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# Challenges and Problems in Charmonium Production at the SPD NICA

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**Abstract**—The SPD NICA is planned to operate as a universal facility for study of the unpolarized and polarized transverse momentum dependent (TMD) gluon distribution functions (PDF) of a nucleon using various hard probes. The first one is the charmonium production processes. The experiment aims to provide access to the gluon TMD PDFs, like the Sivers function and the Boer–Mulders function of a proton. In this article, we present an overview of theoretical predictions for $J/\psi$, $\chi_c$, and $\eta_c$ production in the unpolarized and polarized $pp$-collisions at the $\sqrt{s} = 27$ GeV. We use the TMD parton model as it is postulated in the generalized parton model (GPM) and two models for $c\bar{c}$-pair hadronization into a final charmonium, namely the nonrelativistic QCD (NRQCD) and the Improved Color Evaporation Model (ICEM).

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## 1. INTRODUCTION

Production of different charmonium states in high-energy collisions is a very important process to verify perturbative QCD, heavy quark hadronization models and to get information on collinear and TMD PDFs in a proton [1]. Nowadays, charmonium production is studied by various LHC collaborations (ATLAS, CMS, LHCb) at the energies $\sqrt{s} = 7$–$13$ TeV and by the PHENIX Collaboration at the energy $\sqrt{s} = 200$ GeV. The future SPD NICA experiments at the energy $\sqrt{s} = 27$ GeV with polarized proton beams will be the unique tool to study polarized TMD PDFs in a proton [2, 3]. In this note we draw readers attention on several important charmonium production precesses: $\eta_c$ production as a tool to study gluon TMD PDF, prompt polarized $J/\psi$ production as a tool to study of the heavy quark hadronization model and the $J/\psi$ production in transverse polarized proton collisions to measure the transverse single spin asymmetry (TSSA), denoted as usual as $A_N^{J/\psi}$, which is controlled by the gluon Sivers function (GSF) in the leading twist approximation.

## 2. FACTORIZATION MODELS

The convention approach of the collinear parton model (CPM) may be used to describe charmonium production cross sections at the large transverse momentum of produced charmonium, $p_T \gg M_{\mathcal{C}}$, where the $M_{\mathcal{C}}$ is the mass of the charmonium. In the CPM, the fixed order QCD calculations were made up to full next-to-leading order (NLO) in strong coupling constant approximation [4] and even with the account of real next-to-NLO (NNLO*) corrections [5]. Taking in mind that at the energy of proton-proton collisions about $\sqrt{s} = 27$ GeV, measurable charmonium production events will be mostly in the region of transverse momentum $p_T \leq M_{\mathcal{C}}$, we have to use the TMD PM instead of the CPM. Theoretically proved TMD PM needs small transverse momentum $p_T \ll M_{\mathcal{C}}$ [6] and it involves only $2 \to 1$ amplitudes of the parton subprocesses. To describe data at the $p_T \simeq M_{\mathcal{C}}$ we must perform the non-trivial matching procedure to include contributions from $2 \to 2$ subprocess calculated in the CPM [7]. There is more phenomenological approach for calculations at the $p_T \leq M_{\mathcal{C}}$, namely GPM [8]. The factorization formula of the GPM looks as follows

$$d\sigma(pp \to \mathcal{C}X) = \int dx_1 d^2q_{T1} dx_2 d^2q_{T2} F_g(x_1, \mathbf{q}_{T1}, \mu) \times F_g(x_2, \mathbf{q}_{T2}, \mu) d\hat{\sigma}(gg \to \mathcal{C}), \quad (1)$$

where $F_g(x, \mathbf{q}_T, \mu) = f_g(x, \mu)\Phi(\mathbf{q}_T)$ is the unpolarized gluon TMD PDF and $\Phi(\mathbf{q}_T|) = \left(\frac{1}{\pi a^2}\right)\exp\left(-|\mathbf{q}_T|^2/a^2\right)$ with $a = \sqrt{\langle \mathbf{q}_T^2 \rangle}$, $f_g(x, \mu)$ is the collinear gluon PDF.

The initial partons in the GPM have transverse momenta but they are on the mass shell, i.e.

$$q_1 = x_1 P_1 + \tilde{x}_1 P_2 + q_{1T}, \text{ with } \tilde{x}_1 = \frac{\mathbf{q}_{1T}^2}{x_1 s}, \quad (2)$$

$$q_2 = \tilde{x}_2 P_1 + x_2 P_2 + q_{2T}, \text{ with } \tilde{x}_2 = \frac{\mathbf{q}_{2T}^2}{x_2 s}, \quad (3)$$

where colliding protons 4-momenta are equal to $P_1 = \frac{\sqrt{s}}{2}(1,0,0,1)$, $P_2 = \frac{\sqrt{s}}{2}(1,0,0,-1)$ and $q_{1,2T} = (0, \mathbf{q}_{1,2T}, 0)$.

Let us note that a priori unknown parameter $a$ of Gaussian distribution in the GPM hasn't pure physical meaning and can't be associated with the average value of intrinsic nonperturbative parton transverse momentum, which should be about the $\Lambda_{QCD} \sim 0.1$–$0.2$ GeV. In fact, we obtained in the GPM that $a \sim 1$ GeV after fit of the charmonium transverse momentum spectra at the $0 \le p_T \le M_{\mathcal{C}}$. Despite of this phenomenological restriction, the GPM can be used for an estimation during the calculations in the leading-order approximation in $\alpha_S$.

## 3. NRQCD AND ICEM

The NRQCD framework [9] describes heavy quarkonia in terms of Fock state decompositions. In case of $^3S_1$ charmonium, the wave function can be written as a power series expansion in the velocity parameter $v^2 \sim 0.2$:

$$\begin{aligned} |J/\psi\rangle &= \mathcal{O}(\upsilon^0)|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(\upsilon^1)|c\bar{c}[^3P_J^{(8)}]g\rangle \\ &+ \mathcal{O}(\upsilon^2)|c\bar{c}[^3S_1^{(1,8)}]gg\rangle \\ &+ \mathcal{O}(\upsilon^2)|c\bar{c}[^1S_0^{(8)}]g\rangle + \mathcal{O}(\upsilon^2)|c\bar{c}[^1D_J^{(1,8)}]gg\rangle + \ldots \end{aligned} \quad (4)$$

In the NRQCD effects of short and long distances are separated, and then the cross-section of heavy-quarkonium production via a partonic subprocess $g + g \to J/\psi + X$ can be presented in a factorized form:

$$\begin{aligned} &d\hat{\sigma}(gg \to J/\psi X) \\ &= \sum_n d\hat{\sigma}(gg \to c\bar{c}[n]X)\langle \mathcal{O}^{J/\psi}[n]\rangle. \end{aligned} \quad (5)$$

Color singlet long distance matrix elements (LDME) $\langle \mathcal{O}^{J/\psi}[n]\rangle$ can be calculated within the potential heavy quarkonium model or extracted from the decay widths of the quarkonia, such as $J/\psi \to \mu^+\mu^-, \eta_c \to \gamma\gamma$ et al.

In ICEM [10], it is suggested that all produced $c\bar{c}$ pairs with invariant mass $M_C < M_{c\bar{c}} < 2M_D$ hadronize to charmonium $C$ with the same probability $F^{\mathcal{C}}$. Roughly speaking, ICEM can be viewed as NRQCD factorization without velocity-scaling rules for probabilities $F^{\mathcal{C}}$. Master formula for cross section in the ICEM reads as follows

$$\begin{aligned} &\frac{d\sigma}{d^3p_C}(pp \to \mathcal{C}X) \\ &= F^{\mathcal{C}} \int_{M_C}^{2M_D} dM_{c\bar{c}} \frac{d\sigma}{d^3p_{c\bar{c}}}(pp \to c\bar{c}X), \end{aligned} \quad (6)$$

where $p_{\mathcal{C}} = \frac{M_{\mathcal{C}}}{M_{c\bar{c}}} p_{c\bar{c}}$ and $M_D$ is the mass of lightest $D$-meson. The phenomenological parameter $F^{\mathcal{C}}$ is considered as universal for each charmonium state. However, as it has been shown in [11], this parameter depends on collision energy, that should be taken into account during the calculation.

## 4. $\eta_c$ PRODUCTION

Till now, there is only one measurement for $\eta_c$ production cross section in hadron-hadron collisions, which was done by the LHCb collaboration at the energies $\sqrt{s} = 7$ and 8 TeV [12]. These data contradict theoretical prediction obtained in the NRQCD approach using heavy quark symmetry rules between different LDMEs [13]. Experimental study of $\eta_c$ production at the SPD NICA energies will be an important additional test of the NRQCD. If the color singlet production mechanism of the NRQCD is dominant in $\eta_c$ production, we may use conventional TMD parton model [6] to describe $\eta_c$ production at the small transverse momentum, $p_T \ll M_{\eta_c}$, and extract information about nonperturbative gluon TMD PDF in a proton. In the case of $\eta_c$ production via intermediate color octet states, final state soft gluon interaction destroy the TMD factorization approach and extracted TMD gluon PDF becomes process dependent. Otherwise, to resolve factorization, we must introduce additional TMD dependent factor, so-called the shape function [14]. In Fig. 1 we plot theoretical predictions for $\eta_c$ production at the $\sqrt{s} = 27$ GeV obtained in the G*PM* and two hadronization models, NRQCD and ICEM. In the LHCb experiments, $\eta_c$ production was studied using the proton-antiproton decay channel. We performed estimations considering two decay channels, $\eta_c \to p\bar{p}$ and $\eta_c \to \gamma\gamma$. We found that products of the total cross section and relevant branching fractions are equal $\sigma \times B(\eta_c \to p\bar{p}) \simeq 0.7$ nb and $\sigma \times B(\eta_c \to \gamma\gamma) \simeq 0.1$ nb. In both cases, signal/background ratio, estimated with MC event generator Pythia, is about $10^{-3}$. The search for more preferable $\eta_c$ decay channels is in progress [15].

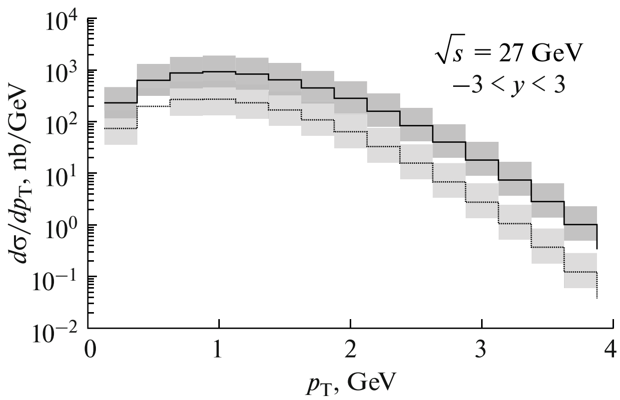


**Fig. 1.** Transverse momentum differential cross section for $\eta_c$-meson at the $\sqrt{s}$ = 27 GeV and $|y| < 3$. Dotted histogram – ICEM calculations, solid histogram—NRQCD calculations, grey boxes around histograms demonstrate the hard scale uncertainties of theoretical models.

## 5. PROMPT POLARIZED $J/\psi$ PRODUCTION

The long time polarization puzzle for the prompt polarized $J/\psi$ production may be studied at the SPD NICA energies. We calculate polarized $J/\psi$ production cross section in the GPM using NRQCD approach with color octet LDMEs, which have been fixed by fit of the data from NA3 Collaboration at $\sqrt{s} = 19.4$ GeV [16]. The parameter $a$ of Gaussian transverse momentum distribution has been fitted too, separately for initial gluons and quarks. The prompt $J/\psi$ production includes direct contribution and $J/\psi$ from cascade processes

$$\sigma^{J/\psi,\mathrm{prompt}} = \sigma^{J/\psi,\mathrm{direct}} + \sigma^{\psi'}\mathrm{Br}(\psi' \to J/\psi + X) \quad (7)$$

$$+ \sigma^{\chi_{c0}}\mathrm{Br}(\chi_{c0} \to J/\psi + \gamma) + \sigma^{\chi_{c1}}\mathrm{Br}(\chi_{c1} \to J/\psi + \gamma) \quad (8)$$

$$+ \sigma^{\chi_{c2}}\mathrm{Br}(\chi_{c2} \to J/\psi + \gamma). \quad (9)$$

The angular distribution of leptons in the $J/\psi \to l\bar{l}$ decays depends on the relation between $J/\psi$ production cross sections in the transverse and longitudinal polarized states ($\sigma_{\mathrm{T,L}}$) and it may be parameterized as follows

$$\frac{d\sigma}{d\Omega_l} \sim 1 + \lambda_\theta \cos^2(\theta_l) + \lambda_\phi \sin^2(\theta_l)\cos(2\phi_l) + \lambda_{\theta\phi_l}\cos(\phi_l), \quad (10)$$

with

$$\lambda_\theta = \frac{\sigma_{\mathrm{T}}^{J/\psi,\mathrm{prompt}} - 2\sigma_{\mathrm{L}}^{J/\psi,\mathrm{prompt}}}{\sigma_{\mathrm{T}}^{J/\psi,\mathrm{prompt}} + 2\sigma_{\mathrm{L}}^{J/\psi,\mathrm{prompt}}}. \quad (11)$$

In Fig. 2, we show transverse momentum spectra of prompt $J/\psi$ calculated in the GPM using NRQCD approach at the energies $\sqrt{s} = 19.4$ and 27 GeV. Color octet LDMEs and parameters of gluon and quark TMD PDFs were obtained via fit of NA3 data and then used to predict cross section at the future SPD NICA experiment.

At the present time, only the spin parameter $\lambda_\theta$ is being under the experimental study in the prompt $J/\psi$ production. We collect our predictions in Fig. 3 as functions of $J/\psi$ transverse momentum and rapidity. The interesting finding is their have sufficient dependence on $J/\psi$ rapidity: $J/\psi$ meson is strongly transverse polarized at the large rapidity modula and unpolarized at the central rapidity region. Taking into account that theoretical predictions based on the NRQCD for polarized $J/\psi$ production at high energies (Tevatron and LHC) don't describe data well, the additional checkup at the energy range of the SPD NICA looks very actual.

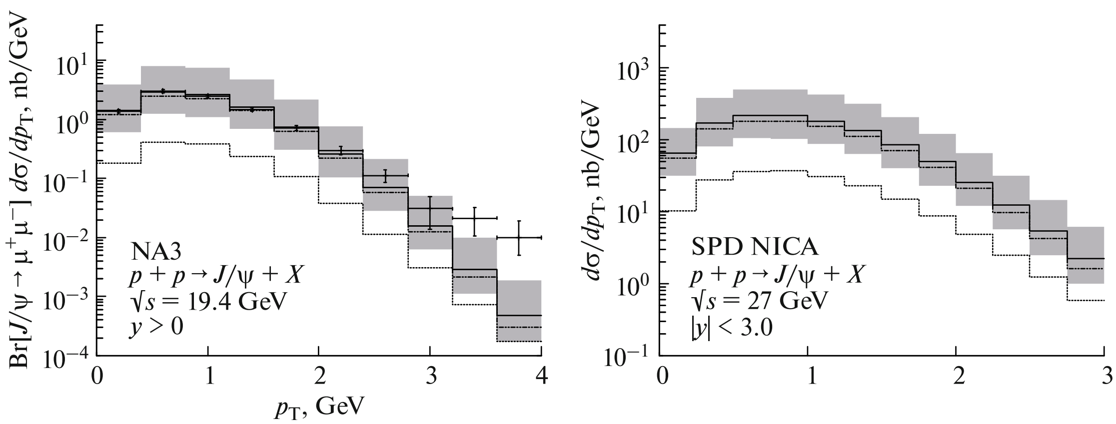


**Fig. 2.** Transverse momentum differential cross section for prompt $J/\psi$ production at the $\sqrt{s}$ = 19.4 GeV (left panel) and $\sqrt{s}$ = 27 GeV (right panel). Dotted histogram is the color octet of NRQCD contribution, dashed histogram is the color singlet of NRQCD contribution. Solid histogram is they sum.

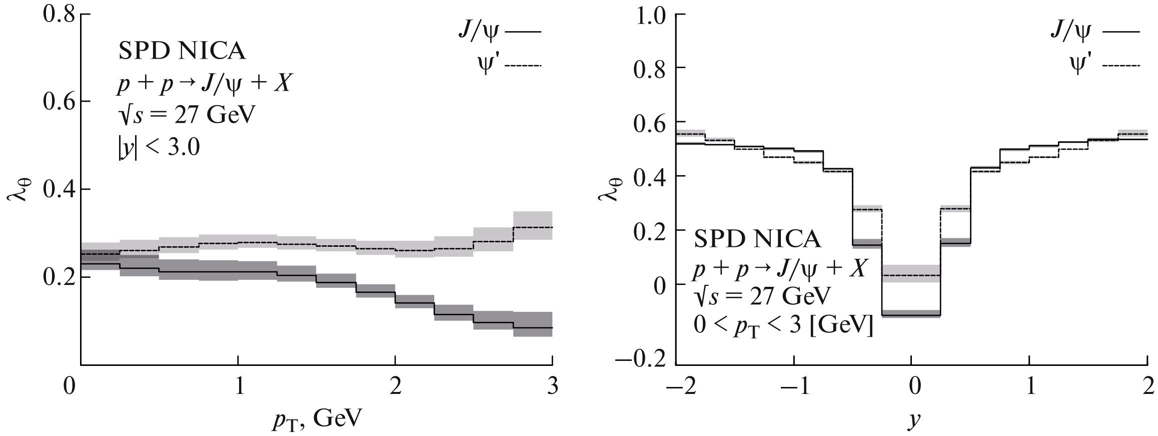


**Fig. 3.** Polarization parameter in the helicity reference frame for prompt $J/\psi$ (solid histogram) and direct $\psi(2S)$ (dashed histogram) production as functions of the $J.\psi$ transverse momentum (left panel) and $J/\psi$ rapidity (right panel) at the $\sqrt{s} = 27$ GeV and $|y| < 3$.

## 6. TSSA IN $J/\psi$ PRODUCTION

The Transverse Single-Spin Asymmetry (TSSA) measured in the process $p + p^{\uparrow} \to J/\psi X$ is defined as

$$A_N^{J/\psi} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\Delta\sigma}{2d\sigma}, \qquad (12)$$

where $p^{\uparrow,\downarrow}$ is transversally polarized proton. The cross section for unpolarized proton–proton collisions is presented as convolution of two gluon TMD PDFs and parton-parton cross section,

$$d\sigma \propto \int dx_1 d^2 q_{1T} dx_2 d^2 q_{2T} F_g(x_1, q_{1T}, \mu_F) \times F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to J/\psi\, g).$$

The numerator of the $A_N^{J/\psi}$ depends on the difference of the polarized gluon TMD PDFs,

$$d\Delta\sigma \propto \int dx_1 d^2 q_{1T} dx_2 d^2 q_{2T} \Delta\hat{F}_g^{\uparrow}(x_1, \mathbf{q}_{1T}, \mu_F) \times F_g(x_2, q_{2T}, \mu_F) d\hat{\sigma}(gg \to J/\psi\, g),$$

where

$$\Delta\hat{F}_g^{\uparrow}(x_1, \mathbf{q}_{1T}, \mu_F) \equiv \hat{F}_g^{(\uparrow)}(x_1, \mathbf{q}_{1T}, \mu_F) - \hat{F}_g^{(\downarrow)}(x_1, \mathbf{q}_{1T}, \mu_F).$$

The gluon Sivers function (GSF) describes the number density of unpolarized gluons (or quarks) with intrinsic transverse-momentum $q_T$ inside a transversely polarized proton $p^{\uparrow}$, with three-momentum $P$ and spin polarization vector $S$,

$$F_g^{\uparrow}(x, \mathbf{q}_T, \mu_F) = F_g(x, \mathbf{q}_T, \mu_F) + \frac{1}{2}\Delta^N F_g^{\uparrow}(x, \mathbf{q}_T, \mu_F)\mathbf{S}\cdot(\hat{\mathbf{P}} \times \hat{\mathbf{q}}_T), \qquad (13)$$

where $x$ is the proton light-cone momentum fraction carried by the gluon, $\Delta^N F_g^{\uparrow}(x, q_T, \mu_F)$ is the GSF, and symbol ( $\hat{}$ ) denotes a unit vector, $a = a/|a|$. Following the Trento conventions [17], GSF can be introduced as

$$\Delta\hat{F}_g^{\uparrow}(x, \mathbf{q}_T, \mu_F)) = \Delta^N F_g^{\uparrow}(x, \mathbf{q}_T, \mu_F)\cos(\phi), \qquad (14)$$

where $\phi$ is the azimuthal angle between the gluon transverse momentum $\mathbf{q}_T$ and reaction plane.

In our numerical calculations we use two different parameterizations for GSF obtained earlier in [18] which we call SIDIS1, and [19] which we refer to as GSF parametrization by D'Alesio et al. Corresponding values of parameters are collected in Table 1 of [20]. In Figs. 4 and 5, we compare predictions for TSSA obtained using two different hadronization mechanisms, NRQCD and ICEM, and we find sufficient difference between them. As we see, absolute values of $A_N^{J/\psi}$ strongly depend on choice of used GSF parameterizations, which is exactly unknown at the present time.

Due to initial-state interaction (ISI) and final-state interaction (FSI) of soft gluons, the Sivers function in the standard TMD PM and in the GPM approaches is process dependent and it is not clear how to extend factorization for the Sivers effect to the processes with colored final states, like $c\bar{c}$-pair in discussed here cases. To solve this problem, the Color Gauge Invariant GPM (CGI-GPM) was suggested [21]. The aim of CGI-GPM formalism is to extract above-mentioned process-dependence from the TMD PDF to the hard-scattering coefficient. The effects of ISI and FSI are included in CGI-GPM via one-gluon exchange approximation [21]. For the case of gluon Sivers effect, this approximation leads to appearance of independent GSFs of $f$-type ($\Delta^N F^{g(f)}$) and $d$-type ($\Delta^N F^{g(d)}$) correspond-

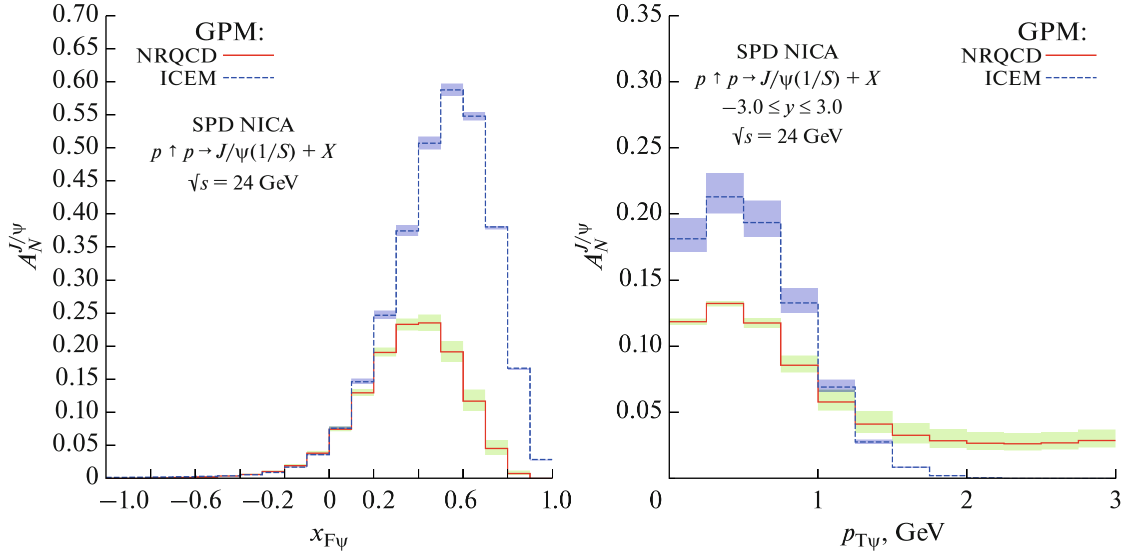


**Fig. 4.** Comparison of predictions in GPM for TSSA $A_N^{J/\psi}$ as function of $x_F$ (left panel) and transverse-momentum (right panel) at $\sqrt{s}$ = 24 GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SDIS1 parametrization of GSF is used.

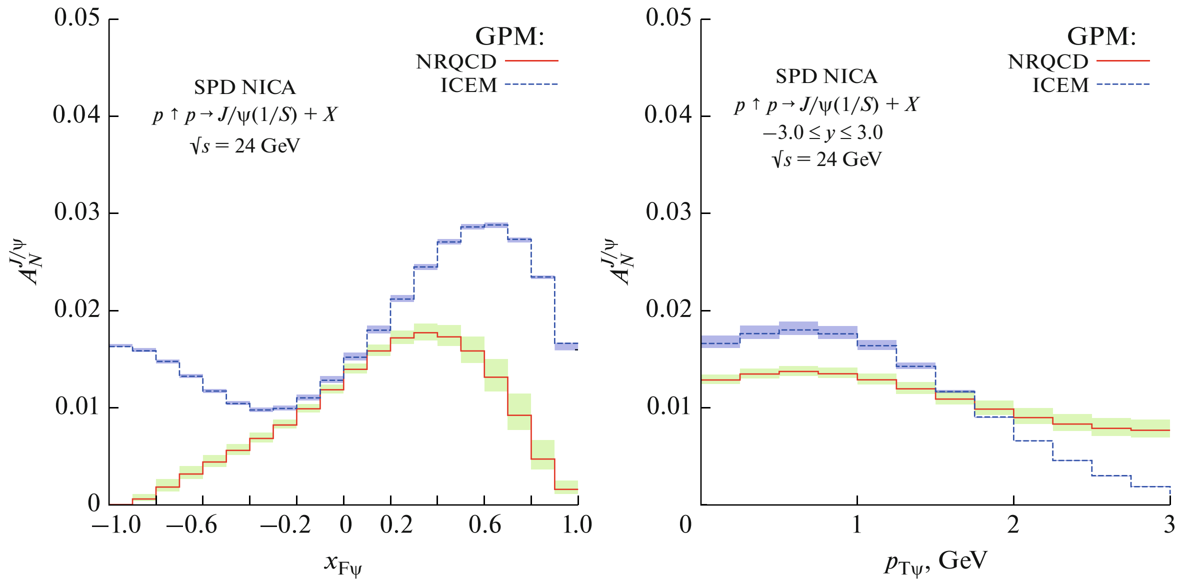


**Fig. 5.** Comparison of predictions in GPM for TSSA $A_N^{J/\psi}$ as function of $x_F$ (left panel) and transverse-momentum (right panel) at $\sqrt{s}$ = 24 GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The D'Alesio et al. parametrization of GSF is used.

ing to two independent ways of combining three gluons into a color-singlet state. The coupling of additional "eikonal" gluon from the GSF to the hard process leads only to modification of the color structure of the latter one. There is no four-momentum transfer from the additional gluon to the hard process, because Sivers effect comes from imaginary part of the loop integral over momentum of exchanged gluon, which is saturated by the contribution of the soft region [22].

Our predictions for $A_N^{J/\psi}$ obtained using the CGI-GPM approach are presented in Figs. 6 and 7. We find that in the CGI-GPM not only absolute value of $A_N^{J/\psi}$ depends on hadronization model, but predictions for

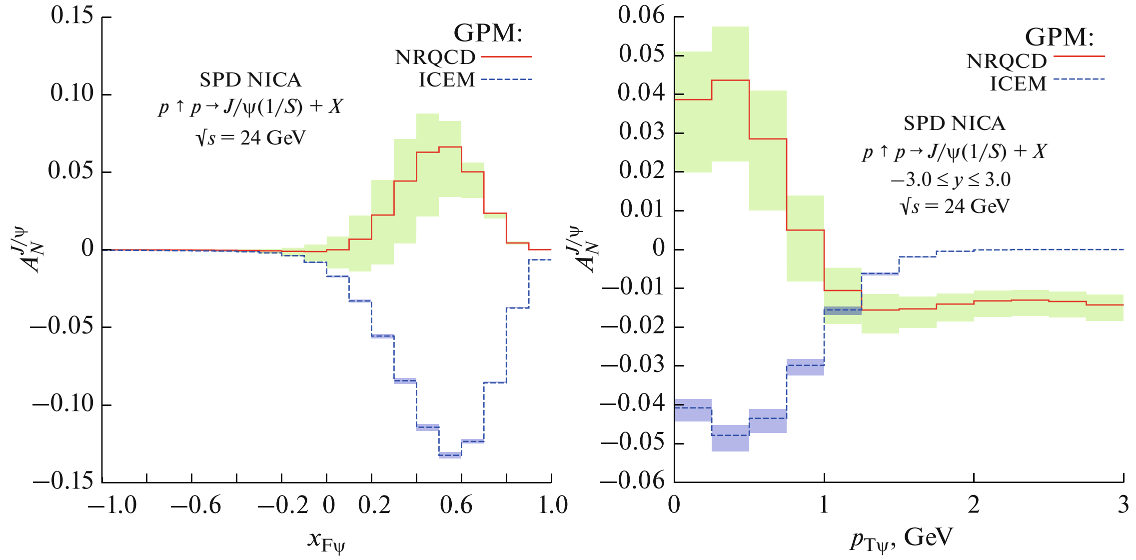


**Fig. 6.** Comparison of predictions in CGI-GPM for TSSA $A_N^{J/\psi}$ as function of $x_F$ (left panel) and transverse-momentum (right panel) at $\sqrt{s}$ = 24 GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The SDIS1 parametrisation of GSFs is used.

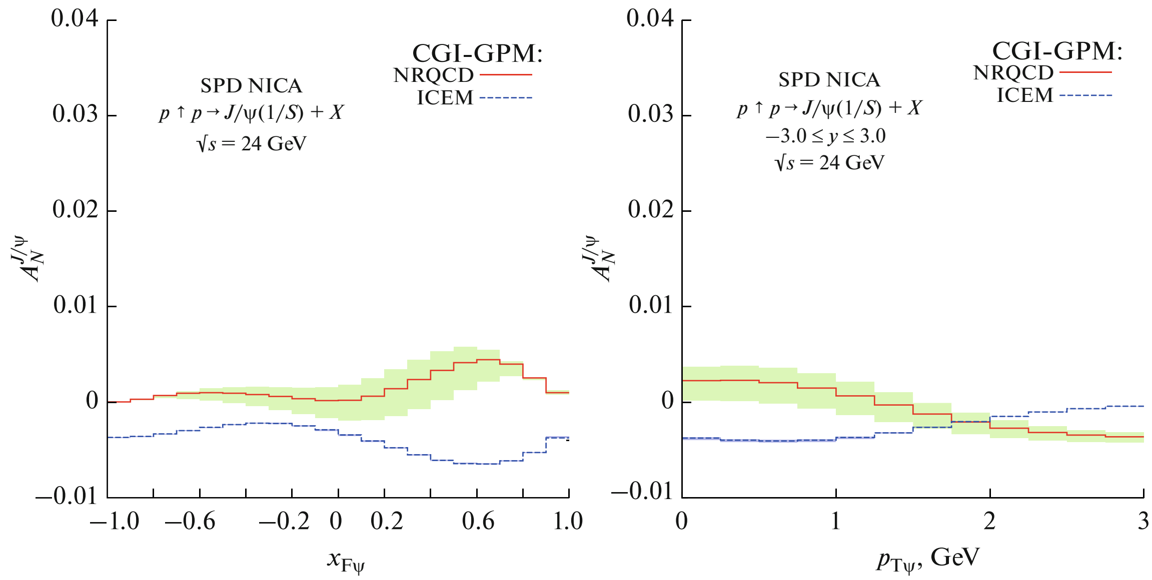


**Fig. 7.** Comparison of predictions in CGI-GPM for TSSA $A_N^{J/\psi}$ as function of $x_F$ (left panel) and transverse-momentum (right panel) at $\sqrt{s}$ = 24 GeV in NRQCD (solid histogram) and ICEM (dashed histogram) approaches. The D'Alesio et al. parametrisation of GSFs is used.

TSSA, obtained in NRQCD and ICEM, have different signs.

## 7. CONCLUSIONS

Taking in mind the main aim of future SPD NICA experiment, to study spin dependent gluon TMD PDFs in a proton using hard probes, we may conclude that prompt $J/\psi$ production as a tool to study proton TMD PDFs needs careful theoretical investigation. First of all, it should be concerned basics of the TMD factorization of the standard PM. The second one, active developments of heavy quark to heavy quarkonium hadronization models beyond the NRQCD or the ICEM should be continued.

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## CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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