ISSN 1063-7788, Physics of Atomic Nuclei, 2025, Vol. 88, No. 2, pp. 338–341. © Pleiades Publishing, Ltd., 2025.

Small-p_T Production of η_c Mesons within the Soft Gluon Resummation Approach

V. A. Saleev[1),2)*] and K. K. Shilyaev[1)]**

Received November 29, 2024; revised November 29, 2024; accepted December 15, 2024

Abstract—In this article we describe the η_c production in proton–proton collisions at small values of the transverse momentum of the charmonium. Soft gluon resummation is used as a TMD factorization approach. At small transverse momenta, the color singlet contribution dominates, allowing us to use η_c as a test of the resummation scheme for the noncollinear gluon PDF. The cross sections of η_c production at $\sqrt{s} = 27$ GeV, 115 GeV and 13 TeV are predicted.

DOI: 10.1134/S1063778825700309

1. INTRODUCTION

Transverse Momentum Dependent (TMD) factorization is an approach to represent a final state cross section as a convolution of a hard scale partonic cross section and TMD parton distribution functions (PDF) [1]. One of the main tasks for the TMD development is the extraction of the TMD PDFs and this is part of the future experimental program of the SPD NICA project, where proton–proton collisions up to energies of $\sqrt{s} = 27$ GeV are planned [2]. The production of η_c mesons is an important probe for TMD analysis and extraction of TMD PDFs, since η_c mesons are produced dominantly via the color singlet channel within the nonrelativistic QCD approach (NRQCD) [3]. That's why some additional degrees of freedom like color octet contributions are reduced in the η_c description as opposed to J/ψ production, for instance. In the current paper we apply the Soft Gluon Resummation (SGR) approach as a TMD factorization framework to η_c production at small transverse momenta.

2. SOFT GLUON RESUMMATION APPROACH

The TMD factorization is a general approach that allows to represent the final state cross section as a convolution of the hard scale partonic subprocess cross section and PDFs in a small transverse momentum domain $p_T \ll Q$, where Q is a hard scale equal to the mass M of the final observable state for heavy quarkonium production [1]. The partonic subprocess refers to the interaction of gluons or quarks from the parent hadron and it is calculated within the perturbative QCD and using some hadronization model. The TMD PDF describes the distribution of initial partons (gluons or quarks) inside a proton with respect to longitudinal and transverse components of their 4-momenta. They depend on two scales: a renormalization scale μ and a rapidity variable ζ. The TMD PDF evolution with these scales is described by the Collins–Soper equations [4]. The SGR approach is a framework for modelling the TMD PDFs and implementing their evolution [5, 6].

The momenta of the initial partons within the TMD can be written as $q_i^\mu = x_i p_i^\mu + q_{iT}^\mu$, where x_i are the longitudinal momentum fractions, p_i^μ are the momenta of the parent protons, $q_{iT}^2 = -\mathbf{q_{iT}^2}$ are transverse components of momenta. The initial partons are prescribed to be on-shell, $q_i^2 = 0$, since corrections of order $(\mathbf{q_{iT}^2}/M^2)$ are neglected here and after.

In the TMD framework, the factorized cross section within the SGR can be written as [1]:

$$d\sigma = \int dx_1\, dx_2\, d\mathbf{q_{1T}}\, d\mathbf{q_{2T}} \times F(x_1, \mathbf{q_{1T}}, \mu, \zeta_1) F(x_2, \mathbf{q_{2T}}, \mu, \zeta_2)\, d\hat{\sigma}, \quad (1)$$

where $F(x, \mathbf{q}_T)$ are TMD PDFs, $d\hat{\sigma}$ is a hard scale cross section of a partonic subprocess $2 \to 1$ which is a leading contribution for small transverse momentum spectrum.

The scale evolution of TMD PDFs can be implemented in a space of an impact parameter $\mathbf{b_T}$ after the Fourier-transform of the PDFs:

$$\hat{F}(x, \mathbf{b_T}, \mu_F, \zeta)$$

1)Samara University, Samara, Russia.
2)JINR, Dubna, Russia.
*E-mail: saleev.va@ssau.ru
**E-mail: kirill.k.shilyaev@gmail.com

$$= \int d^2 q_T \, e^{i\mathbf{q_T b_T}} F(x, \mathbf{q_T}, \mu_F, \zeta). \quad (2)$$

The Collins–Soper equations lead to a solution which realizes the evolution from the initial to the final scale in the perturbative Sudakov factor [5, 7]:

$$S_P(Q, \mu_b, b_T)$$
$$= \int\limits_{\mu_b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \left[A(\mu') \ln \frac{Q^2}{\mu'^2} + B(\mu') \right], \quad (3)$$

where $\mu^2 = \zeta = Q^2$ is a standard choice for the scales. The expansion of the coefficients A and B is performed as follows:

$$A(\mu') = \sum_{n=1}^{\infty} A^{(n)} \left(\frac{\alpha_s(\mu')}{\pi} \right)^n,$$
$$B(\mu') = \sum_{n=1}^{\infty} B^{(n)} \left(\frac{\alpha_s(\mu')}{\pi} \right)^n. \quad (4)$$

In the leading logarithmic (LL) and LO in the α_s approximations of the SGR calculation the coefficients are $A^{(1)} = C_A$ and $B^{(1)} = -(11C_A - 2N_f)/6$, where N_f is a number of flavors and $C_A = N_c$ with a number of colors N_c. The coefficients for the next orders have also been calculated, see [5]. The nonperturbative Sudakov factor S_{NP} suppresses large values of b_T, where S_P is no longer relevant, and contains information about the transverse momentum distribution of the initial partons. We take it in the following form [8]:

$$S_{NP}(b_T, Q) = \left[g_1 \ln \frac{Q}{2Q_{NP}} \right.$$
$$\left. + g_2 \left(1 + 2g_3 \ln \frac{10 x x_0}{x_0 + x} \right) \right] b_T^2 \quad (5)$$

which was extracted from the SIDIS data for initial quarks. But we use the standard color factor C_A/C_F to apply it to gluons. The PDFs within the SGR approach can be expressed with collinear PDFs convoluted with Wilson coefficient functions $C(x, b, \mu')$ [5]:

$$C(x, b, \mu') = \sum_{n=0}^{\infty} C^{(n)}(x, b, \mu') \left(\frac{\alpha_s(\mu')}{\pi} \right)^n. \quad (6)$$

In the LO approximation the SGR PDFs are reduced just to the collinear PDFs at the low initial scale: $\hat{F}(x, b_T) = f(x, \mu_b') + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$. Such expressions for the SGR PDF and its evolution are applicable in an impact parameter interval $b_0/Q \leqslant b_T \leqslant b_{T,\max}$. The lower boundary is prescribed as $\mu_b' = Qb_0/(Qb_T + b_0)$, the upper one is realized with a cutoff $b_T \to b_T^*(b_T) = b_T/\sqrt{1 + (b_T/b_{T,\max})^2}$ [6]. We take 1.5 GeV^{-1} as a $b_{T,\max}$ value [9].

Thus, the final expression for the differential cross section is written as follows

$$\frac{d^2\sigma}{dp_T dy} = \frac{\pi p_T \overline{|\mathcal{M}(2 \to 1)|^2}}{M^2 s} \mathcal{H}(Q^2, M^2)$$
$$\times \int db_T \, b_T \, J_0(p_T b_T) \, e^{-S_P} \, e^{-S_{NP}} \, \hat{F}_1 \, \hat{F}_2, \quad (7)$$

where y and p_T are rapidity and transverse momentum of charmonium, $\sqrt{s}$ is a centre-of-mass energy, J_0 is a zeroth order Bessel function of the first kind, $\hat{F}_i \equiv \hat{F}(x_i, b_T^*)$ is the Fourier-transformed parton distribution, $\mathcal{M}(2 \to 1)$ is the LO amplitude of a hard partonic subprocess and $\mathcal{H}(Q^2, M^2)$ is the hard part including high-order virtual corrections.

3. NONRELATIVISTIC QCD

We use the conventional approach of the nonrelativistic QCD (NRQCD) to describe of a bound state $\mathcal{C}$ formation from a produced $c\bar{c}$-quark pair [3]. The NRQCD allows us to expand the J/ψ wave function into a Fock state series with respect to the relative velocity of the constituent quarks v (for charmonium $v^2 \approx 0.3$). The leading term in the series refers to a color singlet term with the same set of quantum numbers as the quark pair has in an observable charmonium state. This approximation to NRQCD is called the color singlet model (CSM). Therefore, the cross section of the final state production is a sum of the Fock state production terms, and each of them is factorized into the cross section of the quark pair production in that Fock state (it is denoted $[n]$ below) and a corresponding long-distance matrix element $\langle \mathcal{O}^{\mathcal{C}}[n] \rangle$ (LDME) [10]:

$$d\hat{\sigma}(a + b \to \mathcal{C} + X)$$
$$= \sum_n d\hat{\sigma}(a + b \to c\bar{c}[n] + X)$$
$$\times \langle \mathcal{O}^{\mathcal{C}}[n] \rangle / (N_{\text{col}} N_{\text{pol}}), \quad (8)$$

where N_{col} and N_{pol} are for averaging over polarization and color states.

The quark pair production cross section is calculated within the perturbative QCD: an amplitude with the cut quark lines is projected onto states with considered momentum, spin and color quantum numbers. The color singlet LDMEs are calculated within heavy quarkonium potential models or with experimental data on quarkonium decay [11]. The octet color LDMEs have to be extracted directly from experimental data on quarkonium production after subtracting the singlet contribution.

Within the TMD factorization, the $2 \to 1$ partonic subprocesses are the main contributions at small-p_T

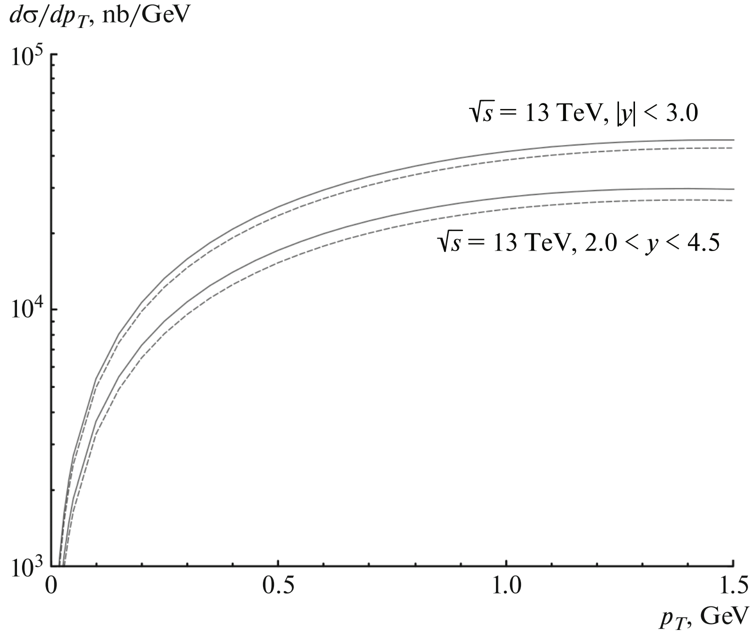

Fig. 1. Differential cross section of the η_c production at forward and mid-rapidity for the LHCb, $\sqrt{s} = 13$ TeV. Solid lines—LL-LO calculations, dashed lines—NLL-LO calculations.

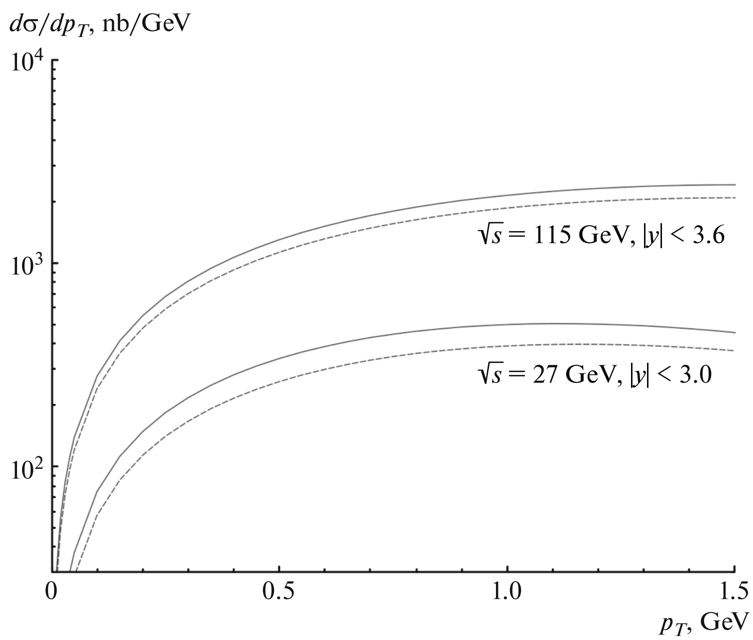

Fig. 2. Differential cross section of the η_c production for $\sqrt{s} = 27$ and 115 GeV. Solid lines—LL-LO calculations, dashed lines—NLL-LO calculations.

production. As for the Fock states, the color singlet state ${}^1S_0^{(1)}$ is considered as the leading one, its matrix element within the NRQCD is

$$\overline{|\mathcal{M}(g+g \to \eta_c[{}^1S_0^{(1)}])|^2} = \frac{2\pi^2\alpha_s^2}{9M}\langle \mathcal{O}^{\eta_c}[{}^1S_0^{(1)}]\rangle, \quad (9)$$

where M is a mass of η_c meson. Only the gluon–gluon fusion subprocesses are considered in our work because of their dominance over quark–antiquark annihilation.

4. CALCULATION RESULTS

We provide here estimations for η_c production at small p_T where the color singlet state contribution

${}^1S_0^{(1)}$ is in the leading order of the NRQCD. Therefore, in contrast to the case of J/ψ production, there is no need to search for the source of the octet LDMEs in order to fit them which is a problematic task due to the narrow fitting region $p_T \ll M$. In any case, there is no data available for the η_c production at the $p_T \ll M$. For η_c we can only evaluate the singlet contribution with a known LDME which we've obtained from the relation

$$\langle \mathcal{O}^{\eta_c}[{}^1S_0^{(1)}] \rangle = \frac{1}{3} \langle \mathcal{O}^{J/\psi}[{}^3S_1^{(1)}] \rangle \tag{10}$$

and the value of $\langle \mathcal{O}^{J/\psi}[{}^3S_1^{(1)}] \rangle = 1.3\ \text{GeV}^3$ [12]. We can only make theoretical predictions for the η_c production cross section and, perhaps, compare its general behaviour with the predictions of some other TMD approaches.

The probes for η_c in the LL-LO and NLL-LO accuracies are shown in Fig. 1 for the LHCb energy of $\sqrt{s} = 13$ TeV. The difference between the NLL and the LL calculations is about 10% which is, for example, less than the expected uncertainty due to the hard scale variation. However, detailed TMD analysis shows that the NNLL-NLO accuracy is only sufficient for a precise η_c description [13]. That's why the NLO calculations and comparison with the LO accuracy should also be done.

Figure 2 shows predictions for the η_c production within the SGR at $\sqrt{s} = 27$ GeV for the SPD NICA and $\sqrt{s} = 115$ GeV for the AFTER experiment. The difference between the LL and the NLL calculations is up to 25% for these rather low energies. We can also compare the AFTER prediction with the corresponding result within another TMD factorization approach—a spectator model [14]. We can see that the NLL calculation within the SGR matches quite well with the spectator model predictions [15].

FUNDING

The work was supported by the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS", grant no. 24-1-1-16-3, and by the grant of the Ministry of Science and Higher Education of the Russian Federation, no. FSSS-2025-0003.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

REFERENCES

1. J. Collins, *Foundations of Perturbative QCD*, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, Vol. 32 (Cambridge University Press, 2011). https://doi.org/10.1017/cbo9780511975592
2. A. Arbuzov, A. Bacchetta, M. Butenschoen, F. G. Celiberto, U. D'Alesio, M. Deka, I. Denisenko, M. G. Echevarria, A. Efremov, N. Ya. Ivanov, A. Guskov, A. Karpishkov, Ya. Klopot, B. A. Kniehl, A. Kotzinian, S. Kumano, J. P. Lansberg, K.-F. Liu, F. Murgia, M. Nefedov, B. Parsamyan, C. Pisano, M. Radici, A. Rymbekova, V. Saleev, A. Shipilova, Q.-T. Song, and O. Teryaev (SPD NICA Collaboration), Prog. Part. Nucl. Phys. **119**, 103858 (2021). https://doi.org/10.1016/j.ppnp.2021.103858
3. G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995). https://doi.org/10.1103/physrevd.51.1125
4. J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. B **250**, 199 (1985). https://doi.org/10.1016/0550-3213(85)90479-1
5. P. Sun, C.-P. Yuan, and F. Yuan, Phys. Rev. D **88**, 54008 (2013). https://doi.org/10.1103/physrevd.88.054008
6. D. Boer and W. J. den Dunnen, Nucl. Phys. B **886**, 421 (2014). https://doi.org/10.1016/j.nuclphysb.2014.07.006
7. J. C. Collins and D. E. Soper, Nucl. Phys. B **193**, 381 (1981). https://doi.org/10.1016/0550-3213(81)90339-4
8. S. M. Aybat and T. C. Rogers, Phys. Rev. D **83**, 114042 (2011). https://doi.org/10.1103/physrevd.83.114042
9. J. Bor and D. Boer, Phys. Rev. D **106**, 14030 (2022). https://doi.org/10.1103/physrevd.106.014030
10. P. Cho and A. K. Leibovich, Phys. Rev. D **53**, 150 (1996). https://doi.org/10.1103/physrevd.53.150
11. E. J. Eichten and Ch. Quigg, Phys. Rev. D **52**, 1726 (1995). https://doi.org/10.1103/physrevd.52.1726
12. E. Braaten, B. A. Kniehl, and J. Lee, Phys. Rev. D **62**, 94005 (2000). https://doi.org/10.1103/physrevd.62.094005
13. A. Signori, PhD Thesis (Vreje Universiteit, 2016).
14. A. Bacchetta, F. G. Celiberto, M. Radici, and P. Taels, Eur. Phys. J. C **80**, 733 (2020). https://doi.org/10.1140/epjc/s10052-020-8327-6
15. A. Bacchetta, F. G. Celiberto, M. Radici, and A. Signori, in *Proceedings of the 29th International Workshop on Deep-Inelastic Scattering and Related Subjects* (2022).

Publisher's Note. Pleiades Publishing remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

AI tools may have been used in the translation or editing of this article.