

Fast simulation of nucleon momenta in deuteron for experiments at NICA SPD

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The Problem

Observation weakly excited (below π production threshold) deuteron states in $d-d \rightarrow d^*+d$ reactions at the colliding beams of the NICA SPD facility at JINR.

For what? Experimental detection of possible transformations in small-nucleon systems.

This may provide clues to understanding phase transitions in nuclear matter.

Two types of dibaryons

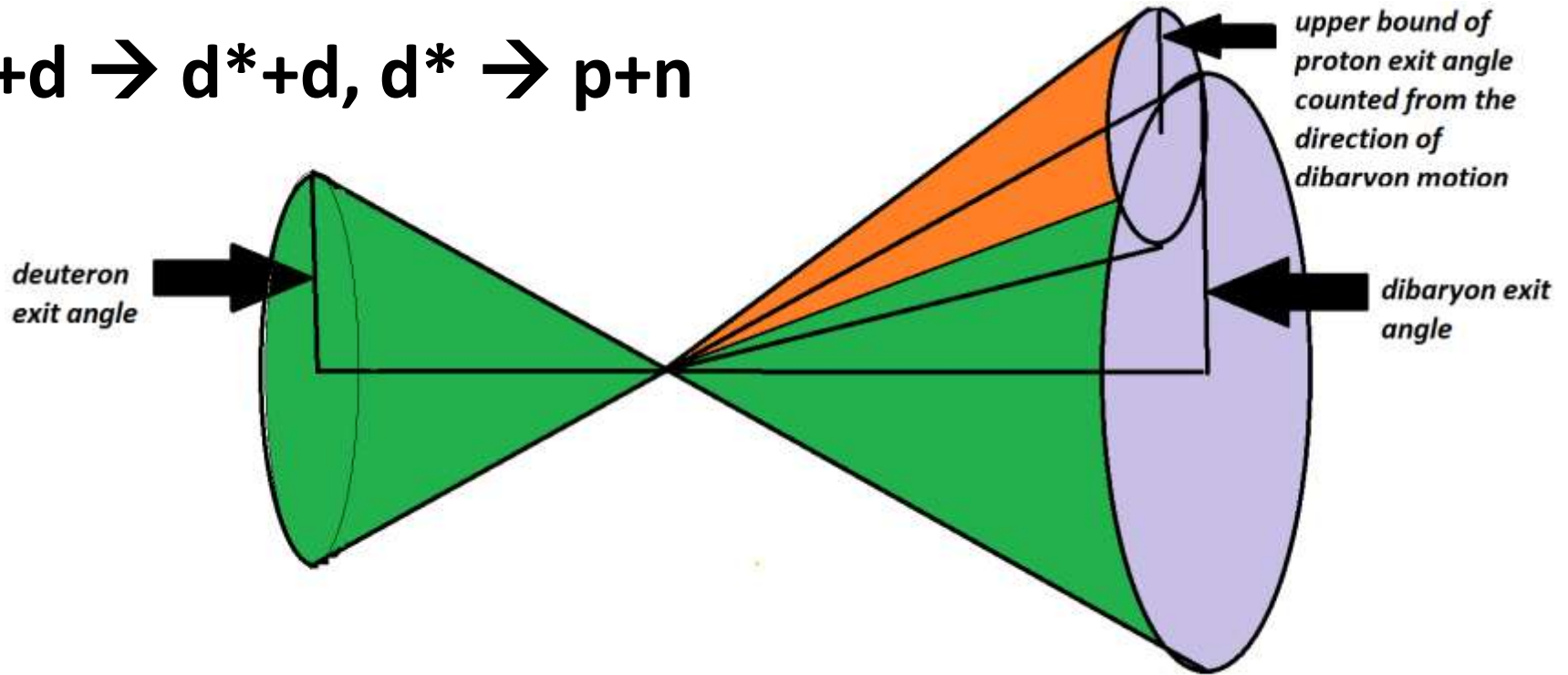
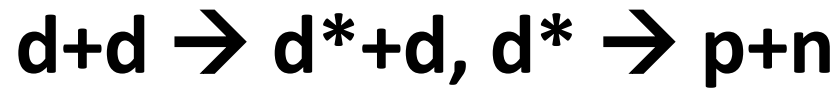
1. Six-quark systems possessing definite quantum numbers (among them mass, decay width, baryon number, spatial parity), as is the case for meson or baryon resonances

[Ю.А. Троян, ЭЧАЯ (1993)].

1. Small pieces of quark-gluon plasma, whose properties depend on the temperature inside them

[B. M. Abramov, Z. Phys. C (1996)].

Kinematic signature of dibaryon production



It is assumed that dibaryons can decay in collinear kinematics (along the direction of motion of the parent dibaryon).

Experiment in coincidence of hits d and p in counters located on opposite sides.

Results of calculations at $v_s=6.7$, $t=-0.5$

Cannot be used in any way

Great potential for
experiment improvement

We need to distinguish the proton
from the deuteron (TOF detector)

$$\langle \sigma \rangle = 0.033^\circ$$

Gauss distribution ↓

E_{ex} MeV	Θ_d °	P_d , GeV	P_{p1} GeV	P_{p2} GeV	t_d ns	t_{p1} ns	t_{p2} ns
10	14.6447	2.7723	1.537	1.233	4.025	3.905	4.188
20	14.6536	2.7689	1.613	1.154	4.026	3.856	4.296
30	14.6624	2.7655	1.668	1.096	4.028	3.824	4.387
40	14.6712	2.7620	1.712	1.048	4.029	3.801	4.473
50	14.6800	2.7586	1.751	1.006	4.031	3.781	4.558
60	14.6887	2.7551	1.785	0.968	4.032	3.766	4.642
70	14.6974	<u>2.7516</u> *)	1.816	0.934	4.034	3.752	4.724
80	14.7060	<u>2.7481</u>	1.844	0.902	4.036	3.740	4.809
90	14.7146	<u>2.7445</u>	1.870	0.873	4.037	3.729	4.893
100	14.7231	<u>2.7410</u>	1.896	0.845	4.039	3.719	4.980
110	14.7315	<u>2.7374</u>	1.917	0.818	4.041	3.711	5.072
120	14.7399	<u>2.7338</u>	1.939	0.793	4.042	3.703	5.163
130	14.7483	<u>2.7301</u>	1.959	0.769	4.044	3.696	5.355
140	14.7565	<u>2.7265</u>	1.979	0.746	4.046	3.689	5.355

$d+d \rightarrow d+d$: $\theta_d^{\text{el}} = 14.636^\circ$, $p_d = 2.7757$ GeV

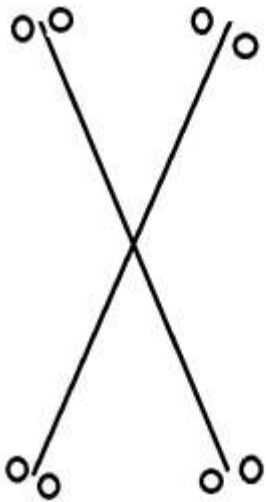
*) V.S.Kurbatov

The first and second stages of a possible experiment

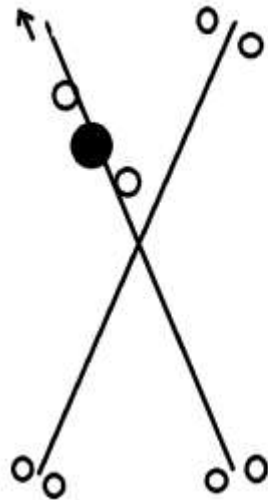
- At the **first stage**, which does not require a radical increase in measurement accuracy, one can verify the **very existence of the kinematic signature of dibaryons**.
- The **second stage** requires the improvement of the measurement accuracy of the proton momentum to 1%. If this can be done, then it will be possible to determine **whether the dibaryons are hot pieces of quark-gluon plasma, which have a temperature, or they are characterized by certain masses and widths**. If it turns out that the second possibility is realized, then a proton momentum measurement **accuracy of 1% would be sufficient to determine these characteristics**.

Why do we need a nucleon momentum generator at NICA SPD ?

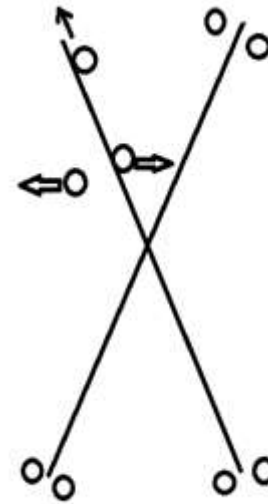
The kinematic **signature can be forged!**



Elastic d-d scattering

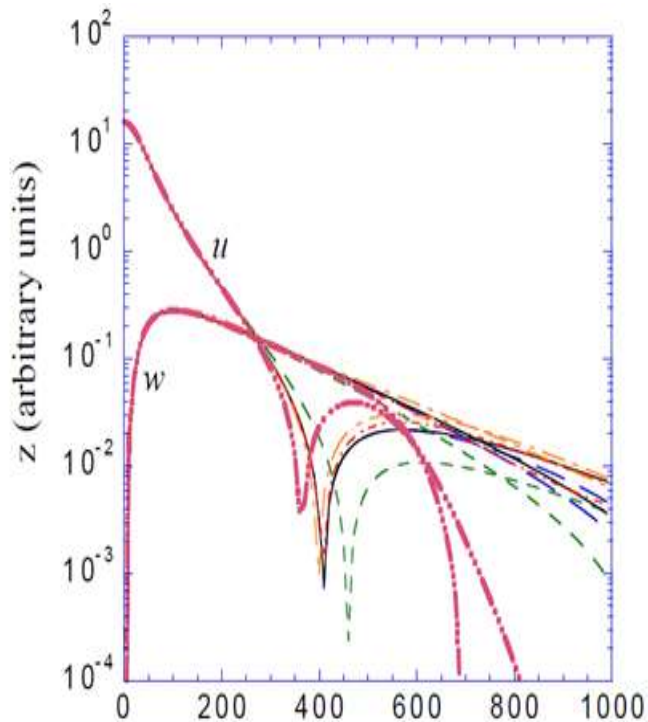


Kinematic signature of dibaryon decay: the proton flies into the region of elastic d-d scattering angles



Imitation (Fake): knocking out a high-momentum nucleon from deuteron by another colliding deuteron. The proton flies into the region of elastic d-d scattering angles, the scattered deuteron also flies approximately into this region

Challenges of Monte Carlo modeling



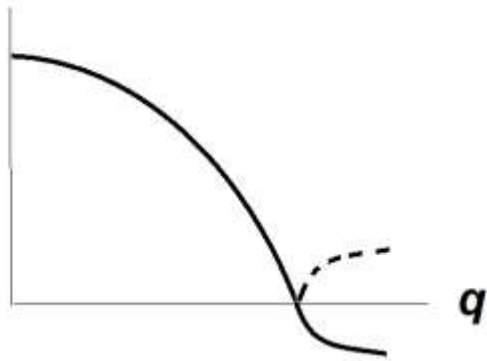
The u and w wavefunctions in p space for six models discussed in the text.

The von Neumann exclusion method is extremely ineffective here due to a **wide range of variation of probabilities** of admissible momentum values.

The **lack of an explicit analytical expression** for the cumulative distribution function seems to be an insurmountable obstacle to the use of another general approach - the method of inverse transformation of N. V. Smirnov.

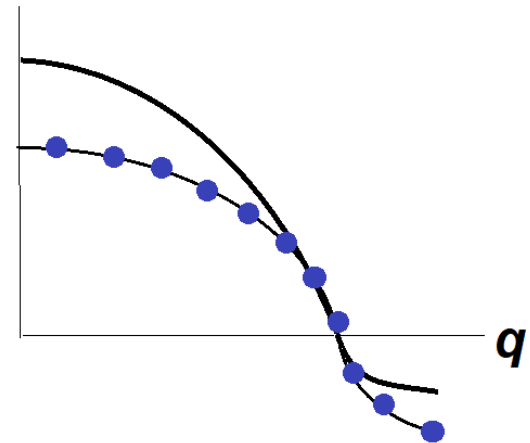
Nonlinear (smoothing) wave function transformation $\psi \rightarrow \phi$

wave function



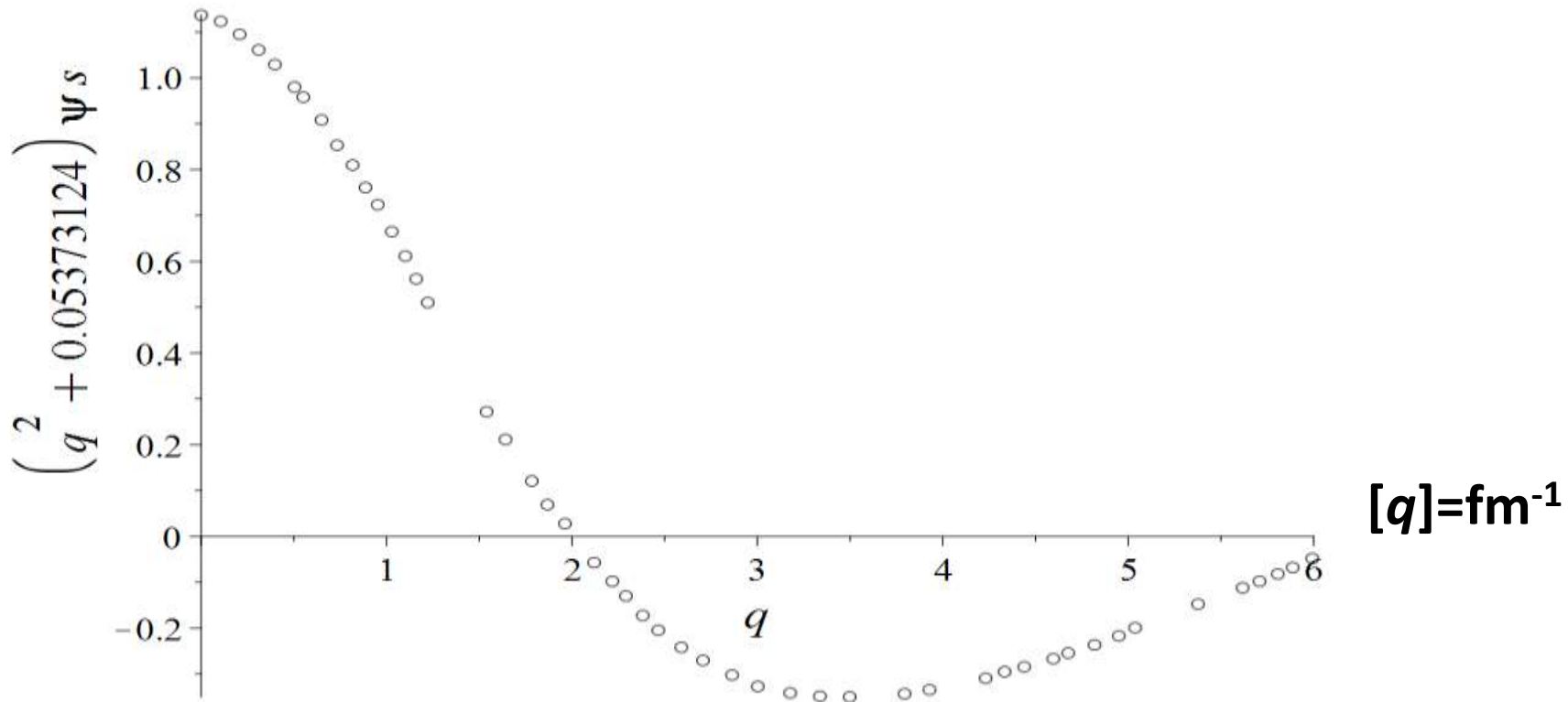
Getting rid of cusp by **reflection** into the region of negative values of the wave function.

wave function



Then we perform a nonlinear **smoothing** transformation and, after that, **split** (by eye) the curve into approximately linearly varying segments.

Deuteron's **transformed** wave function,
 $\phi = \psi \cdot (q^2 + a^2)$, for S-state after segmentation
 looks like this ($a = m_N \cdot E_d \text{ binding}$)



Linear interpolation in
 each partition interval:

Tangent of the slope
 of a line segment

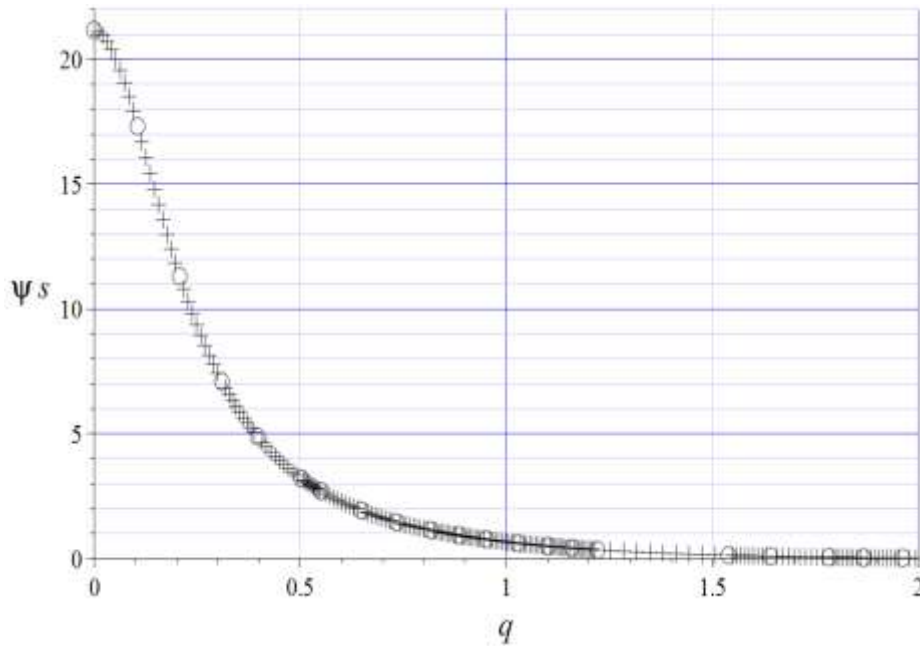
$$u(q[i]+x) = u(q[i]) + k_i \cdot x,$$

where $0 \leq x \leq q[i+1] - q[i]$,

$$k_i = (u(q[i+1]) - u(q[i])) / (q[i+1] - q[i]).$$

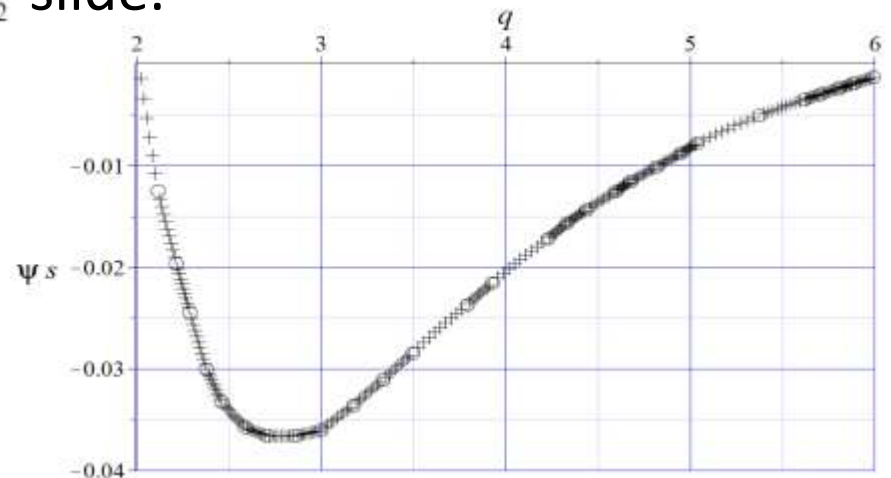
Return to the **true** wave function ψ :

$$\psi = \phi / (q^2 + 0.2318^2)$$



The **circles show digitized values** of the wave function from [Brown, Jackson. Nucleon-nucleon interactions](#). The crosses show the values of the wave function obtained by the **linear interpolation for ϕ** described on the previous slide.


It seems surprising that the same analytic formula (shown in the slide title) depicts very different behavior of the wave function between tabulated points (circles).



Cumulative probability function for the i -th interval

Geometric multiplier for integration in 3-D momentum space

Interpolation for wave function ψ is in curly brackets

$$\int_{q_i}^{q_i + x} \left\{ \frac{(u_i + k_i \cdot x)^2}{((q_i + x)^2 + 0.2318^2)^2} \right\} \cdot (q_i + x)^2 dx$$


The integral **can be computed analytically** and is cumbersome.

Obtained analytical expression, $F(u, q, k, x)$, for the integral admits a numerical solution of the equation $\mathbf{q} = \mathbf{F}^{-1}(\xi)$. Thus, the problem of drawing the values of the random variable q in accordance with its cumulative probability distribution may be solved by the inverse transformation method.

Program of drawing random values of nucleon momentum according to the deuteron wave function is very short

```
num := 1; for i from 1 to 100000 do if  $y[i] \leq \text{Scaled}[num, 1]$  then interval := 1 elif  $y[i]$   
   $\leq \text{Scaled}[num, 2]$  then interval := 2 elif  $y[i] \leq \text{Scaled}[num, 3]$  then interval := 3 elif  $y[i]$   
   $\leq \text{Scaled}[num, 4]$  then interval := 4 elif  $y[i] \leq \text{Scaled}[num, 5]$  then interval := 5 elif  $y[i]$   
   $\leq \text{Scaled}[num, 6]$  then interval := 6 elif  $y[i] \leq \text{Scaled}[num, 7]$  then interval := 7 elif  $y[i]$   
   $\leq \text{Scaled}[num, 8]$  then interval := 8 elif  $y[i] \leq \text{Scaled}[num, 9]$  then interval := 9 elif  $y[i]$   
   $\leq \text{Scaled}[num, 10]$  then interval := 10 elif  $y[i] \leq \text{Scaled}[num, 11]$  then interval := 11 elif  $y[i]$   
   $\leq \text{Scaled}[num, 12]$  then interval := 12 end if; ind := 4 interval - 3; for j from 1 to 3 while  $y[i]$   
   $> \text{PFi}[num, ind]$  do ind := ind + 1 od;  $q[i] := \text{fsolve}(\text{PFi}[1, ind] + F(ui[1, ind], qi[1, ind],$   
   $ki[1, ind], X) - Fi[1, ind] = y[i], X)$  end do:
```

A personal computer (RAM 3.5 GB, processor with two cores at 3 GHz) calculates 100 000 random values of the nucleon momentum in the deuterium over the entire admissible range of its variations in only 16 minutes.

Thank you for your attention!