



# *“Active Role of Gluons in Multiparticle Production”*

**Е.Кокouлина, А. Кutov, V. Nikitin et al.**  
**(JINR, RF и GSTU, Gomel, RB)**

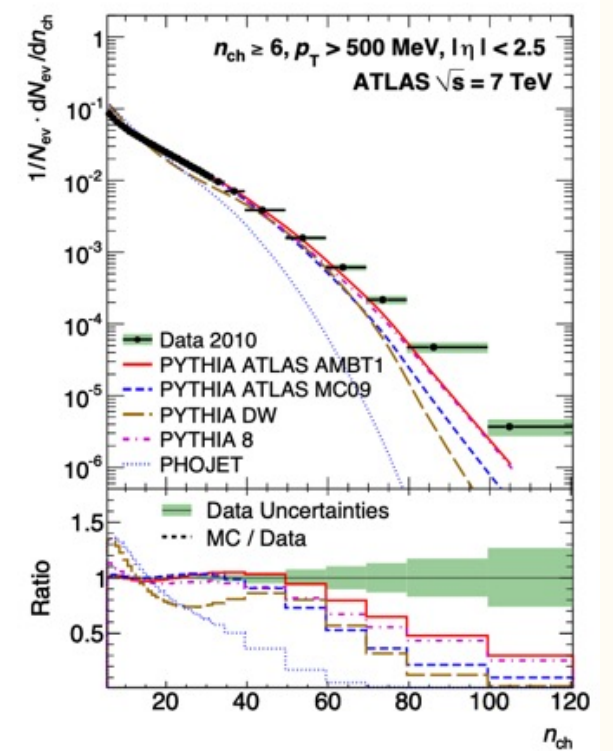
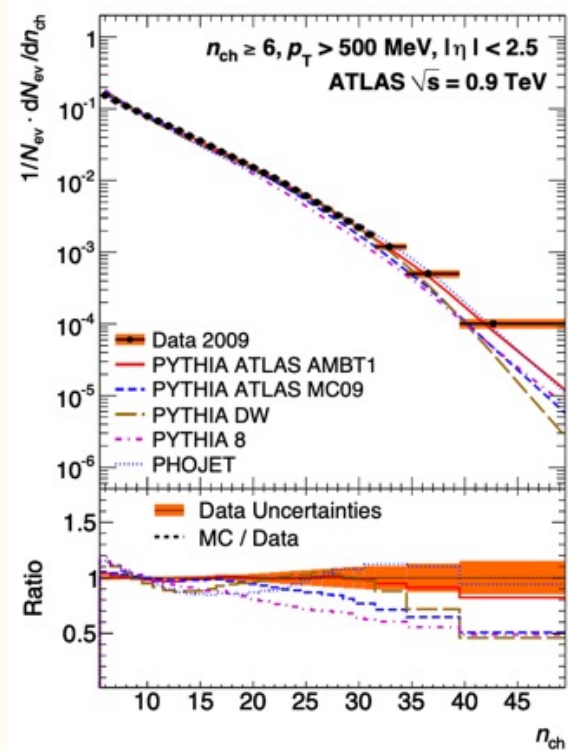
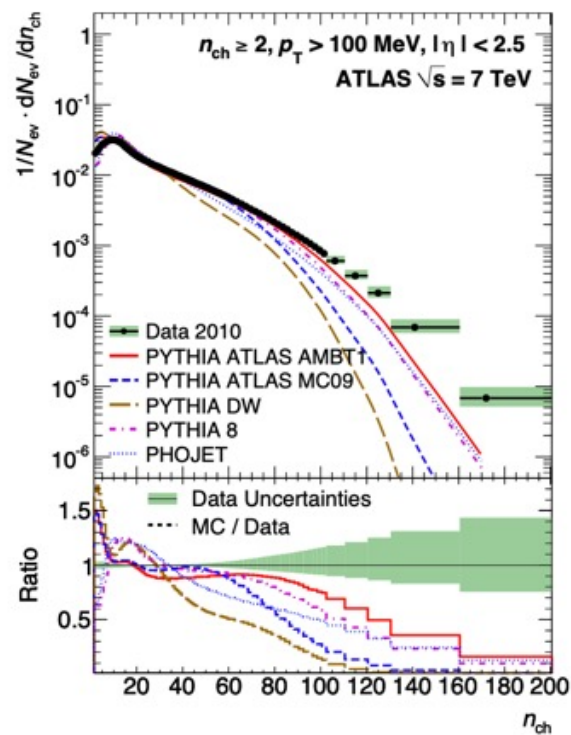
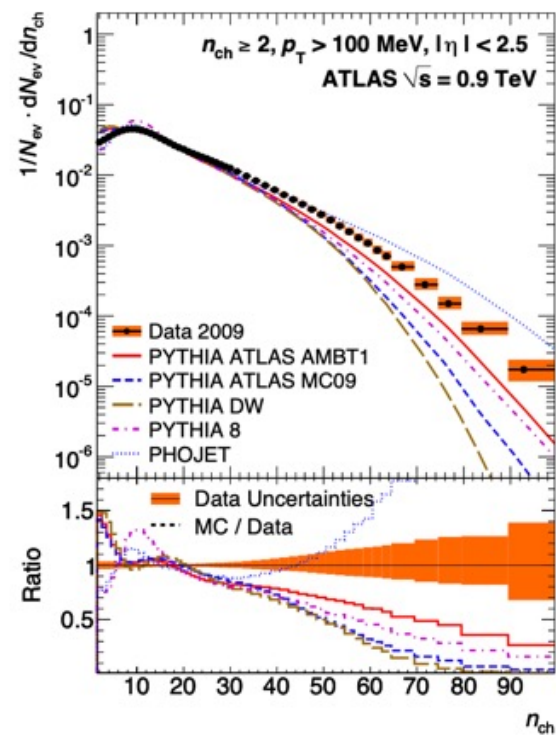
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# Multiparticle processes studying in HEP

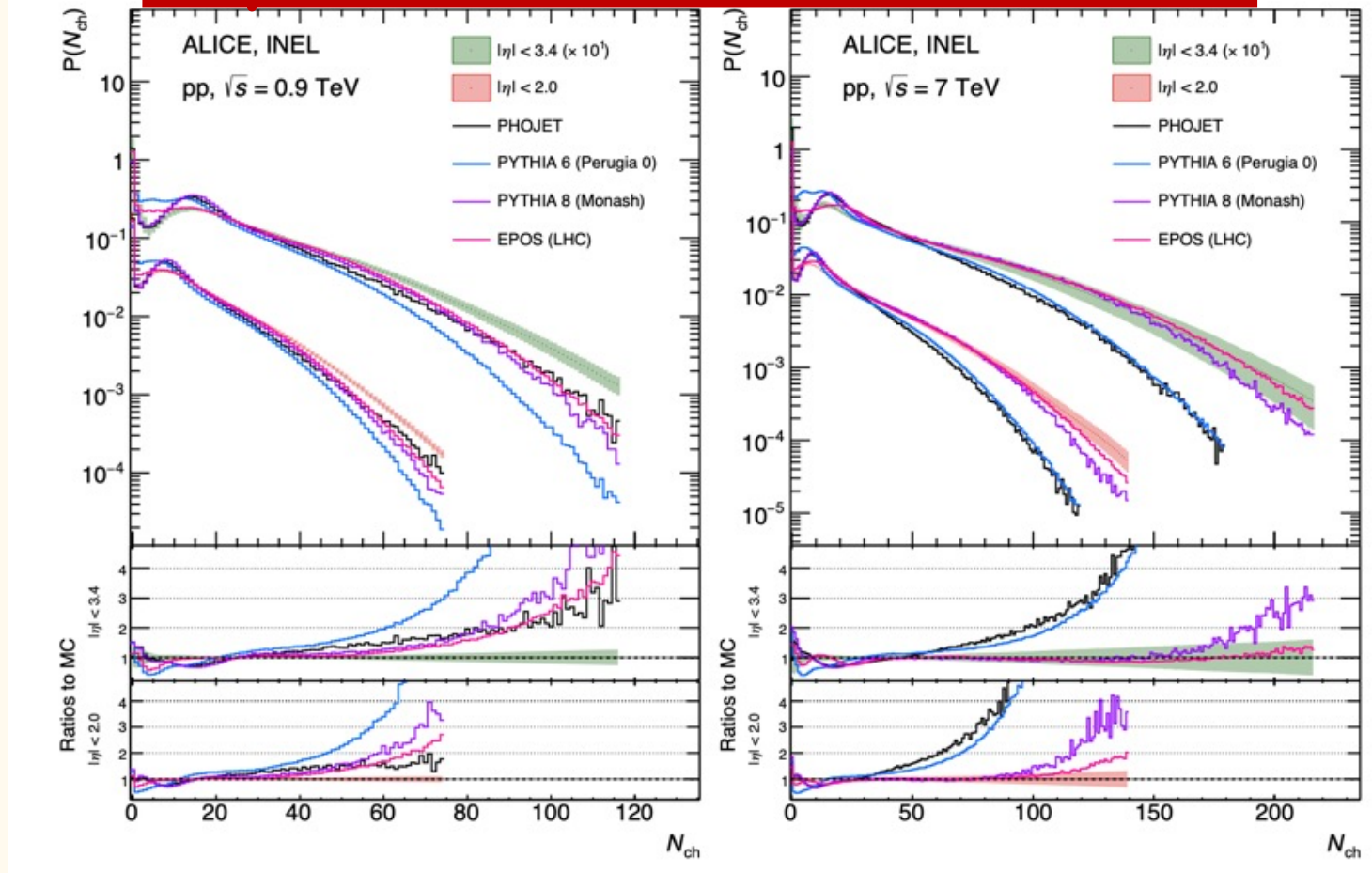
1. Electron-positron annihilation ( $e^+e^-$ ).
2. Proton & proton-antiproton interactions.  
Bottomonium decay.
3. "Thermalization" project (high multiplicity).  
Collective phenomena.
4. Relativistic nuclear physics (AA).

# Experiments at LHC (ATLAS)



ATLAS's data [2010] for MD in pp interactions at 0.9 and 7 TeV  
Comparisons with Monte Carlo generators (PYTHIA, PHOJET).

# Experiments at LHC (ALICE)



[arXiv:1708.01435, ALICE Coll.]: MD of charged particles in high pseudo-rapidity region (pp collisions at 0.9 и 7 TeV).

## $e^+e^-$ annihilation. I stage

Konishi et al. & Giovannini [NP, 1979] describe quark-gluon cascade in pQCD in view of elementary processes

- 1) Quark emission of gluon -  $q \rightarrow q + g$ , ( $\tilde{A}$ )
- 2) Gluon fission -  $g \rightarrow g + g$ , ( $A$ )
- 3) Quark-antiquark pair creation from gluon -  $g \rightarrow q + \bar{q}$ .

$$\left\{ \begin{array}{l} \frac{\partial G}{\partial Y} = -AG + AG^2, \\ \frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG. \end{array} \right. \quad \text{System of diff. eq. describing branching processes, leads to Polia (NBD) for } q\text{-jet and Farry MD for } g\text{-jet:}$$

$$P_m^g = \frac{1}{\bar{m}} \left(1 - \frac{1}{\bar{m}}\right)^{m-1}, \quad P_m^q = \frac{k_p(k_p+1)\dots(k_p+m-1)}{m!} \left(\frac{\bar{m}}{\bar{m}+k_p}\right)^m \left(\frac{k_p}{\bar{m}+k_p}\right)^{k_p}.$$

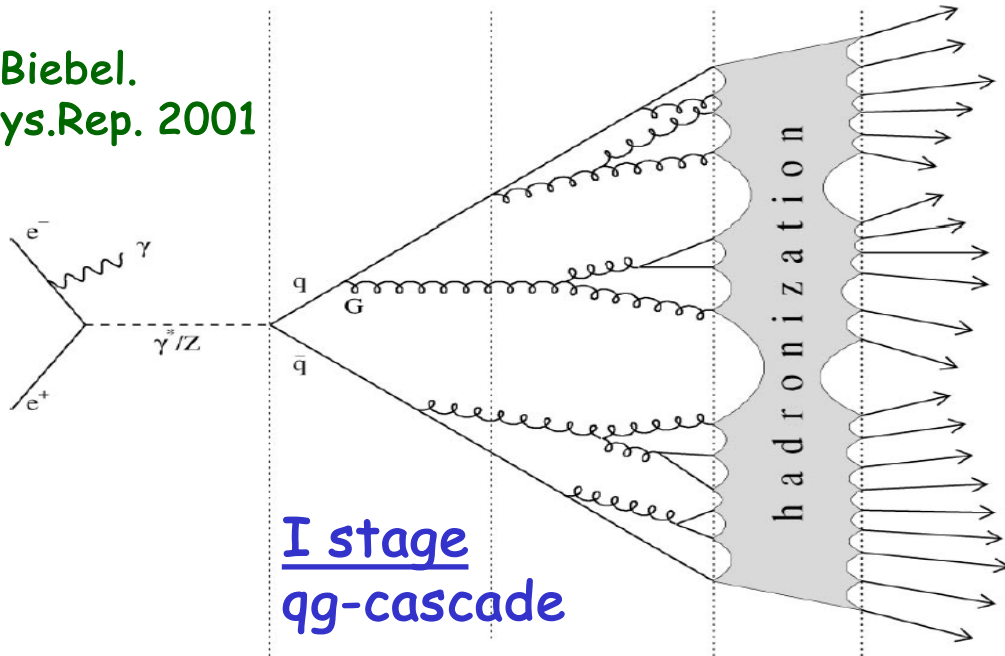
Evolutinary parameter:

$$Y = \frac{1}{2\pi b} \ln[1 + ab \ln(Q^2 / \mu^2)], \quad \tilde{A} \text{ и } A - \text{вероятности процессов 1) и 2), } k_p = \tilde{A}/A.$$

# $e^+e^-$ - annihilation

$$e^+e^- \rightarrow \gamma(Z^0) \rightarrow q\bar{q} \rightarrow (q, g) \rightarrow ? \rightarrow \text{hadrons}$$

O.Biebel.  
Phys.Rep. 2001



I stage  
qq-cascade

II stage  
(hadronization)

**Multiplicity  
Distribution (MD)**

$$P_n(s) = \frac{\sigma_n}{\sum_m \sigma_m}$$

**Generation  
function (GF):**

$$Q(s, z) = \sum_n P_n(s) z^n$$

**GF  $\leftrightarrow$  MD**

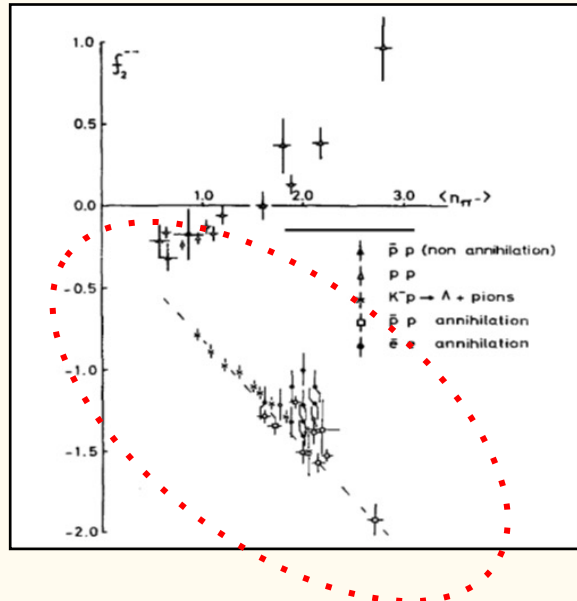
$$P_n(s) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} Q(s, z) \Big|_{z=0}$$

**Correlative moments,  $F_k$ :**

$$F_k(s) = \overline{n(n-1)\dots(n-k+1)} = \frac{\partial^k}{\partial z^k} Q(s, z) \Big|_{z=1}$$

## $e^+e^-$ annihilation - II stage

pQCD is unable to describe hadronization. The choice of MD at this stage is based on experimental behavior of the second correlative moment  $f_2 = \langle n(n-1) \rangle - \langle n \rangle^2 = D_2 - \langle n \rangle$  ( $D_2$  - variance).



The independent formation of particles is described by the Poisson distribution with  $f_2 = 0$ . Polya MD at the first stage corresponds to **negative binomial distribution (NBD)**:

$$f_2 = \overline{n(n-1)} - \bar{n}^2 \rightarrow \frac{\bar{m}^2}{k_p} > 0$$

We chose **binomial MD** (Bernoulli) for II-stage:

$$P_P^H(n) = C_{N_p}^n \left( \frac{\bar{n}_p^h}{N_p} \right)^n \left( 1 - \frac{\bar{n}_p^h}{N_p} \right)^{N_p - n}, P = q, g.$$

## Convolution of two stages

$$Q(s, z) = \sum_m P_m^P Q^H(m, s, z) \quad (\text{soft decoloration}).$$

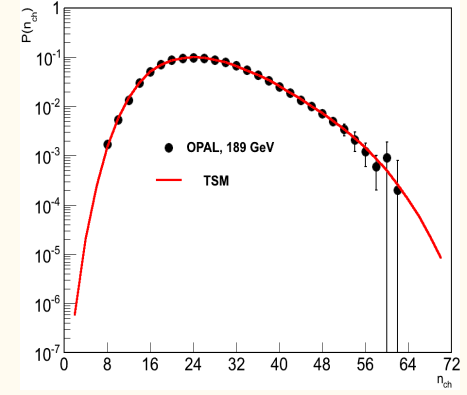
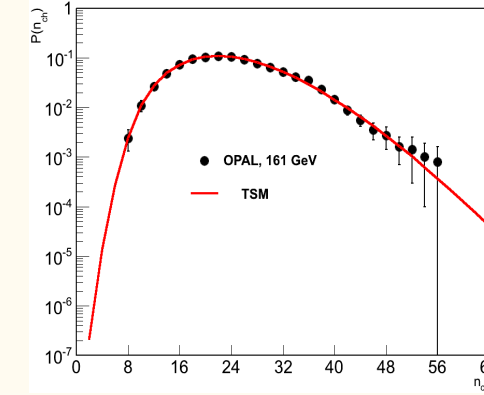
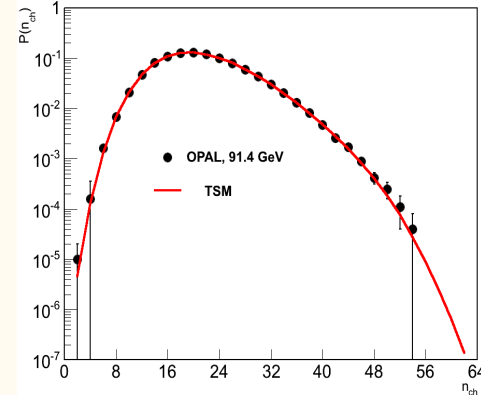
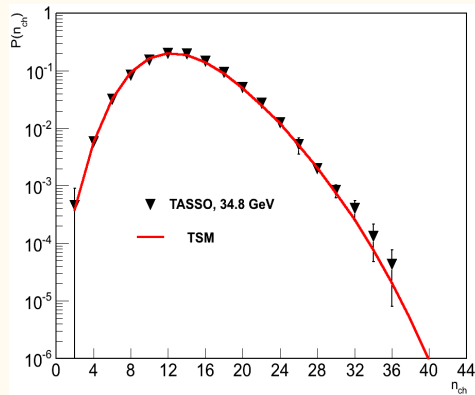
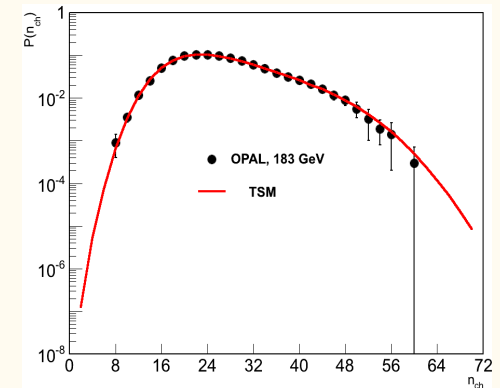
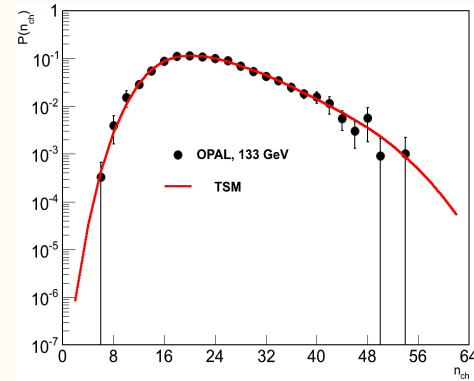
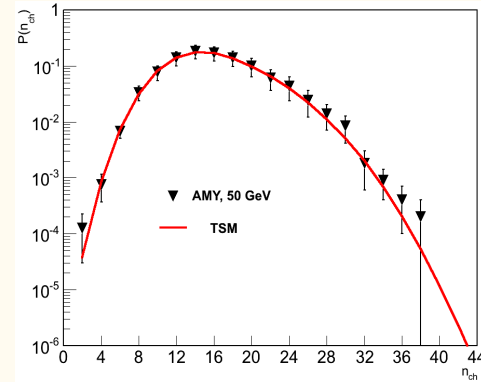
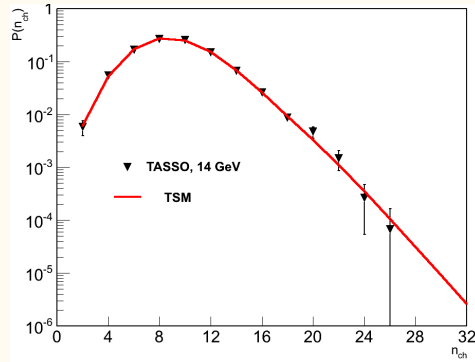
$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C_{(2+\alpha m)N}^n \left( \frac{\bar{n}^h}{N} \right)^n \left( 1 - \frac{\bar{n}^h}{N} \right)^{(2+\alpha m)N-n}$$

$$Q_p^H = \left[ 1 + \frac{\bar{n}_p^h}{N_p} (z-1) \right]^{N_p}, \quad \mathbf{p} = \mathbf{q}, \mathbf{g}, \quad f_2 = -\frac{(\bar{n}_p^h)^2}{N_p} < 0.$$

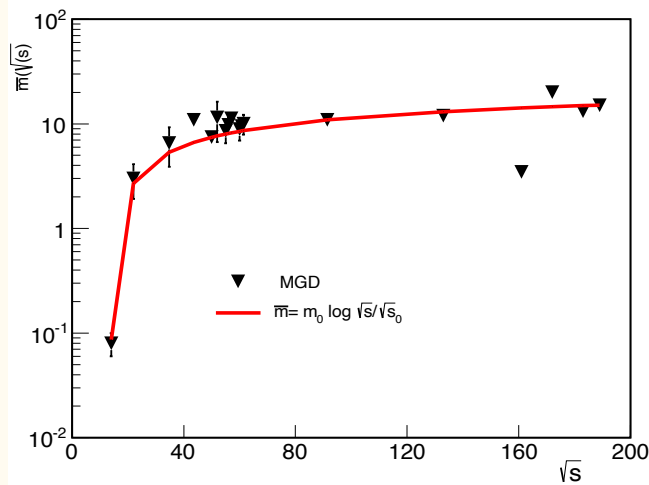
Model parameters:  $N_q = N$ ,  $\alpha = N_g/N$ ,  $\bar{n}^h$ .



# MD in $e^+e^-$ annihilation (14 - 189 $\Gamma_{\text{эВ}}$ )

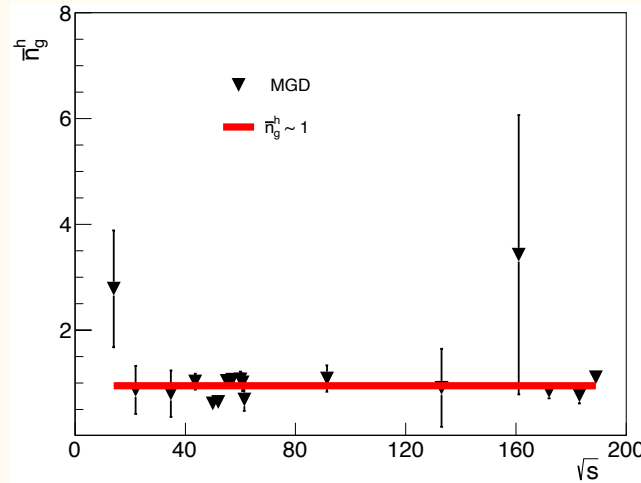


# Parameters of Model

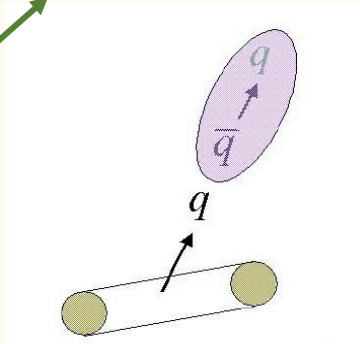


$$\bar{m} \sim \log s.$$

Hypothesis of parton-hadronic duality (LoPHD)  
 $\langle m \rangle = \rho \langle n \rangle, \rho \sim 1.$



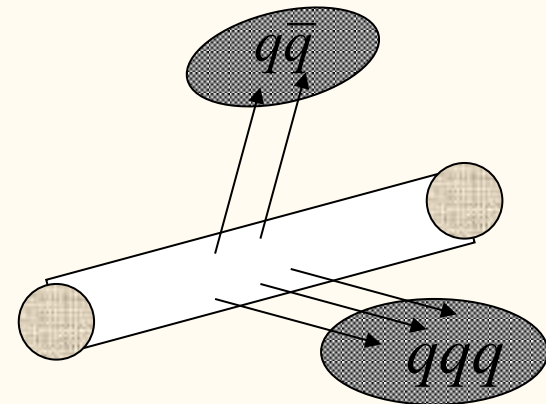
Average number of hadrons,  $\bar{n}_g^h$ , formed from gluon, is close to 1, which testifies the **fragmentation** mechanism of hadronization.



Fragmentation mechanism (in vacuum)

Recombination mechanism (in qg-medium)

[ B. Muller. 2004 ]



## Parameter of model $k_p$

$k_p$  is determined by the ratio of the bremsstrahlung contributions of active gluons ( $q \rightarrow q + g$ ) to their division ( $g \rightarrow g + g$ ),  $\tilde{A}/A$ , at the cascade stage. It takes on values more than 1, which indicates the predominance of bremsstrahlung over fission. It tends to decrease with increasing energy. Within the statistical bootstrap model, it was shown,  $k_p^{-1}$  can be interpreted as the "temperature" at that stage

$$k_p^{-1} = T_0 + \frac{1}{c} E ,$$

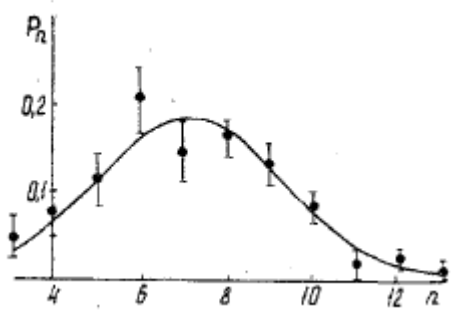
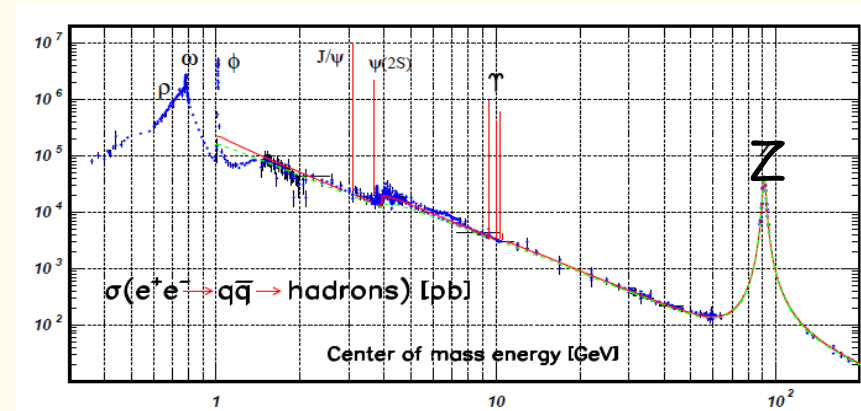
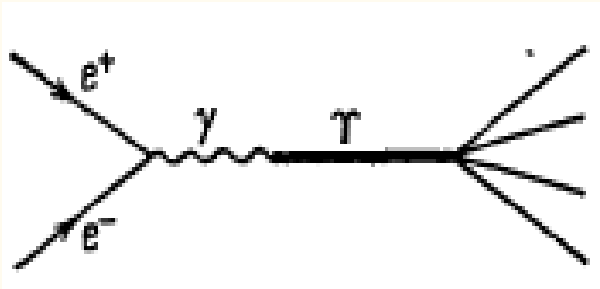
it rises with energy.

## $f_2$ Sign changing with energy growth from "-" to "+"

$$f_2 = F_2 - F_1^2 = \sum_{m=0} (2 + \alpha m) \left( 2 + \alpha m - \frac{1}{N} \right) P_m^q \cdot (\bar{n}^h)^2 - \left[ (2 + \alpha \bar{m}) \bar{n}^h \right]^2 =$$
$$= \left[ \alpha^2 \frac{\bar{m}^2}{k_p} + \alpha^2 \bar{m} - \frac{2 + \alpha \bar{m}}{N} \right] (\bar{n}^h)^2.$$

Parameters:  $\alpha \sim 1$ ,  $N \sim 6$ . At  $\sqrt{s} < 5$   $\bar{m} \ll 1$  and  $f_2 < 0$ .  
At  $\sqrt{s} \gtrsim 10$ ,  $\bar{m} > 10$  and the sign of the second  
correlative moment is changed :  $f_2 > 0$ .

# Three-gluon decay of quarkoniums $\Upsilon(9.46)$ , $\Upsilon(10.02)$

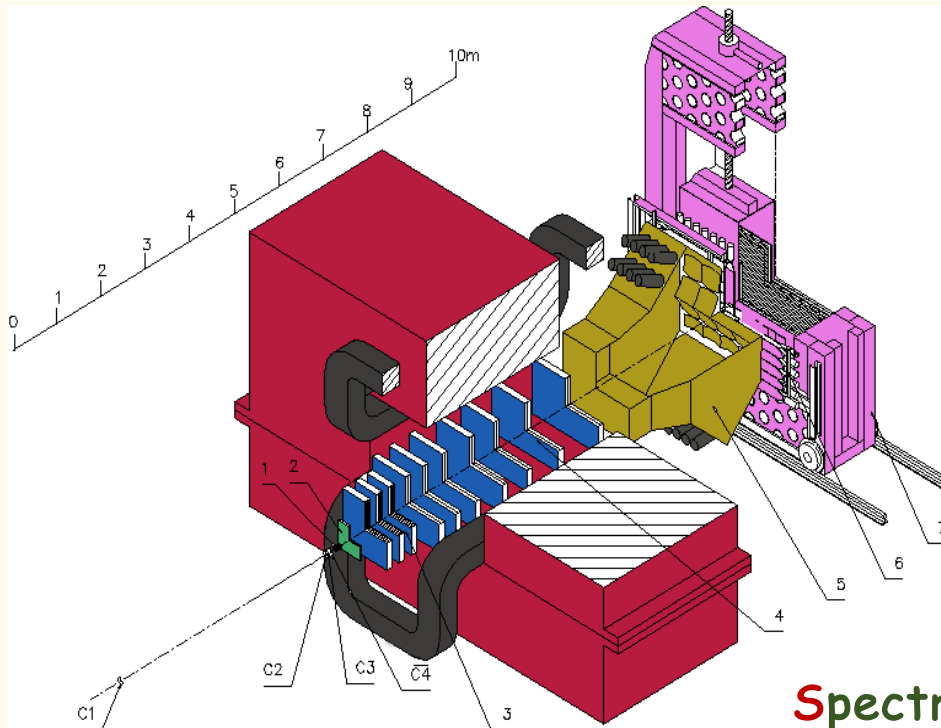


$$P_n(s) = \sum_{m'=0} \frac{(m'-1)(m'-2)}{2(\bar{m}/3)^2} \left(1 - \frac{1}{\bar{m}/3}\right)^{m'} C_{(3+m')N_g}^n \left(\frac{\bar{n}_g^h}{N_g}\right)^n \left(1 - \frac{\bar{n}_g^h}{N_g}\right)^{(3+m')N_g - n}$$

$$\Delta \bar{n}_{TSM}(s) = \left[ \alpha(\bar{m}' - \bar{m}_{(q)}) - 3(\alpha - 2/3) \right] \bar{n}_q^h \quad \Delta \bar{n}_{\text{exp}}(s) \approx \Delta \bar{n}_{TSM}(s) \approx 0.8$$

# Hadron Interactions with High Multiplicity (HM). "Thermalization" project (IHEP+SINP MSU+JINR)

$$p + p \rightarrow 2N + \pi_1 + \pi_2 + \dots + \pi_n$$



Project started in 2005, SVD-2 setup at U-70 accelerator, 50-GeV p-beam, H<sub>2</sub> target, high multiplicity trigger. **Kinematical limit** (total energy spent on secondary particle production at rest) **is close to 59 pions.**

Spectrometer with **V**ertex **D**etector  
(SVD-2)

## Gluon Dominance Model (GDM)

Modification of the two stage model has been carried out: at the initial stage valence quarks and nascent gluons in accordance with pQCD branch due elementary fission processes (1-3). Then they fragment into real hadrons in accordance with the binomial distribution confirmed in  $e^+e^-$  annihilation. The scheme of convolution combines both stages.

A comparison of this model with data (Mirabelle, U-70) showed that the hadronization parameters differ considerably from the values obtained in  $e^+e^-$  annihilation, they were much less than 1. It meant that the number of valence quarks involved in branching should be reduced. Participation of all three pairs of quarks is unlikely.

## Gluon Dominance Model (GDM)

Our study showed: hadronization parameters grow with decreasing of valence quark pairs. Only their complete exclusion improved considerably  $\chi^2$  for MD description, but hadronization parameter, the average number of hadrons produced from gluon at the second stage increases with energy, exceeding 1.

Valence quarks are staying in leading particles, secondaries are created by active gluons, which decay into quark-antiquark pairs, and form colorless hadrons.

Soft gluons (~50%) remain in the  $qg$ -system and partly re-emit soft photons leading to their excess yield.



# GDM, g-fission:

$$P_k \otimes P_m^P \otimes P_n^h$$



$$P_k = \frac{e^{\bar{k}^k \bar{k}^k}}{k!}; \quad k > 1, \quad P_m^P = \frac{1}{m} \left(1 - \frac{1}{m}\right)^{m-1},$$

$$P_n^h = C_{\delta m N_g}^{n-2} \left(\frac{\bar{n}_g^h}{N_g}\right)^{n-2} \left(1 - \frac{\bar{n}_g^h}{N_g}\right)^{\delta m N_g - (n-2)};$$

Almost a half of gluons (~ 47 %) remain in  $qg$ -system, without being fragmented into hadrons (A.H. Mueller has the same result, 2003).

# GDM, scheme without gluon fission

$$P_n(s) = \Omega \sum_{m=1}^{ME} \frac{\bar{m}^m e^{-\bar{m}}}{m!} \cdot C_{mN}^{n-2} \left( \frac{\bar{n}^h}{N} \right)^{n-2} \left( 1 - \frac{\bar{n}^h}{N} \right)^{mN-(n-2)} .$$

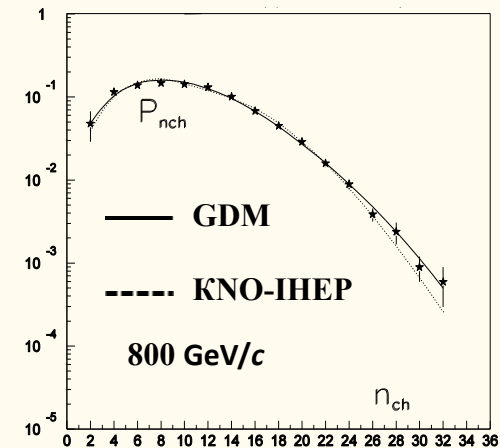
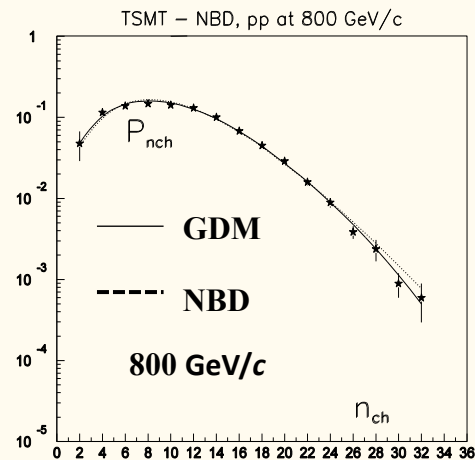
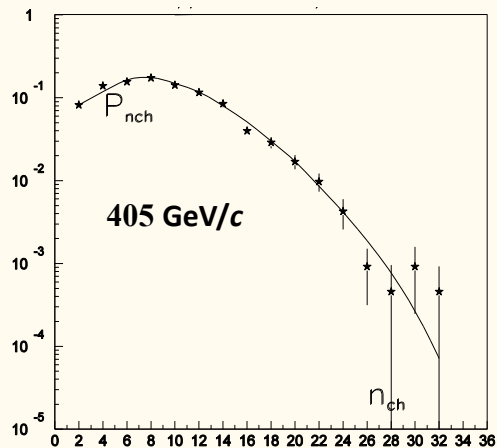
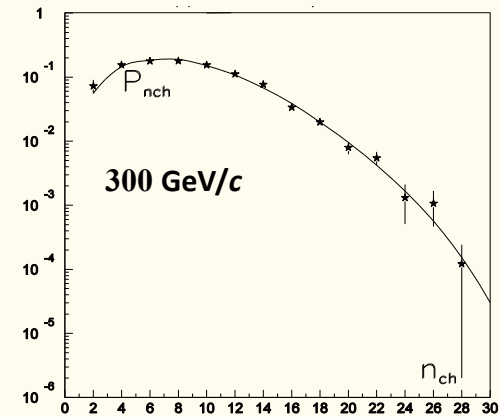
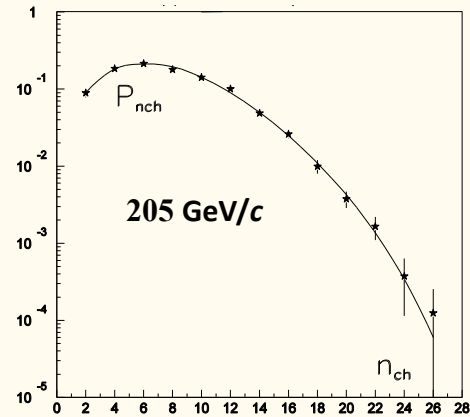
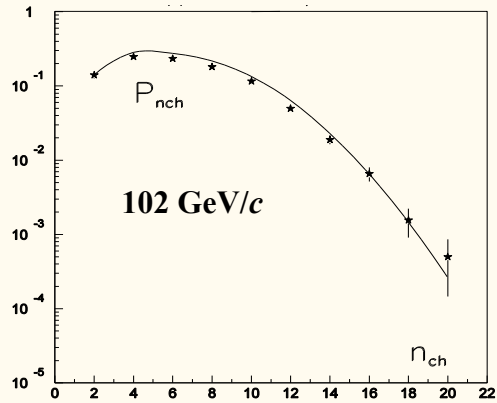
Parameters:  $N = 4.24 \pm .13$ ,  $\bar{m} = \bar{m}(s) = 2.48 \pm .20$ ,  $\bar{n}^h = 1.63 \pm .12$ ,  $\chi^2 = 2.0$ .

$p$ $\Gamma_{\Xi B/c}$	$\bar{m}$	$M_g$	$N$	$\bar{n}_g^h$	$\Omega$	$\chi^2/\text{ndf}$
102	2.75±0.08	8	3.13±0.56	1.64±0.04	1.92±0.08	2.2/5
205	2.82±0.20	8	4.50±0.10	2.02±0.12	2.00±0.07	2.0/8
300	2.94±0.34	10	4.07±0.86	2.22±0.23	1.97±0.05	9.8/9
405	2.70±0.30	9	4.60±0.24	2.66±0.22	1.98±0.07	16.4/12
800	3.41±2.55	10	20.30±10.40	2.41±1.69	2.01±0.08	10.8/12

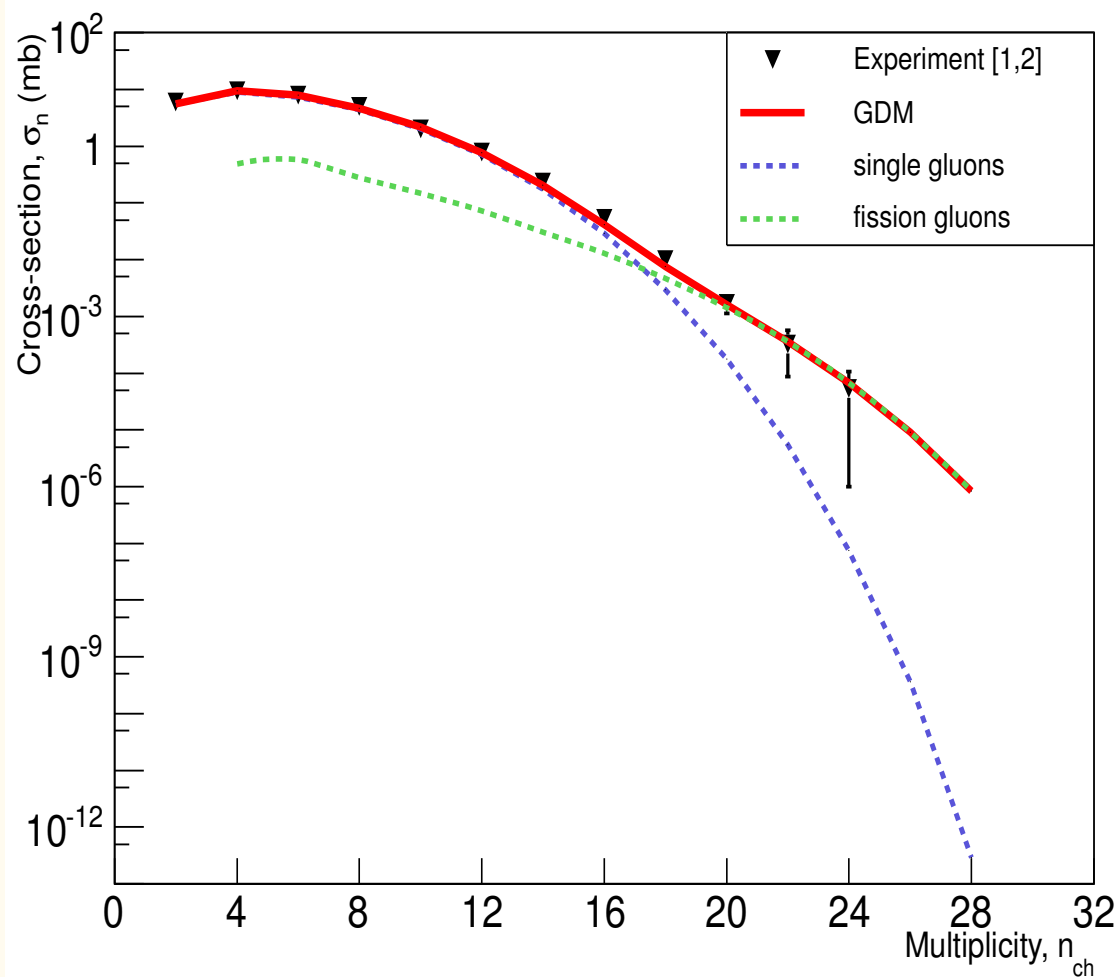
Observed growth of  $\bar{n}_g^h$  evidences a change of mechanism from fragmentation to recombination

ISR energy 60 GeV:  $\bar{n}_g^h \approx 3.3$

# GDM describes MD in pp at 100-800 GeV/c



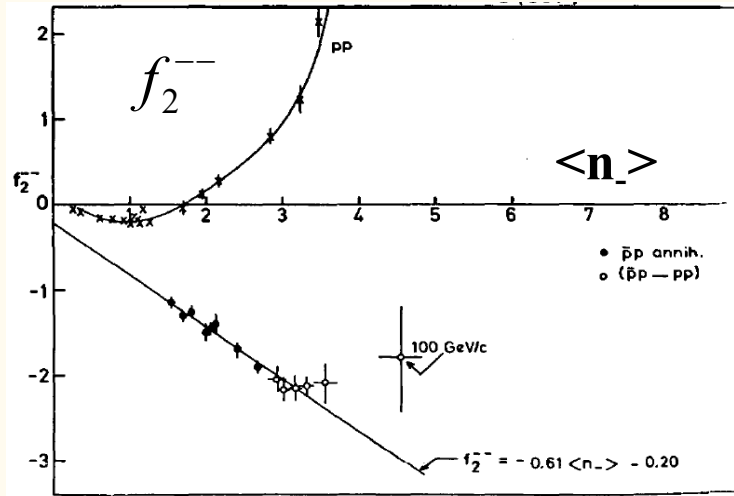
## GDM with gluon fission



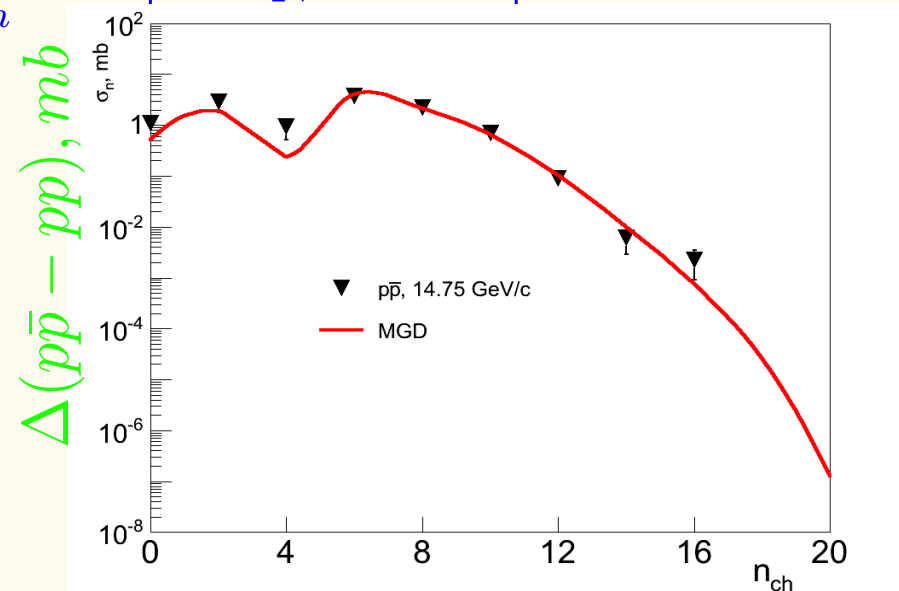
Data [Mirabelle & SVD-2] pp at 50 GeV/c has been stitched along topolog. cross sections,  $\sigma_n$ . GDM takes into account two types of hadronization: gluon c without their fission (green line) and with fission (blue line). Superposition of those contributions is shown by red line. HM stipulates namely by gluon fission. Their ratio: **bremsstrahlung to gluon fission is equal to  $\sim 1/10$** .

# Proton-antiproton annihilation in GDM

$$Q(z) = c_0 \sum_m P_m^G \left[ 1 + \frac{\bar{n}^h}{N} (z - 1) \right]^{mN} + c_2 z^2 \sum_m P_m^G \left[ 1 + \frac{\bar{n}^h}{N} (z - 1) \right]^{mN} + c_4 z^4 \sum_m P_m^G \left[ 1 + \frac{\bar{n}^h}{N} (z - 1) \right]^{mN},$$



J.G. Rushbrooke, B.R. Webber.  
Phys.Rep. 44 (1978) 1



$$\Delta(p\bar{p} - pp) = \sigma_n(p\bar{p}) - \sigma_n(pp)$$

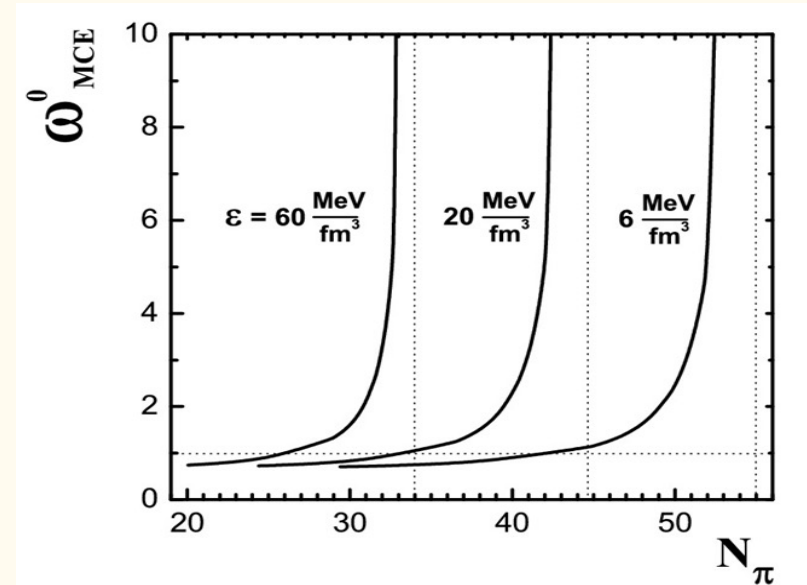
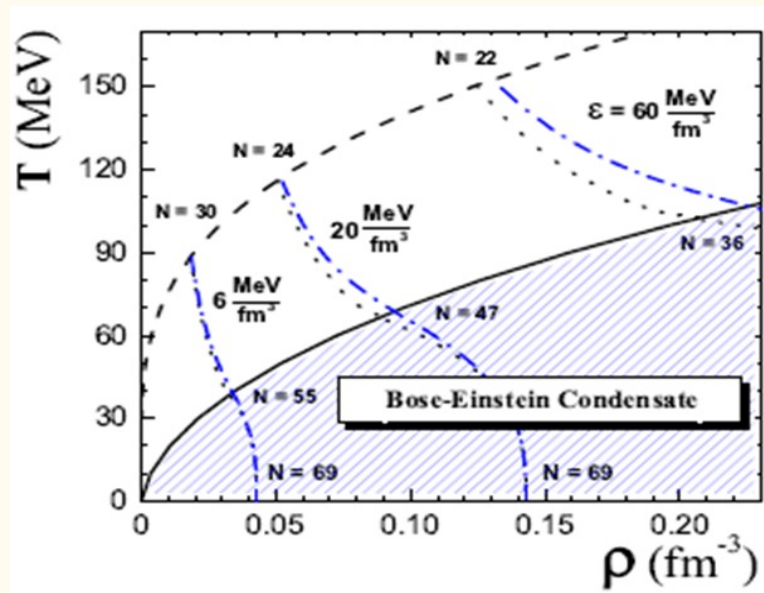
# Neutral pion fluctuations at high total multiplicity

V. Begun & M. Gorenstein put us the task on searching for pionic (Bose-Einstein, BEC) condensate [Phys.Lett., 2007, Phys.Rev. 2008] in pp interactions at U-70 for HM. For this purpose, we only had to measure the scaled variance

$$\omega^0 = D / \langle N_0(N_{\text{tot}}) \rangle, \quad D = \langle N_0^2 \rangle - \langle N_0 \rangle^2,$$

of  $\pi^0$ -meson number with growth of total multiplicity ( $n_{\text{tot}} = n_{\text{ch}} + n_0$ ). Abrupt growth of  $\omega^0$  would be signal of BEC formation.

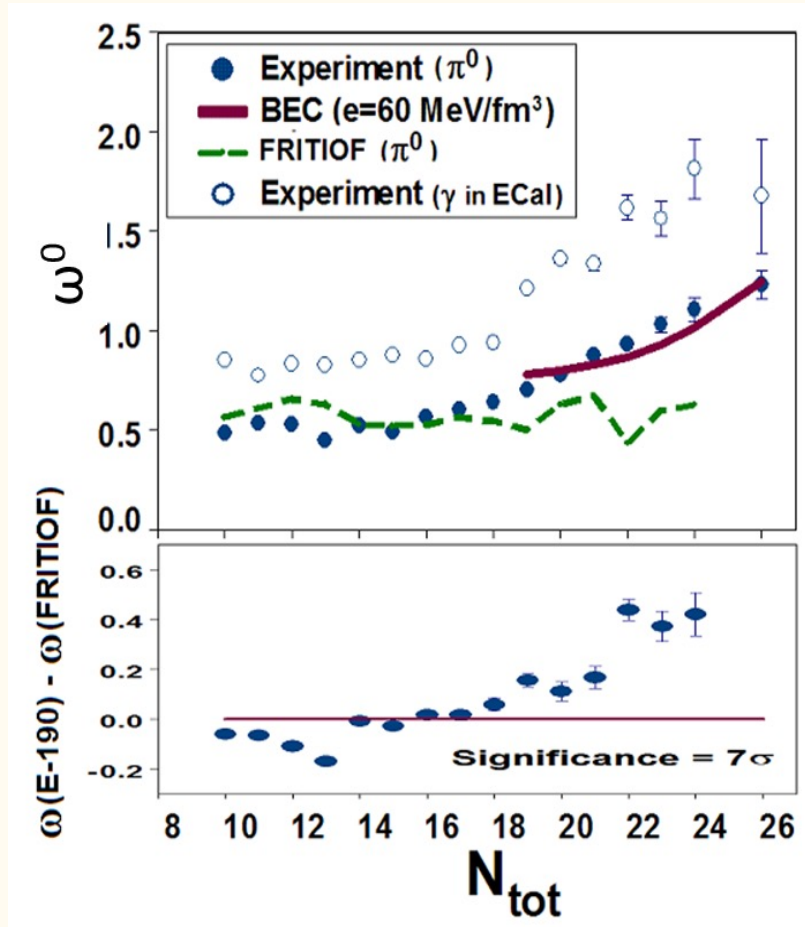
# Fluctuations of $\pi^0$ -mesons at High multiplicity



Phase diagram of pionic gas at  $\mu_Q = 0$ . Dash line corresponds to  $\rho_\pi(T, \mu_\pi = 0)$ , solid - BEC. Energy densities 6, 20 и 60 Mev/fm<sup>3</sup>.

$$\frac{T_C(\pi)}{T_C(A)} \approx \frac{m_A}{m} \left( \frac{r_A}{r_\pi} \right)^2 \cong \frac{m_A}{m} 10^{10} \rightarrow T_C(\pi) \gg T_C(A).$$

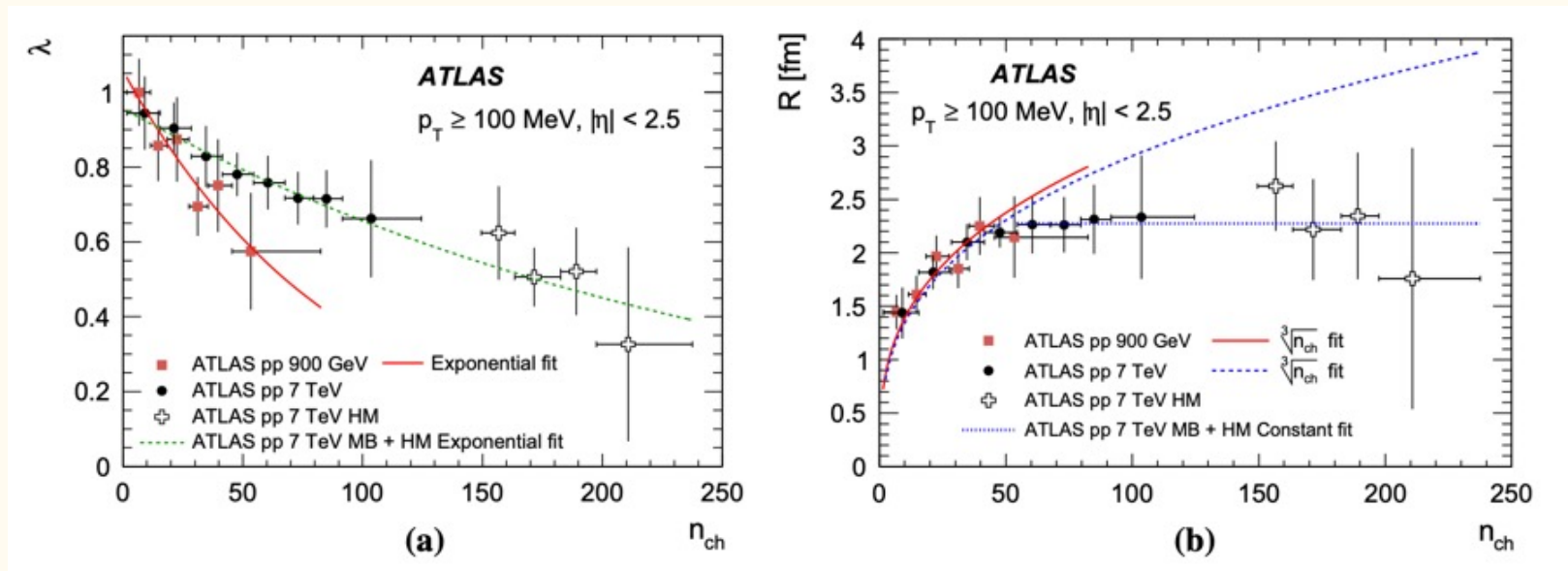
# Fluctuations of $\pi^0$ -mesons at High multiplicity



The deviation of the scaled variance,  $\omega^0$  measured on the SVD-2 from the Monte Carlo predictions in the HM region is  $7\sigma$  at  $N_{\text{tot}} \sim 25$  [EPJ, 2012, ICHEP 2012].



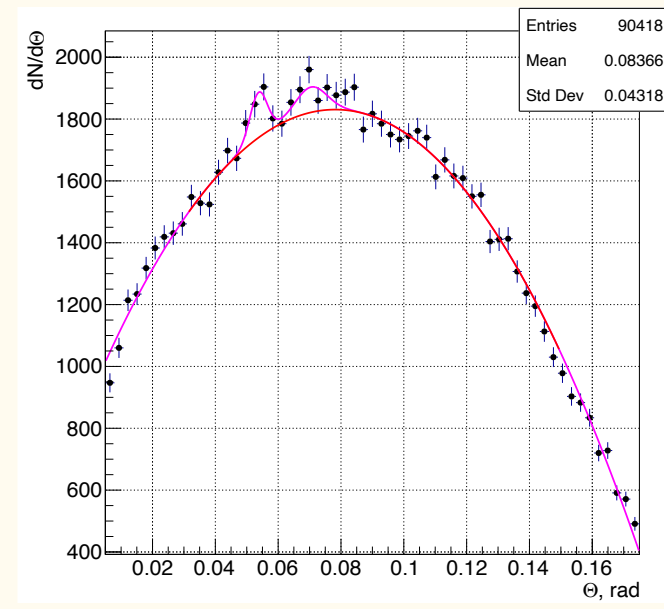
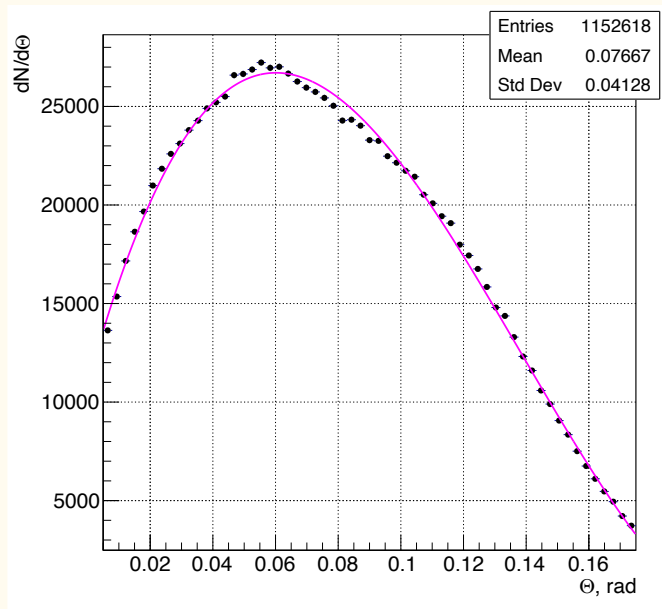
# Bose-Einstein correlations in pp at HM (ATLAS)



Two-particle Bose-Einstein correlations in pp at 0.9 и 7 TeV. ATLAS Collab. EPJ, 75 (2015).  $C_2(Q) = \rho(Q)/\rho_0(Q) = C_0 [1 + \Omega(\lambda, QR)](1 + \epsilon Q)$ ,  $Q^2 = (p_1 - p_2)^2$ .

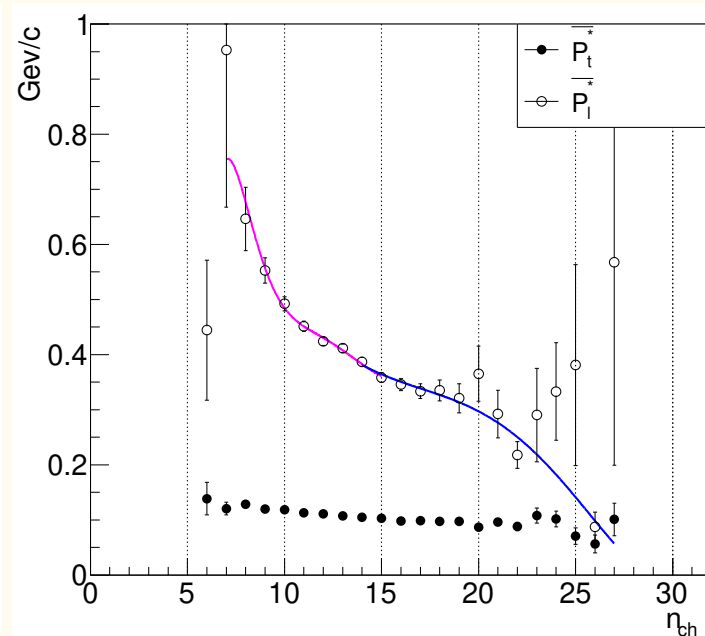
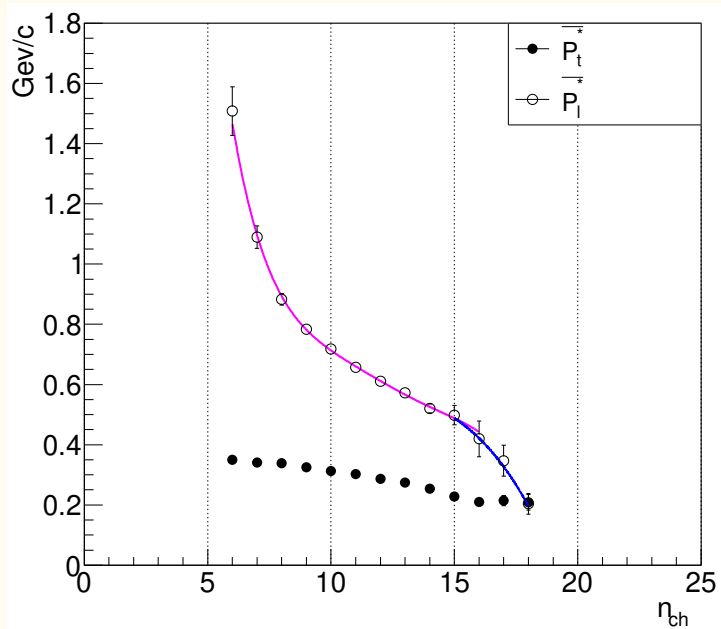
$\lambda$  close to 1 characterizes chaotic emission of particles,  $\lambda$  close to 0 - coherent emission (is characteristic for BEC),  $R$  defines the size of emission region, for BEC it's hard time [hep-ph 1501.04530]. Their wave functions entangle. LHCb confirms 1709.01769 [hep-ex].

# Polar angle ( $\theta$ ) distributions. All and HM.



Angle distributions on the polar angle  $\Theta$ . In HM region we observe two-humped structure, which it's interpreted as Cherenkov radiation gluon by quark.  $\Theta_{\text{Cher}} = 0.05377 \pm 0.00273$  rad with  $CL_{3.1} \sigma$ . For gluon rings  $\cos \Theta = 1/\beta n_r$ , where  $n_r$  refraction coefficient  $n_r = 1.0016 \pm 0.0001(4)$ , close to 1. It testifies about rarity of  $qg$ -medium.

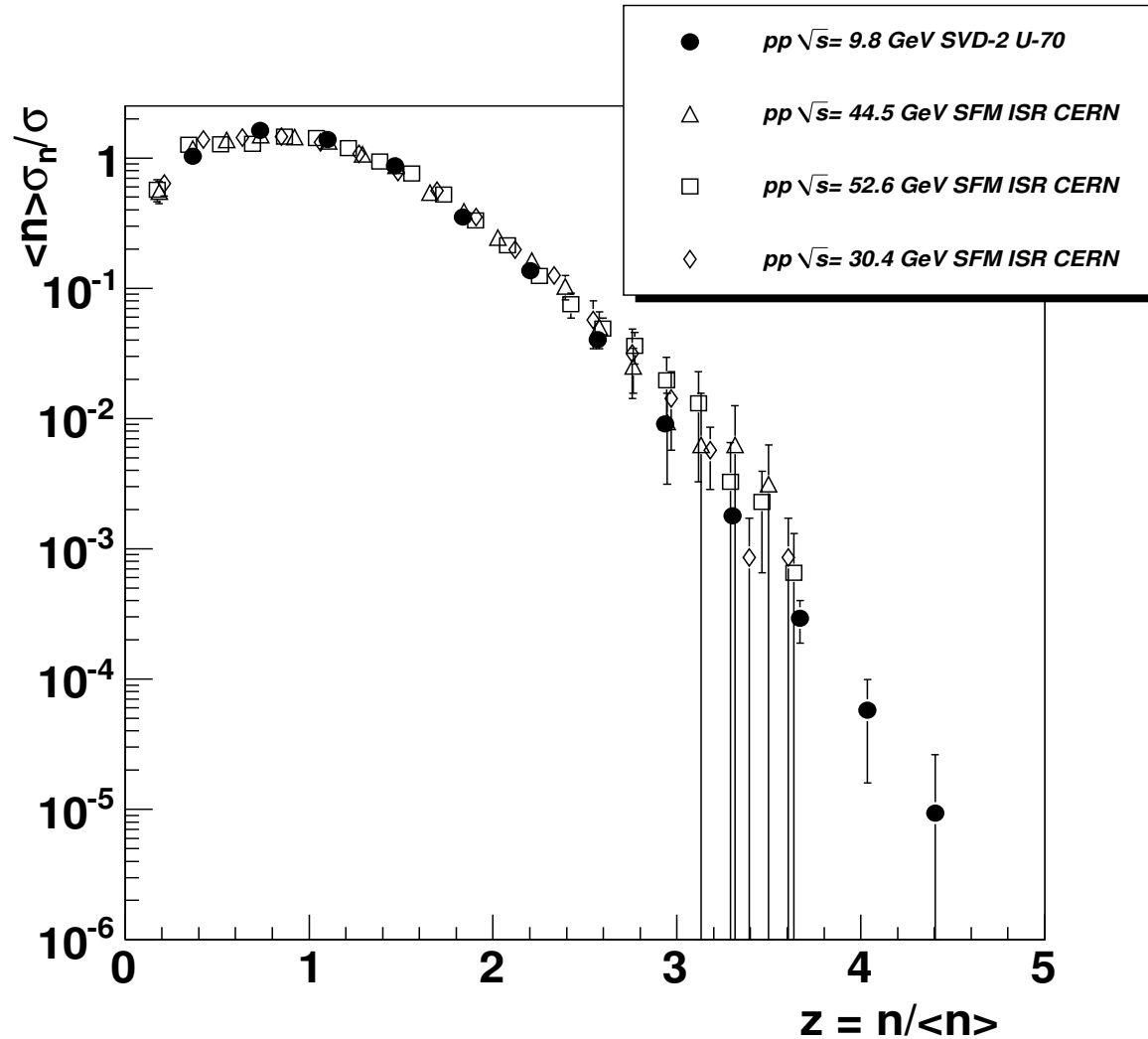
# Longitudinal & transversal components of p at HM



$\langle p_{\perp} \rangle$  and  $\langle p_{\parallel} \rangle$   
components of charged  
particles.  
Left: M.C.-simulation,  
right: experimental  
data.

BEC formation starts to form from  $n_{ch} \sim 16$  (inflection point).  
At  $n_{tot} > 18$   $\omega^0$  rises, leading particles disappear, hadron system  
becomes isotropic in all directions.

# KNO scaling



The world KNO distribution with addition of the SVD-2 four points ( $\sqrt{s} = 9.8 \text{ GeV}$ ).

## RESULTS

The study of events with HM allowed us to develop and supplement the mechanism of multiple production with a description of the hadronization stage by the Bernoulli distribution for various processes:  $e^+e^-$  annihilation, bottomonium decays, pp interactions, and proton and antiproton annihilation.

In the HM region, the collective behaviour of secondary particles has been discovered and confirmed, which gives us new ideas about the mechanism of multiple production, in particular, the active role of the gluon component in this process.

## Afterwords

"Perhaps there are no discoveries in elementary or higher mathematics, or even, perhaps, in any other field that could be made ... without analogy."

George Poiya.

# DNA Replication ~ gluons -> hadrons

