Small- $p_T$  production of  $J/\psi$  mesons within the Soft Gluon Resummation approach

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# Outline

- Hadronisation model: NRQCD
- Soft Gluon Resummation approach
- ▶  $J/\psi$  production at low- $p_T$
- Conclusion

#### Hadronisation model: NRQCD

•  $J/\psi$  wave function as a series with respect to relative constituent quarks velocity v:

$$\begin{split} |J/\psi\rangle &= \mathcal{O}(\upsilon^0) \, |c\bar{c}[{}^3S_1^{(1)}]\rangle + \mathcal{O}(\upsilon^1) \, |c\bar{c}[{}^3P_J^{(8)}]g\rangle + \mathcal{O}(\upsilon^2) \, |c\bar{c}[{}^3S_1^{(1,8)}]gg\rangle + \\ &+ \mathcal{O}(\upsilon^2) \, |c\bar{c}[{}^1S_0^{(8)}]g\rangle + \mathcal{O}(\upsilon^2) \, |c\bar{c}[{}^1D_J^{(1,8)}]gg\rangle + \dots \end{split}$$

- $\blacktriangleright$  Approximate v-scaling due to  $v^2\approx 0.2$
- Hard cross section factorisation:

$$d\hat{\sigma}(ab \to \mathcal{C}X) = \sum_{n} d\hat{\sigma}(ab \to c\bar{c}[n]X) \langle \mathcal{O}^{\mathcal{C}}[n] \rangle$$

Nonperturbative (hadronisation) factors:

 $\langle \mathcal{O}^{\mathcal{C}}[n] \rangle$  – long-distance matrix elements (LDME):

color singlet LDMEs — potential models, data for leptonic decay

color octet LDMEs - lattice QCD calculation or experimental data fitting

#### General remarks on our approximations in calculations of prompt $J/\psi$ production

▶ Direct production:  $g + g \rightarrow J/\psi + X$ , feed-down contributions from  $\psi(2S) \rightarrow J/\psi + X$  and  $\chi_{cJ} \rightarrow J/\psi + \gamma$ 

Prompt = Direct + Feed-down contributions

- We study the direct production & the P-wave feed-down. At  $\sqrt{s} = 200$  GeV (PHENIX data), feed-down contribution is estimated at 30%
- We study gluon-gluon fusion & quark-antiquark annihilation, quark-antiquark subprocesses may contribute about 10% to the total cross section at  $\sqrt{s} = 200$  GeV (within the Improved Color Evaporation model) [Saleev, Chernyshev, 2022]
- Our preliminary calculations were done in the LO approximation of the pQCD in  $\alpha_S$

#### TMD factorisation and initial parton transverse momenta

Fransverse Momentum Dependent (TMD) factorization:  $q_T, k_T \ll \mu_F \sim M$ ,

TMD parton distribution functions  $F(x, q_T, \mu_F, \zeta) \Rightarrow$  two-scale **Collins-Soper** equations,

$$q_1^{\mu} = x_1 p_1^{\mu} + y_1 p_2^{\mu} + q_{1T}^{\mu}, \qquad q_2^{\mu} = x_2 p_2^{\mu} + y_2 p_1^{\mu} + q_{2T}^{\mu},$$

▶ Preserving  $\mathcal{O}(q_T/M)$  terms, neglecting  $\mathcal{O}(q_T^2/M^2)$  terms and, therefore, assuming  $y_{1,2} \rightarrow 0$ :

$$q_1 \approx \left(\frac{x_1\sqrt{s}}{2}, \boldsymbol{q_{1T}}, \frac{x_1\sqrt{s}}{2}\right), \quad q_2 \approx \left(\frac{x_2\sqrt{s}}{2}, \boldsymbol{q_{2T}}, -\frac{x_2\sqrt{s}}{2}\right),$$

▶ Relevant processes only  $2 \rightarrow 1$ , intermediate  $c\bar{c}$ -states can be

- color-octet  $^1S_0^{(8)}$  ,  $^3P_{0,2}^{(8)}$  and  $^3S_1^{(8)}$  for  $J/\psi$
- color-sinlget  $^3P_{0,2}^{(1)}$  and color-octet  $^3S_1^{(8)}$  for  $\chi_{cJ}$

#### TMD factorisation and TMD PDFs

General formula of TMD factorization [TMD Handbook, arXiv:2304.03302]:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int d\boldsymbol{q_{1T}} \, d\boldsymbol{q_{2T}} \, F(x_1, \boldsymbol{q_{1T}}, \boldsymbol{\mu_F}, \zeta_1) \, F(x_2, \boldsymbol{q_{1T}}, \boldsymbol{\mu_F}, \zeta_2) \, \delta(\boldsymbol{q_{1T}} + \boldsymbol{q_{2T}} - \boldsymbol{p_T})$$

 $\blacktriangleright$  To implement CS evolution, the transfer to impact parameter  $b_T$  space by 2D Fourier transform is done:

$$\frac{d\sigma}{d\boldsymbol{p_T}} = \sigma_0 \int \frac{d\boldsymbol{b_T}}{(2\pi)^2} e^{i\boldsymbol{p_T}\boldsymbol{b_T}} \, \hat{F}(x_1, \boldsymbol{b_T}, \boldsymbol{\mu_F}, \zeta_1) \, \hat{F}(x_2, \boldsymbol{b_T}, \boldsymbol{\mu_F}, \zeta_2)$$

•  $\sigma_0$  is calculated as series in small  $\alpha_S$ 

#### Soft Gluon Resummation Approach

Soft and collinear gluon resummation approach by [J. Collins, D. Soper, 1981]:

$$\frac{d\sigma(J/\psi)}{d\boldsymbol{p_T}} = \sigma_0 \int_0^\infty db_T \, b_T \, J_0(\boldsymbol{p_T} \boldsymbol{b_T}) \, e^{-S_P(\boldsymbol{b_T}, \boldsymbol{\mu_F}, Q)} \, e^{-S_{NP}(\boldsymbol{b_T})} \, \hat{F}(\boldsymbol{x_1}, \boldsymbol{\mu_{b^*}'}, \boldsymbol{b_T'}) \, \hat{F}(\boldsymbol{x_2}, \boldsymbol{\mu_{b^*}'}, \boldsymbol{b_T'}) \, \hat{$$

Sudakov factor in LL-LO perturbative calculations [J. Collins, D. Soper (1982)]:

$$S_{P}(b_{T},\mu_{F},Q) = \frac{C_{A}}{\pi} \int_{\mu_{b}^{2}}^{Q^{2}} \frac{d\mu'^{2}}{\mu'^{2}} \alpha_{s}(\mu') \left[ \ln \frac{Q^{2}}{\mu'^{2}} - \left(\frac{11 - 2N_{f}/C_{A}}{6}\right) \right] + \mathcal{O}(\alpha_{s})$$

Sudakov factor expression is valid only on region  $b_0/Q \leq b_T \leq b_{T, \max}$  which is being controlled with [D. Boer, W. J. den Dunnen (2014); J. Collins, D. Soper, G. Sterman (1985)]

$$\mu_b \to \mu_b' = \frac{Q b_0}{Q b_T + b_0} \quad \text{and} \quad b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\,\text{max}})^2}}$$

#### Soft Gluon Resummation approach

Master formula for soft gluon resummation:

$$\frac{d\sigma(J/\psi)}{dp_T} = \sigma_0 \int_0^\infty db_T \, b_T \, J_0(p_T b_T) \, e^{-S_P(b_T,\mu_F,Q)} \, e^{-S_{NP}(b_T)} \, \hat{F}(x_1,\mu_{b^*}',b_T^*) \, \hat{F}(x_2,\mu_{b^*}',b_T^*)$$

Nonperturbative quark factor obtained in SIDIS data fitting: [S. Aybat, T. Rogers (2011)]:

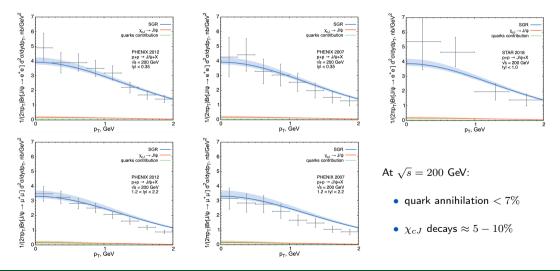
$$S_{NP}(b_T, Q) = \left[g_1 \ln \frac{Q}{2Q_{NP}} + g_2 \left(1 + 2g_3 \ln \frac{10xx_0}{x_0 + x}\right)\right] b_T^2$$

- it should be scaled by  ${\cal C}_A/{\cal C}_F$  for gluons

▶ In the leading order of  $\alpha_{s}$ , the perturbative tail of TMD PDF is expressed with collinear PDF:

$$\hat{F}(x,\mu_{b^*}',b_T^*) = f(x,\mu_{b^*}') + \mathcal{O}(\alpha_S) + \mathcal{O}(b_T\Lambda_{\mathsf{QCD}})$$

# Extraction of CO LDME in the Soft Gluon Resummation approach at $p_T < 1$ GeV



### Extraction of CO LDME in the Soft Gluon Resummation approach at $p_T < 1$ GeV

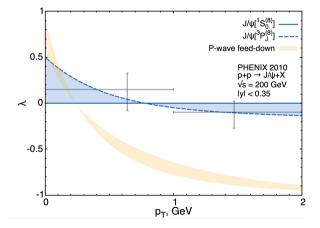
• 
$$M_7^{J/\psi} = \langle \mathcal{O}[{}^1S_0^{(8)}] \rangle + 7 \cdot \langle \mathcal{O}[{}^3P_0^{(8)}] \rangle / m_c^2 = (1.39 \pm 0.06) \cdot 10^{-1} \text{ GeV}^3$$
,

- $\langle \mathcal{O}^{J/\psi}[{}^3S_1^{(8)}] \rangle = (0.00 \pm 3.15) \cdot 10^{-3} \ {\rm GeV^3}$
- $\langle \mathcal{O}^{\chi_{c0}}[{}^3S_1^{(8)}] \rangle = (0.00 \pm 3.59) \cdot 10^{-3} \text{ GeV}^3$  $\chi^2/\text{d.o.f.} = 0.44$

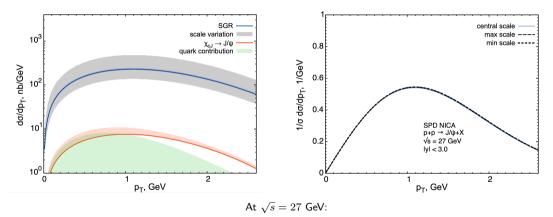
• 
$$\langle \mathcal{O}^{\chi_{c0}}[{}^{3}P_{0}^{(1)}] \rangle = (8.9 \pm 1.3) \cdot 10^{-2} \text{ GeV}^{5}$$

CO LDME	LO CPM [Cho, Leibovich (1996)]	NLO CPM [Butenschön, Kniehl (2011)]	NLO CPM [Ma, Wang, Chao (2011)]
$M_3^{J/\psi}$	$(6.6\pm1.5)\cdot10^{-2}~{\rm GeV^3}$	$(1.83\pm 0.56)\cdot 10^{-2}~{\rm GeV^3}$	$(-1.18 \pm 2.94) \cdot 10^{-2} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}[{}^3S_1^{(8)}]\rangle$	$(6.6\pm2.1)\cdot10^{-3}~{\rm GeV^3}$	$(1.68\pm0.46)\cdot10^{-3}~{ m GeV^3}$	$(8.86\pm 3.91)\cdot 10^{-2}~{\rm GeV^3}$
$\langle \mathcal{O}^{\chi_{c0}}[{}^3S_1^{(8)}]\rangle$	$(3.3\pm 0.5)\cdot 10^{-3}~{\rm GeV^3}$	_	_

## Polarisation test for CO LDME at PHENIX



### Predictions for SPD NICA using the Soft Gluon Resummation approach



• quark annihilation contribution < 5%

•  $\chi_{cJ}$  decays contribution  $\approx 5\%$ 

# Conclusion

- We have used the Soft Gluon Resummation approach to calculate low- $p_T J/\psi$  production in the TMD factorisation
- CO LDMEs of NRQCD are necessary to describe  $J/\psi$  production using the TMD factorisation, where they are major contributions
- ▶ Soft Gluon Resummation approach for gluon and quark TMD PDF satisfyingly describe experimental data for unpolarised  $J/\psi$  production at  $\sqrt{s} = 200$  GeV in the TMD domain of  $p_T < 1$  GeV
- However, quite biased result for CO LDME fitting may be a sign of a narrow fitting region for TMD and a necessity to include collinear factorisation for common LDME fitting
- ▶ We estimate the perspective region for the extraction of TMD PDF in the  $J/\psi$  production at the SPD NICA experiment as  $p_T \leq 1$  GeV