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Application of the maximum likelihood method to estimate the parameters of elementary particle trajectories in the reconstruction problem of the internal detector of the SPD NICA experiment.

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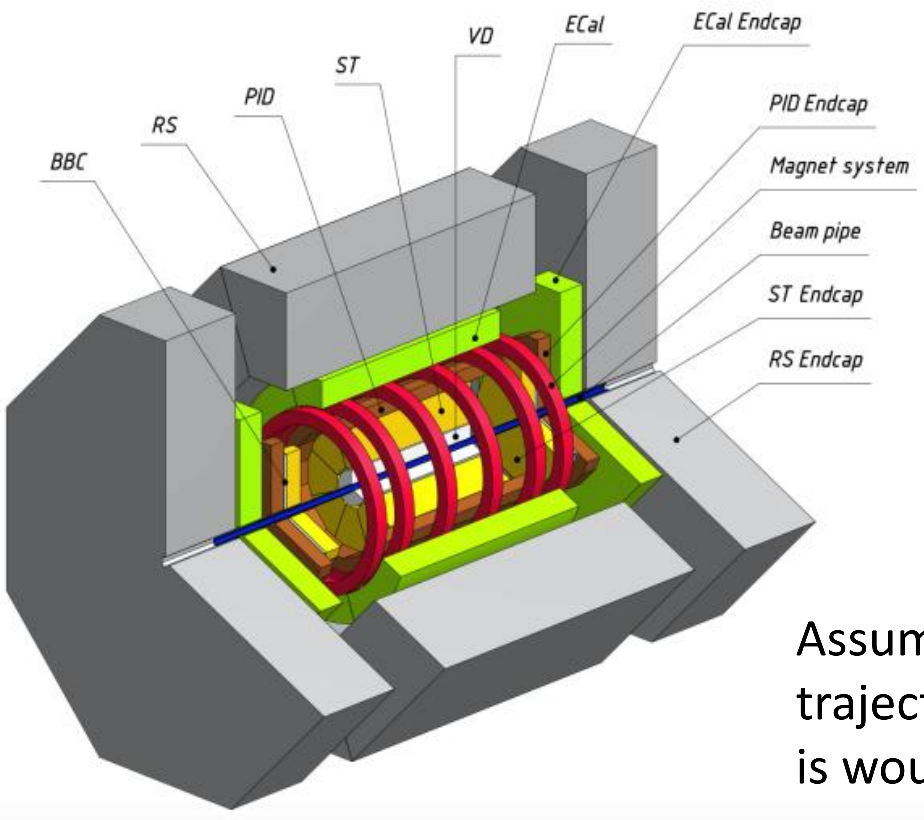
SPD setup, magnetic field and tracking detectors

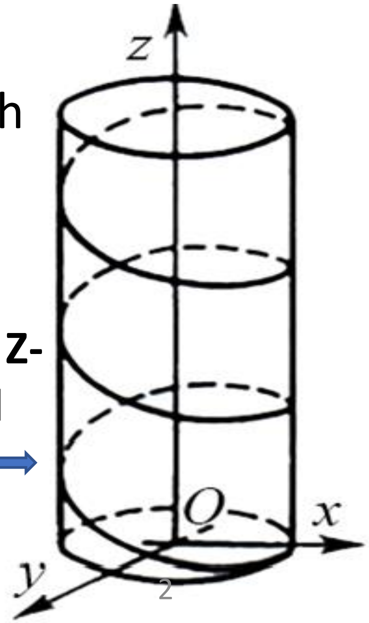


The SPD (Spin Physics Detector) is a planned spin physics experiment in the second interaction point of the NICA collider that is under construction at JINR. The main goal of the experiment is the test of basic of the QCD via the study of polarized structure of nucleon and spin related phenomena in the collision of polarized protons and deuterons.
Due to the low mass of interacting particles, the multiplicity of events is also relatively small, not exceeding 20 tracks per event

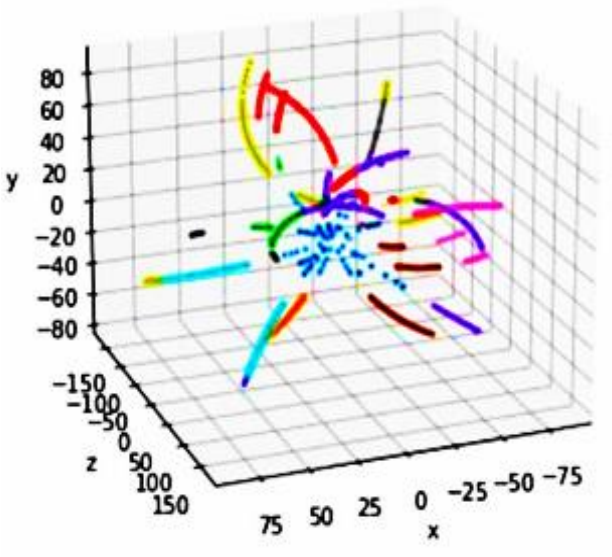
General layout of the SPD setup.
VD- pixel Vertex Detector;
ST- Strow-Tracker.

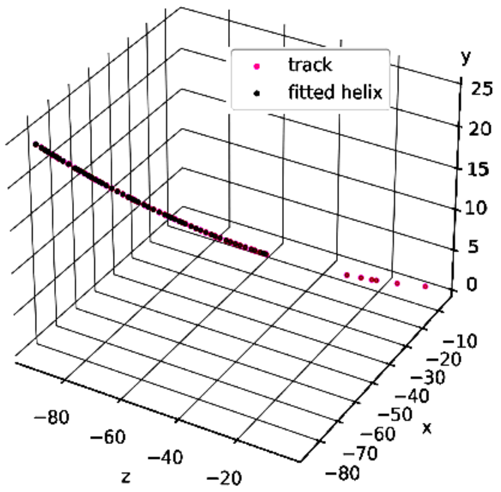
Assuming the **magnetic field homogeneity** of the detector, the trajectories of the particles must be close to a **helical line**, which is wound on a cylinder with its axis pointing along the **Oz axis according to the magnetic field direction.**

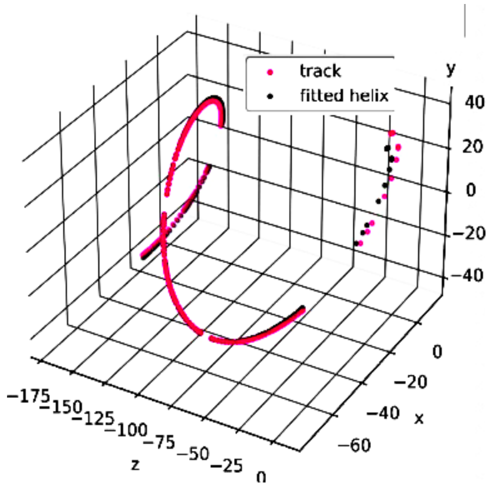
The beam direction coincides with Z-axis, which is the axis of cylindrical helix line



Simulated data and particle tracking







Tracking or track finding is a process of reconstructing **particle trajectories** in a high-energy physics detector by connecting the dots - hits - that each particle leaves behind as it passes through the detector's sensitive elements.

Examples of simulated events, tracks registered in different detectors are shown different colors. On the right are views of two separate tracks

We use for tracking TrackNetv3 program, based on deep recurrent neural network. It is trained and tested on a sample obtained by Monte Carlo simulation of events for simplified detector geometry.

However, during the neural network reconstruction of tracks the **appearance of false track-candidates**, formed from pieces of close neighboring tracks, noise samples, etc., **is inevitable**.

Our problem: remove false track-candidates

Solution: **develop a threshold criterion** that responds to violation of the smoothness of the particle trajectory, the presence of kinks, outliers of individual measurements, etc. **by exceeding a specially selected threshold**

Equations of the helix and its parameter estimates by maximum likelihood method

Given that the projection of the helical line on the xOy plane forms a circle of radius R centered at (x0,y0), which touches the origin, and the helical line itself is inclined to this plane at an angle with tangent λ, we obtain the following equations

$$\begin{cases} (x-x_0)^2+(y-y_0)^2 = R^2 \\ y_0^2 = R^2-x_0^2 \\ z= \lambda \, \mathrm{arctg}((y-y_0)/(x-x_0)) \end{cases}$$

The values of the helical line parameters for each track with n measured points on it (x_i, y_i, z_i); i=1,2,...n were estimated separately. The parameters R, x_0, y_0 were estimated from the first two equations, and the estimate of the parameter λ was obtained from the third equation. Assuming the normality distribution of the measured point deviations from the helical line, **the maximum likelihood method was reduced to the least squares method (LSM).**

To estimate the parameters of the circle R, x_0, y_0 in the xOy plane, we had to minimize a functional of the form

$$F(R, x_0, y_0) = \sum_{i-1}^{n} ((x_i - x_0)^2 + (y_i - y_0)^2 - R^2)^2$$

Since the measured part of the track is only a small arc of the circle, it was necessary to normalize this functional by dividing it by the approximate circle gradient R^2. Subsequently, a suitable substitution of variables and the application of Newton's method allowed us to obtain estimates of the circle parameters in 2-5 iterations with accuracy on the order of 10^{-3}.

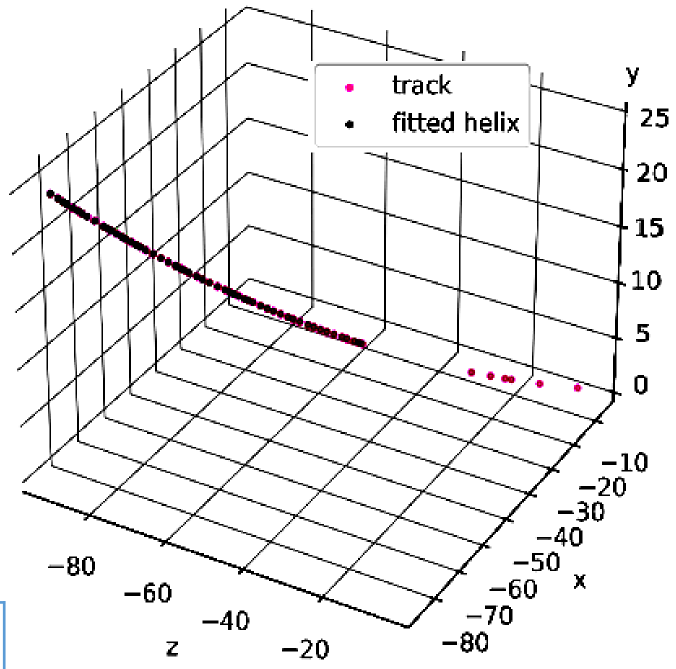
The estimate of the parameter λ from the third equation using the LSM is obtained as $\boldsymbol{\lambda = \frac{\sum_{i=1}^{n} z_i(\varphi_i - \varphi_0)}{\sum_{i=1}^{n}(\varphi_i - \varphi_0)^2}}$,

where $\varphi_i = arctg(y_i\text{-}y_0)/(\,x_i - x_0)$, and $\varphi_0 = 0$.

Hi-square a metric to distinguish between true and false track-candidates

The root-mean-square error of the helix fit to measured points (x_i, y_i, z_i); i=1,2,...n is calculated as follows

$$\chi^2 = \frac{1}{n-3}\sum_{i=1}^{n}\left((x_i - x_0)^2 + \left(y_i - \sqrt{R^2 - x_0^2}\right)^2 - R^2\right) + ((z_i - \lambda\varphi_i))^2$$



As an example, Table 2 presents the results of estimation of R, x_0, y_0, λ and calculated values of χ^2 for track №5 of one of the model events shown in Figure

Table2. Parameters x_0,y_0, R, λ, χ^2 for track №5

Track number	Number of hits	x_0	y_0	R	lambda	chi2
5	53	85.126934	47.200887	97.100687	286.866152	0.000524

Calculations for the optimal threshold search

No	Mean χ2	Root-mean-square χ2	number of items
0	4.517840	0.357003	121
1	**0.000584**	**0.001576**	**16839**
2	0.022471	0.008011	1091
3	0.058520	0.013088	677
4	0.121041	0.023318	419
5	0.226017	0.037765	277
6	0.384177	0.058412	259
7	0.618191	0.080733	211
8	0.941860	0.100519	182
9	1.324644	0.132633	161
10	1.864959	0.173176	144

Table. Clustering results for the first 10 clusters with the lowest chi-squared values

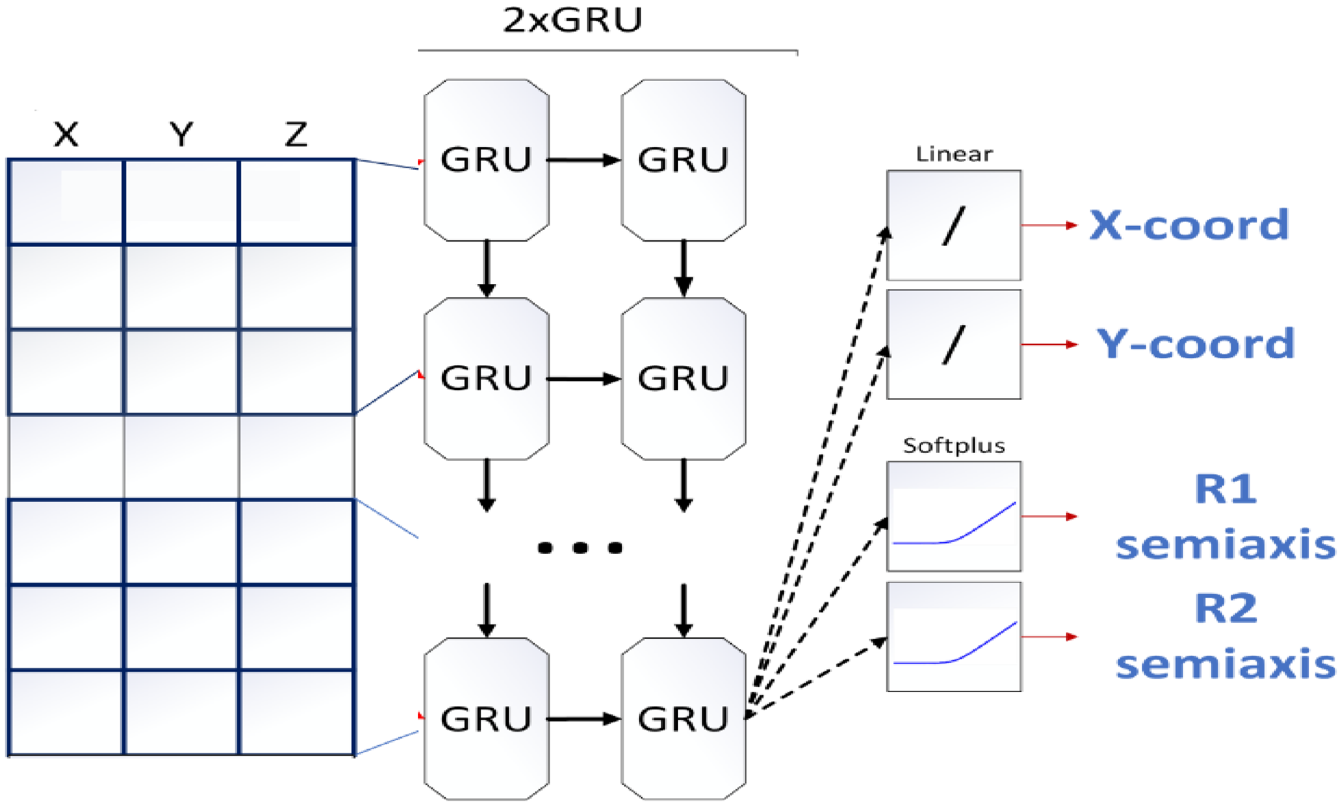
We clustered the tracks of all events according to the value of χ^2 of the helix line fit to these tracks. The results of clustering can be judged from the table on the left, which presents the first 10 clusters in descending order of the number of tracks in them

An analysis of the clusters by root-mean-square of χ^2 values for every cluster and number of elements in them showed **that 83% of the tracks fell into cluster №1,** which has the lowest mean and root-mean-square values of χ^2 .

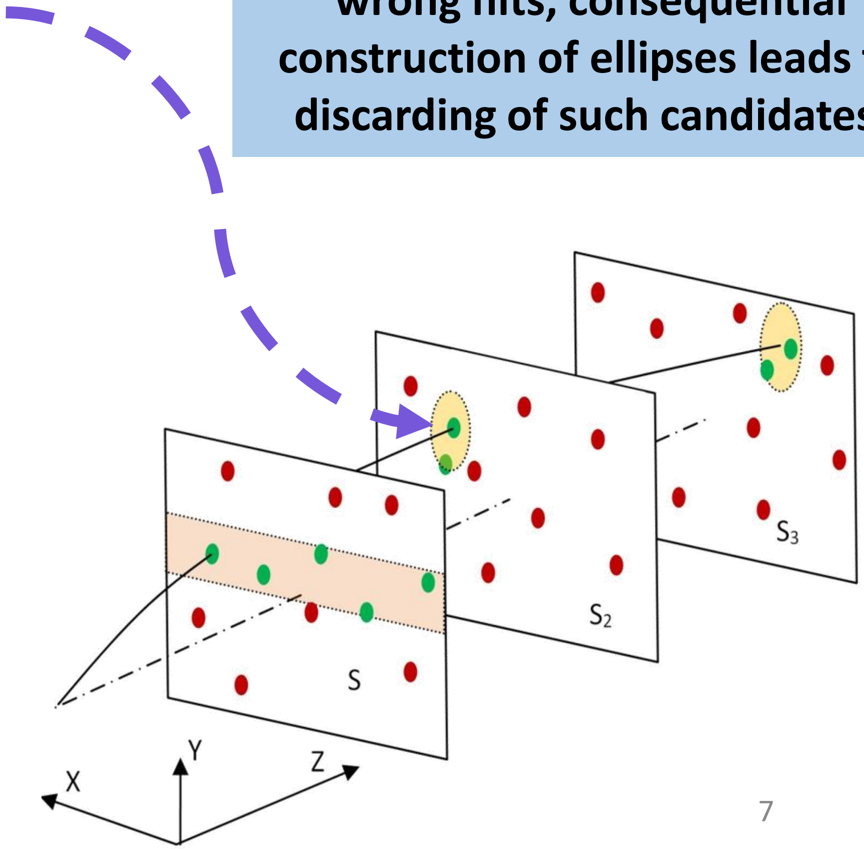
The other clusters have noticeably higher errors in the helical fitting procedure for the tracks.

Therefore we use the events from cluster #1 to obtain the needed threshold for our criterion

SPD track reconstruction using TrackNETv3 neural network



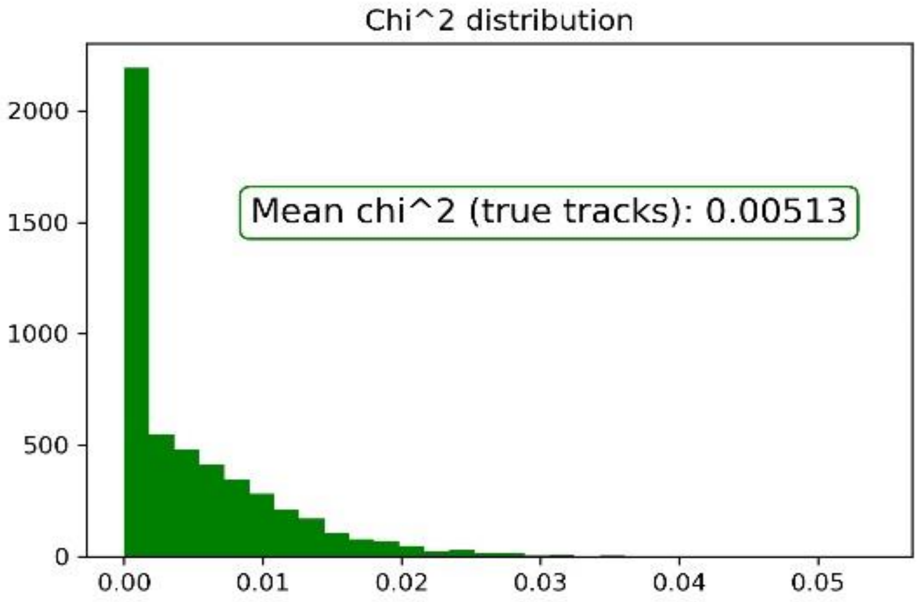
The prediction is an ellipse in which we search for the next candidate hit. So, one by one, we construct candidate tracks, and even if early ellipses contain wrong hits, consequential construction of ellipses leads to discarding of such candidates.



TrackNETv3 is an algorithm for local tracking that allows us to restore each track station by station.

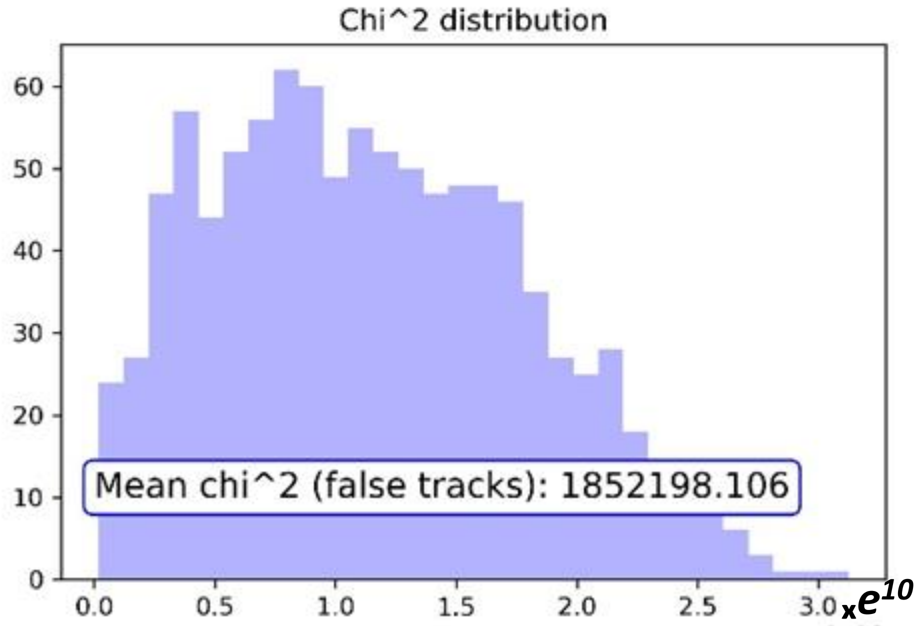
TrackNETv3 uses recurrent architecture to extract temporal features from data.

χ^2 – distribution for true and false tracks



χ^2 **distribution for true tracks**

The trained model is able to reconstruct more then 90% of the tracks. But at the same time, along with the true tracks, the neural network outputs false tracks, which number is much higher.



χ^2 **distribution for false tracks**

Thus, in the total number of reconstructed tracks, the **share of true tracks is only 2%** .

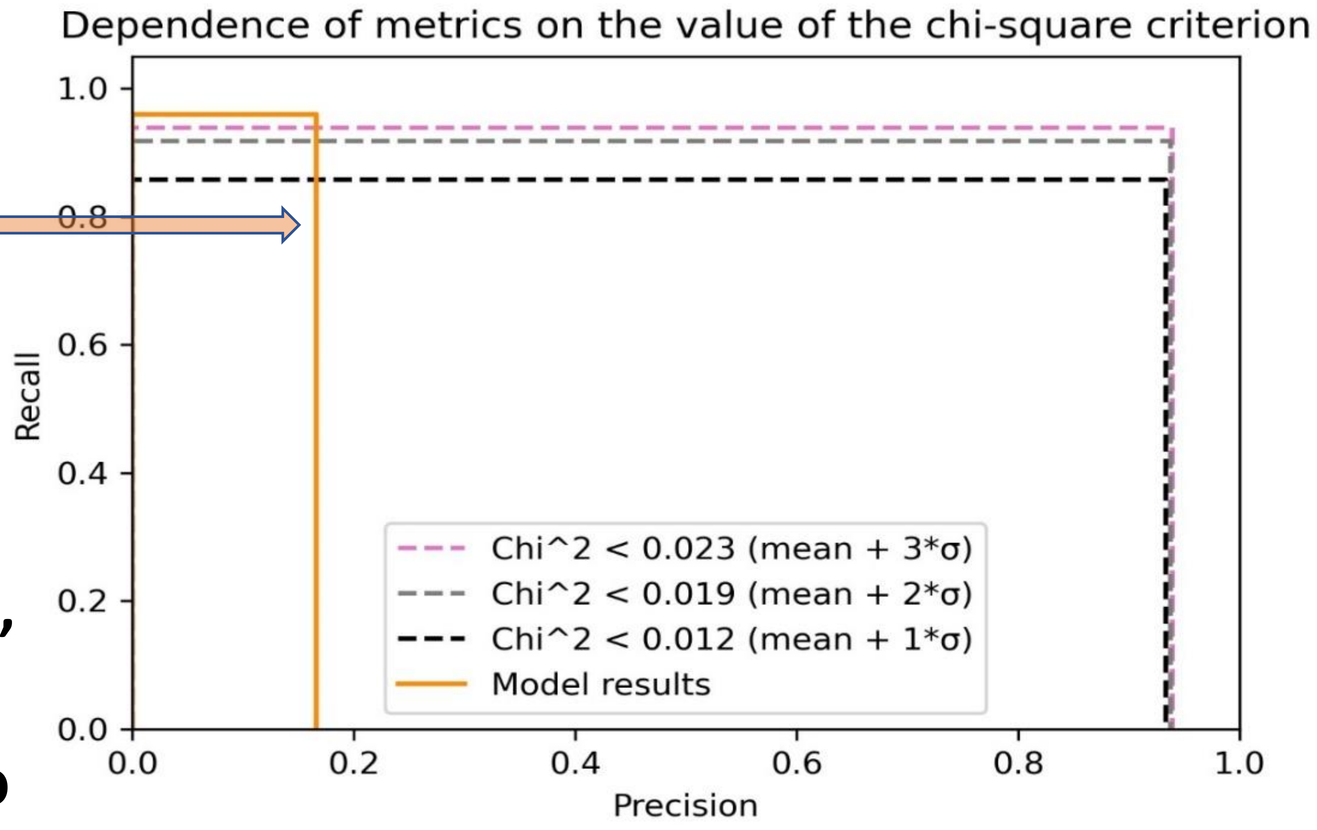
In order to increase the proportion of true tracks in the total number of all reconstructed tracks, a value of χ^2 **for true tracks** can be used as a screening criterion. We calculate for all true tracks in cluster #1 χ^2 **mean** and root mean square (***σ***) and construct the threshold as ***t=mean + Kσ.*** The multiplier ***K*** must be selected very carefully in order to avoid some unwanted recall downgrade.

Results of applying χ^2 criterion for false track removing

The quality of track recovery is assessed using two metrics: recall and precision.
Recall is the percentage of true tracks that have been restored completely.
Precision reflects the proportion of true tracks among all tracks found by the model.

Result BEFORE the criterion applying

Recall	0,93
Precision	0,16



After applying criterion *t* with *K*=3 we obtain Precision increase from 16% to 93%, keeping Recall on the same level

Inference speed on a **single Nvidia Tesla V100**
Mean gpu time per event 0.1952 sec

Conclusion

- **RMSE of helical line fitting to the tracks reconstructed by TrackNETv3 can be used as a criterion for screening out false tracks.**
- **An appropriate choice of threshold for χ^2 increases precision metric from 16% to 93%.**
- **The high speed of the helical fitting algorithm allows this method to be used for real-time data filtering.**

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Thank you for the attention!

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