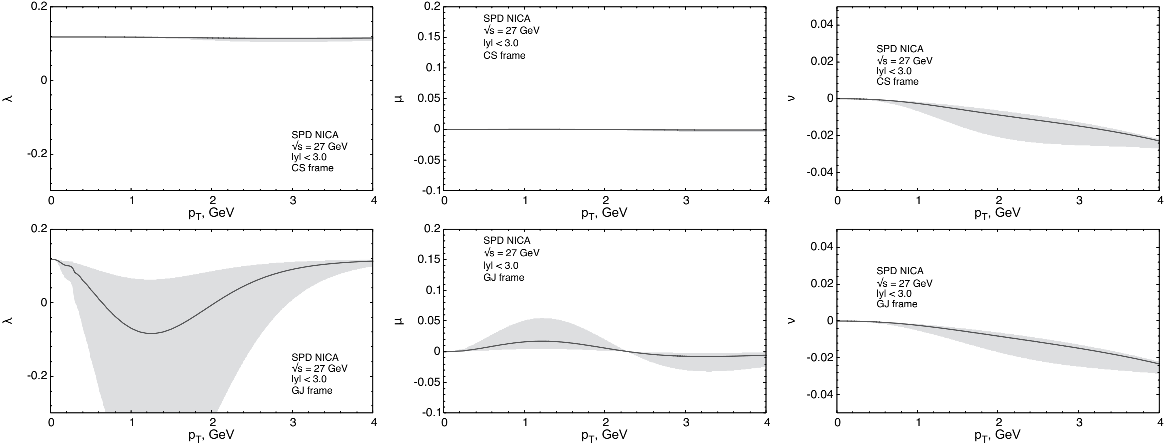
Leptonic angular coefficients, HERA-B data



Leptonic angular coefficients, PHENIX data



Leptonic angular coefficients, SPD NICA predictions



Summary

- We used the Soft Gluon Resummation approach to calculate small-p_T unpolarised J/ψ production in the TMD factorisation
- We found agreement between our preductions for unpolarised J/ψ production at SPD NICA with our previous NRQCD & ICEM calculations
- We estimate decreasing contribution of BM PDFs with decrease of $\sqrt{s}$
- Quite difficult BM PDFs estimations on basis of unpolarised cross sections at relatively small energies $\sqrt{s}$
- BM PDFs (only in convolution with unpolarised PDF) defines angular coefficient ν which could be of sufficient magnitude even at small $\sqrt{s}$

THANK YOU FOR ATTENTION!

Backup slides. Perturbative Sudakov factor

- Perturbative Sudakov factor:

$$S_P(\mu, \mu_b, b_T) = \int_{\mu_b^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \left[A(\mu') \ln \frac{\mu^2}{\mu'^2} + B(\mu') \right]$$

- Coefficients A and B are calculated perturbatively:

$$A(\mu') = \sum_{n=1}^{\infty} A^{(n)} \left(\frac{\alpha_s(\mu')}{\pi} \right)^n \qquad B(\mu') = \sum_{n=1}^{\infty} B^{(n)} \left(\frac{\alpha_s(\mu')}{\pi} \right)^n$$

- First perturbative coefficients are as follows [P. Sun, C.-P. Yuan, F. Yuan (2013)]:

$$A^{(1)} = C_A$$

$$A^{(2)} = \frac{C_A}{2} \left[\left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{5}{9} N_f \right]$$

$$B^{(1)} = -\frac{11 C_A - 2 N_f}{6} - \frac{C_A}{2} \delta_{c8}$$

Backup slides. LDME fit results

CO LDME	Our fit LL-LO, MSTW2008	Our fit NLL-LO, NNPDF4.0	LO CPM [Cho, Leibovich (1996)]	NLO CPM, global fit [Butenschön, Kniehl (2011)]
$\langle\mathcal{O}^{J/\psi}[^1S_0^{(8)}]\rangle$, GeV^3	$(9.7 \pm 0.5) \cdot 10^{-2}$	$(4.0 \pm 0.2) \cdot 10^{-1}$	–	$(3.0 \pm 0.4) \cdot 10^{-2}$
$\langle\mathcal{O}^{J/\psi}[^3P_0^{(8)}]\rangle$, GeV^5	$(1.3 \pm 0.2) \cdot 10^{-2}$	$(4.8 \pm 0.6) \cdot 10^{-2}$	–	$(-9.1 \pm 1.6) \cdot 10^{-3}$
$M_3^{J/\psi}$, GeV^3	$(1.1 \pm 0.1) \cdot 10^{-1}$	$(4.6 \pm 0.3) \cdot 10^{-1}$	$(6.6 \pm 1.5) \cdot 10^{-2}$	$(1.8 \pm 0.6) \cdot 10^{-2}$
$\langle\mathcal{O}^{J/\psi}[^3S_1^{(8)}]\rangle$, GeV^3	$(2.0 \pm 1.6) \cdot 10^{-3}$	$(2.0 \pm 0.6) \cdot 10^{-2}$	$(6.6 \pm 2.1) \cdot 10^{-3}$	$(1.7 \pm 0.5) \cdot 10^{-3}$
$\langle\mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}]\rangle$, GeV^3	$(8.6 \pm 2.9) \cdot 10^{-3}$	–	$(3.3 \pm 0.5) \cdot 10^{-3}$	–
χ^2/n.d.f.	0.76	0.86	0.9	3.74