# Physics. Generalized Parton Distributions, polarization phenomena at high pT, other polarization phenomena

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JINR, Dubna

# Outline

- QCD factorization and hadron spin structure: types of spin-dependent NPQCD functions

- GPDs, pressure in proton and exclusive DY
- High $p_T$ - vector and TENSOR (related to SHEAR forces) polarization

- Transitions: Exclusive-Inclusive; Hadronic - HIC

# SPD – 3D hadronic spin structure

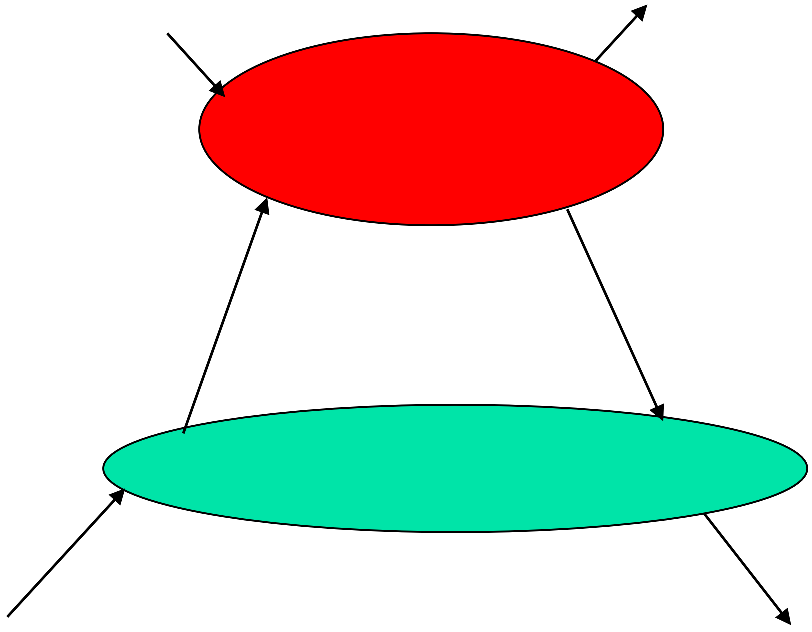
- Modern description – Wigner function

- Quantum mechanical measurement – probe is crucial

- Various complementary probes are important

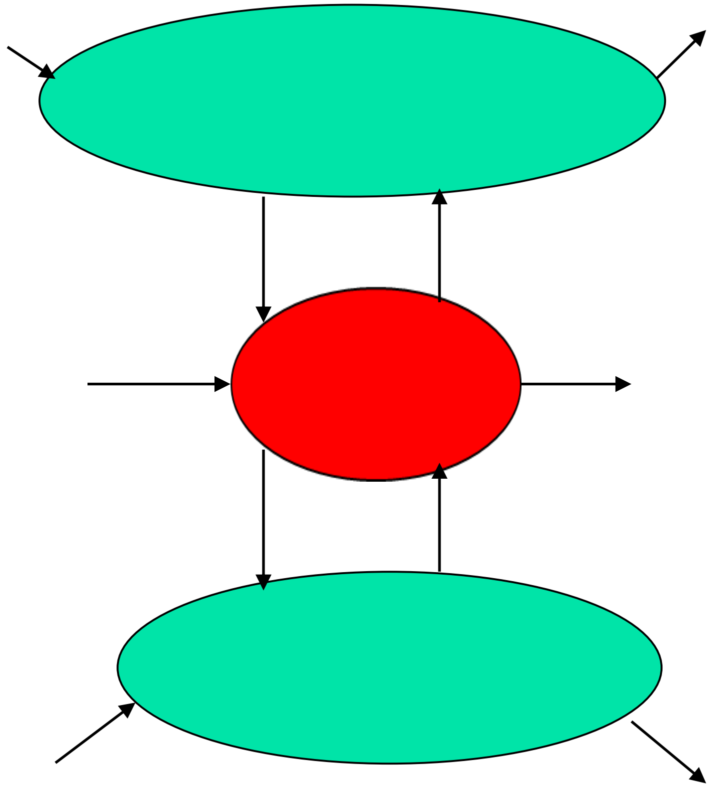
# Factorization (lh-> DIS, DVCS)

- Short and hard distances separated (JINR – Efremov, Radyushkin; Higher twist – Efremov,OT; DVCS-Anikin,OT)



# Factorization for DY – type Inclusive and Exclusive

- 2 hadrons participate



# Recent progress

**Exclusive Drell-Yan process with two GPDs**

S.Goloskokov, P.Kroll and O.Teryaev in progress.

We consider quark-gluon and quark-quark effects

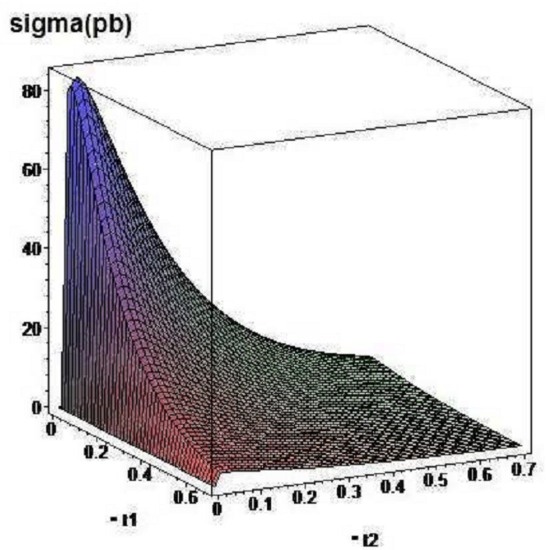
Problem- some divergencies like double pole appear in the amplitudes

Regularization procedure

$$\frac{1}{(x_1-\xi_1)(x_2-\xi_2)+i\epsilon} \to \frac{1}{[(x_1-\xi_1)+i\epsilon][(x_2-\xi_2)+i\epsilon]}$$

# First numerical results

Cross section is integrated over $s_1$ and $s_2$ was calculated at NICA energies
Preliminary result for cross section of $p\,p \to p\,p\,l^+\,l^-$ process at NICA energies




Preliminary results for cross section of exclusive Drell-Yan process over $t_1$ and $t_2$ at NICA energies. $\frac{d\sigma}{dQ^2 dt_1 dt_2}$ -in $pb/GeV^6$. Estimations show that such contribution might be visible.

Both final protons should be detected

Integrated over $t_1$ and $t_2$ cross section $d\sigma/dQ^2 \sim 5.5$ pb/GeV$^2$ at $Q^2 = 5$GeV$^2$ (NICA energies)

# Other development

- J/Ψ – enhancement by ~ $10^2$
- Both quark and gluonic couplings
- Interference with EM
- Ap (AA) UPC – photoproduction of J/Ψ – another probe of GPDs

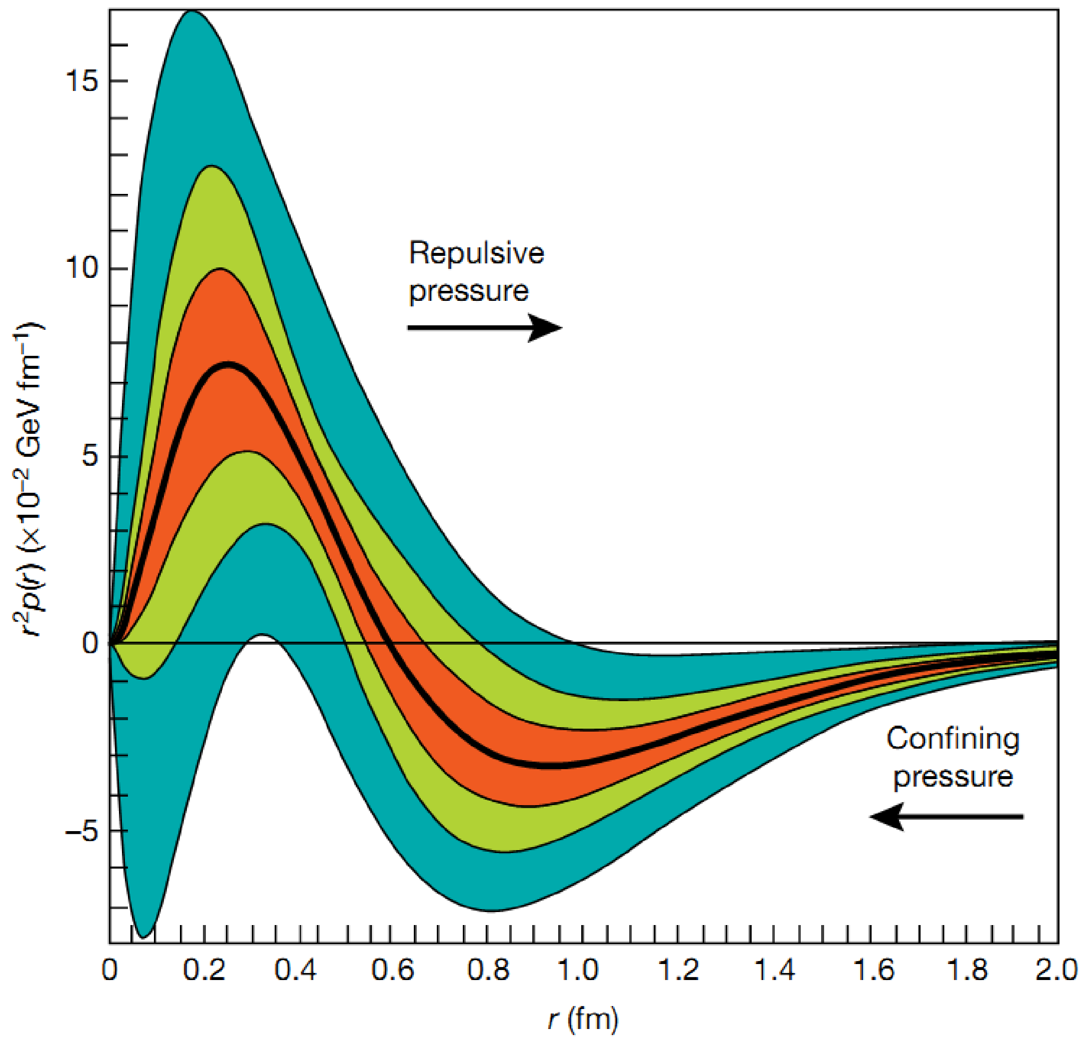
LETTER

NATURE | www.nature.com/nature

https://doi.org/10.1038/s41586-018-0060-z

# The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]



# Gravitational Formfactors (spin ½)

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

# Electromagnetism vs Gravity (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q) \qquad M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu \qquad \sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q) \qquad M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

# Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

  $$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

  spin dragging twice smaller than EM
- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

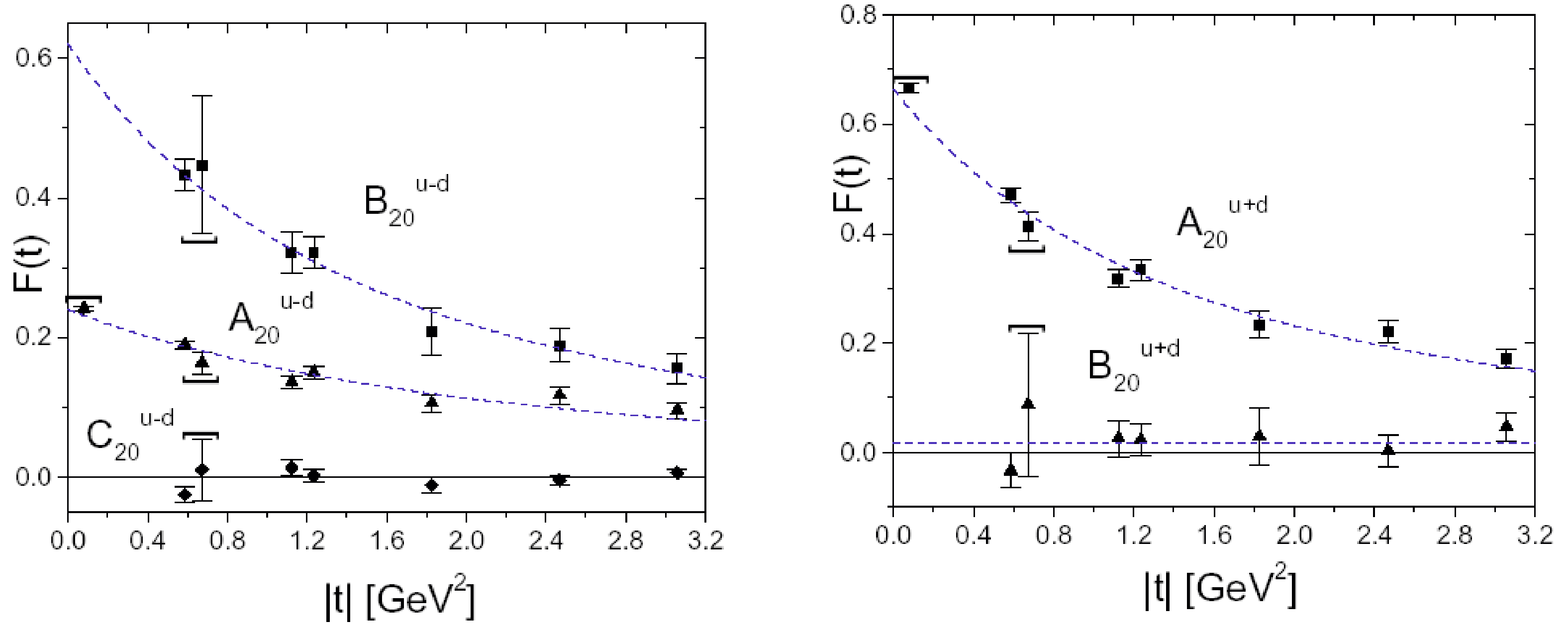
  $$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$
- Orbital and Spin momenta dragging – the same - Equivalence principle

# Equivalence principle

- Newtonian – "Falling elevator" – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun'; rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- Gravitational analog of Ji's SR $\int dx\ x\ \ (\Sigma\ E_q + E_G) = 0$!

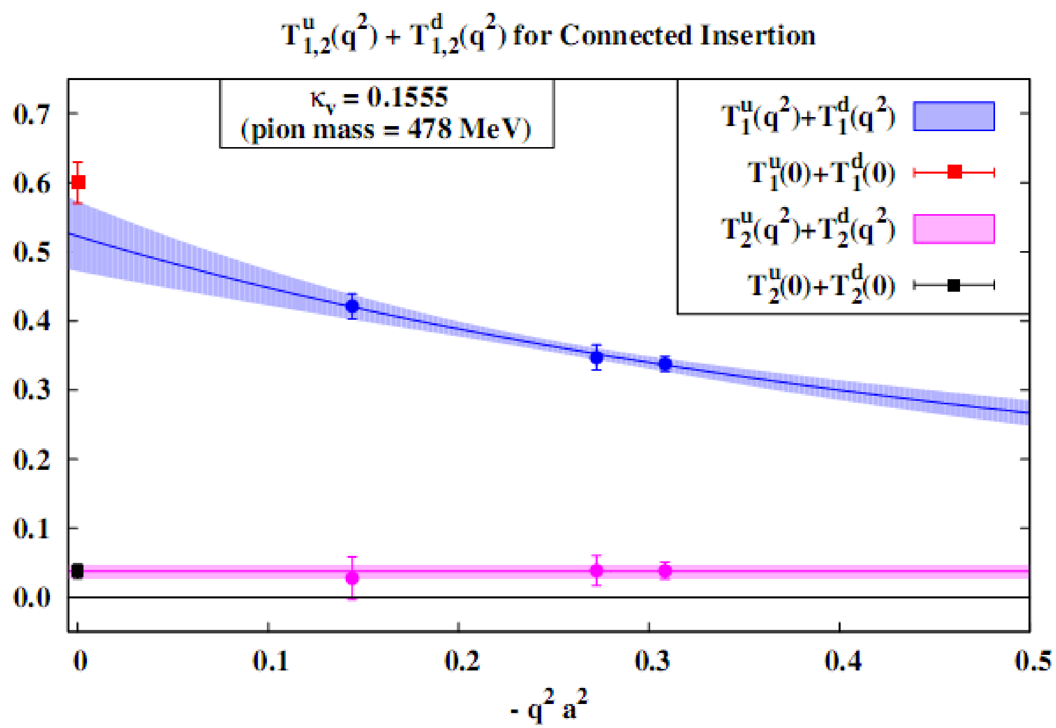
# Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



# Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



# Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of  ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observation of smalness of Cbar

# Spin-1 hadrons

- MANY new FFs!
- Recent extensive analysis
- Cosyn,Cotogno, Freese,Lorce: 1903.00408
- Polyakov, Sun: 1903.02738
- A lot of integral relations between GPDs and FFs

# Spin 1 EMT and inclusive processes

- Forward matrix element ->density matrix
- Contains P-even term: tensor polarization
- Symmetric and traceless: correspond to (average) shear forces
- For spin ½: P-odd vector polarization requires another vector (q) to form vector product

# Spin 1 in QCD

- Tensor polarization in QCD: Frankfurt, Strikman (81), Efremov,OT (81)
- Spin ½: kinematically enhanced longitudinal polarization transverse-twist 3
- Spin 1: LL/TT related by tracelessness

# SUM RULES

- We (A.V. Efremov,OT'81) derived zero sum rules:
- 1st moment: also in parton model by Close and Kumano (90)
- 2nd moment (forward analog of Ji's SR)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT'09,19

# Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

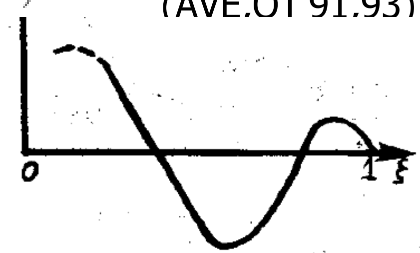
- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

(AVE,OT'91,93)




$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

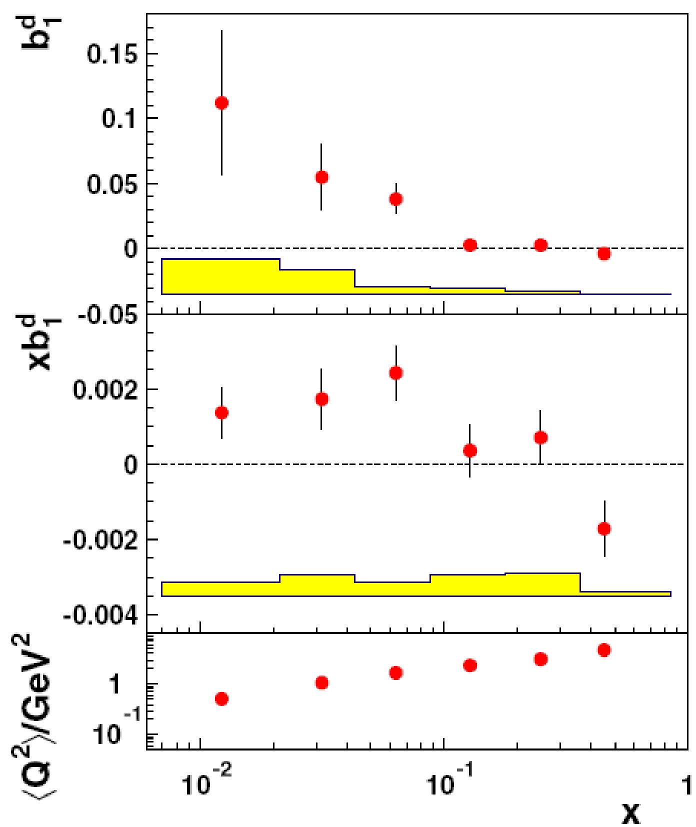
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2) = 0$$ for ExEP

# HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective glue << sea)



# Where else to test?

- COMPASS (2021)
- EIC
- DY@J-PARC: (Song,Kumano:1902.04712)
- However: ET'81-any hard process
- $f_{Al} \sim b_1$

$$P_{zz} = -2P_{yy} = -2P_{xx} = \frac{2\int_0^1 d\xi\, f_{Al}(\xi)\, Sp[\hat{P} E(\xi, P)]}{3\int_0^1 d\xi\, f(\xi)\, Sp[\hat{P} E(\xi, P)]} = \frac{2F_{Al}(x_1^*, x_2)}{3F(x_1, x_2)},$$

- Suggestion: hadronic tensor SSA

# Vector vs Tensor SSA

- Vector: $A = (\sigma(+)-\sigma(-))/(\sigma(+)+\sigma(-))$

- Tensor:
  $A = (\sigma(+)+\sigma(-))/(\sigma(+)+\sigma(-)+\sigma(0))$

- Inclusive pion production: (T-odd) vector SSA may be also excluded by summing $\sigma(L)+\sigma(R)$

# Tensor polarized beams

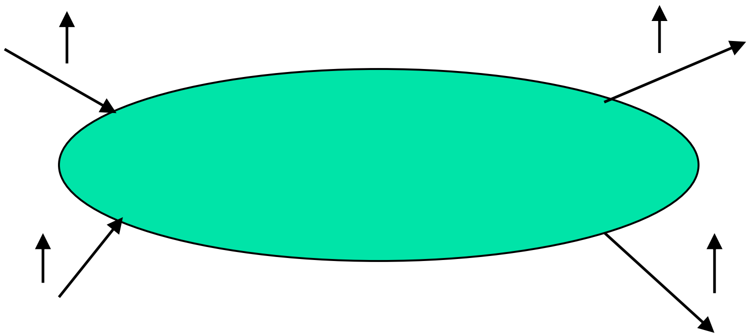
- Opportunity: NICA@JINR with polarized hadronic beams
- Polarized deuterons is easier to accelerate: no depolarizing resonances
- DY, J/Ψ (+hadronic SSA)

# Transitions

- Inclusive -> exclusive
- GPDs <-> TMDs
- P->d->A (pressure, shear…

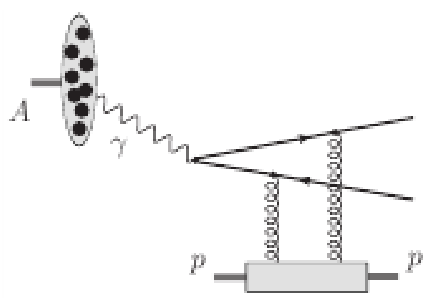
# Types of parton distributions

- Most general – Wigner function (: non-symmetric partonic and hadronic momenta with transverse components
- The spin of both hadrons and partons fixed



# Measurement of Wigner (GTMD) function

- Small x – Ip (Hatta,Xiao,Yuan'16) or Ap UP (Hagiwara, Hatta, Pasechnik, Tasevsky, OT'17) collisions




- Larger x – UPC at SPD (R.Tsenov)
- UPC – light exotics (axions) – DIS'19

# Types of parton distributions - II

- Too rich structure of Wigner function
- Simplifications – Putting some (transverse) momenta to zero or average over some variables
- Hadronic moments equal - inclusive
- Allow for proof of QCD factorization is some cases (perturbative corrections are taken into account by some kind of evolution)

# Collinear vs $k_T$ factorization

- Collinear: NP longitudinal and pQCD transverse (GLAPD) evolution
- BFKL (also perturbative origin!) NP transverse and pQCD longitudinal evolution
- GI for off-shell partons? $(xP + k_T)^2 < 0$
- Special BFKL vertices, effective action

# TMD factorization

- BFKL (with non-linear unitarizaing modifications – CGC, BK) – low x regions
- $k_T$ for larger x  (relevant for SPD) – TMD factorization
- Another approach to GI: transverse momentum only in parton distributions
- Transition? Application of effective action at larger x
- Possible reason (Soffer,OT) : convex $x^a(1-x)^b$
- Approximate validity of Regge ~ $x^a$ at rather large x~0.1

# TMDs and GPDs

- Hadronic and partonic transverse momenta
- Variables $k_T{}^2$ vs $t$
- Models (AdS/QCD) using overlap of LCWF – relation (Maji, Mondal, Chakrabarti, OT'15)

$$\frac{\partial}{\partial |t|}[\ln(\mathrm{GPD})] = \frac{(1-x)^2}{4}\frac{\partial}{\partial p_\perp^2}[\ln(\mathrm{TMD})].$$

# Special interest to GPDs: pressure in proton

- Universal concept at all scales
- Similarity to stable macroscopic objects in all known cases
- Transition to HIC – similarity to hadronic physics (c.f. “Ridge”)

# Pressure –related to D-term (Poyakov'03) and to holographic SR (OT'05)

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y))\delta(z - x - \xi y)$$

- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Also for exclusive DY! – OT'05 and work in progress

# SR in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta \qquad \Delta = 2\int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1}$$

$$\Delta^p_{\mathrm{CQM}}(2) = \Delta^n_{\mathrm{CQM}}(2) \approx 4.4, \qquad \Delta^p_{\mathrm{latt}} \approx \Delta^n_{\mathrm{latt}} \approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

# From D-term to pressure

- Inverse -> 1$^{st}$ moment (model)
- Kinematical factor – moment of pressure $C \sim <p\, r^4>$ ($<p\, r^2> = 0$)

M.Polyakov'03

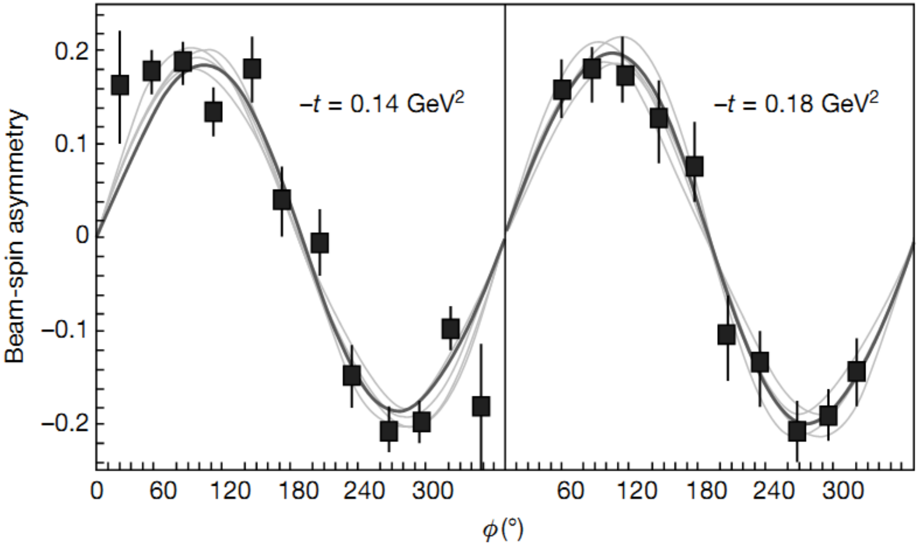
$$T^{Q}_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^{Q}_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- Stable equilibrium $C>0$:

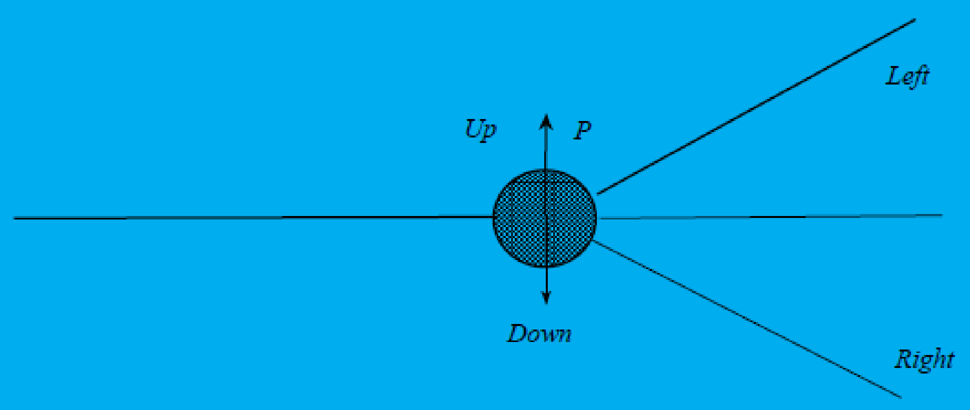
# Experiment

- Jlab, TJNAF, CEBAF
- Very accurate data
- Imaginary part from Single Spin Asymmetry



# Single Spin Asymmetries: simplest example

Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi\vec{N} \to \pi N$




$M = a + ib(\vec{\sigma}\vec{n})$ $\vec{n}$ is the normal to the scattering plane.

Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n})$, $A = \frac{2Im(ab^*)}{|a|^2+|b|^2}$

# Single Spin Asymmetries

Main properties:

– Parity: transverse polarization

– Imaginary phase – can be seen from T-invariance or technically - from the imaginary $i$ in the (quark) density matrix

Various mechanisms – various sources of phases

# Phases in QCD

- QCD factorization – soft and hard parts-
- Phases form soft, hard and overlap
- Assume (generalized) optical theorem – phase due to on-shell intermediate states – positive kinematic variable (= their invariant mass)
- Hard: Perturbative (a la QED: Barut, Fronsdal

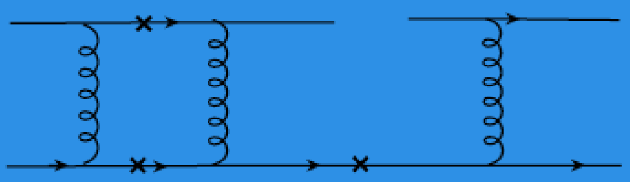
(1960):

Kane, Pumplin, Repko (78) Efremov (78)

# Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q - e$ scattering in DIS):

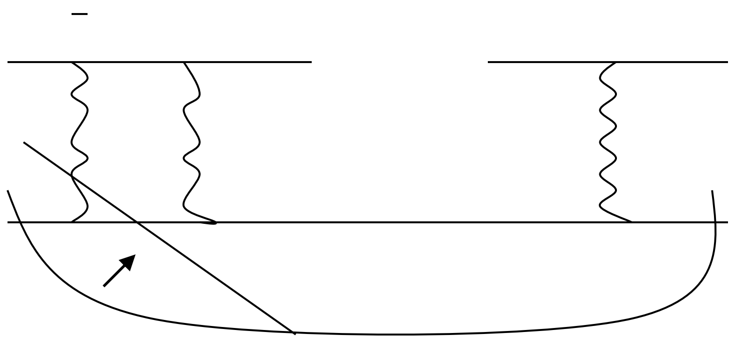


$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

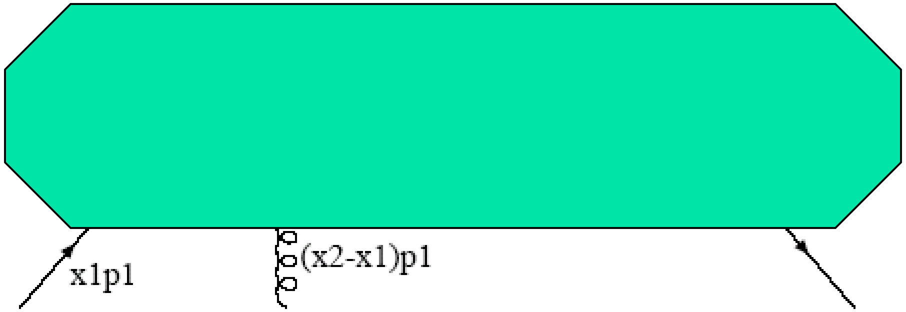
# Short+ large overlap– twist 3

- Quarks – only from hadrons
- Various options for factorization – shift of SH separation (prototype of duality)



- New option for SSA: Instead of 1-loop twist 2 – Born twist 3: Efremov, OT (85, Ferminonc poles); Qiu, Sterman (91, GLUONIC poles)

# Quark-gluon correlators

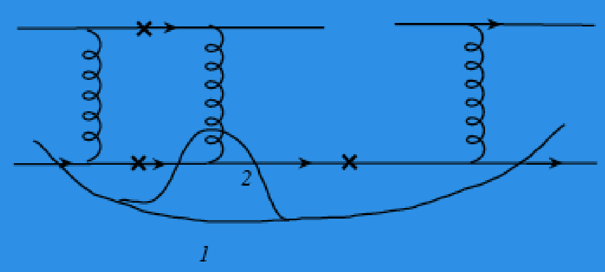



- Non-perturbative NUCLEON structure – physically mean the quark scattering in external gluon field of the HADRON.
- Depend on TWO parton momentum fractions
- For small transverse momenta – quark momentum fractions are close to each other- gluonic pole; probed if :
  $Q >> P_T >> M$

$$x_2 - x_1 = \delta = \frac{p_T^2 x_B}{Q^2 z}$$

# Twist 3 correlators

Escape: QCD factorization - possibility to shift the borderline between large and short distances




At short distances - Loop $\rightarrow$ Born diagram

At Large distances - quark distribution $\rightarrow$ quark-gluon correlator.

Physically - process proceeds in the external gluon field of the hadron.

Leads to the shift of $\alpha_S$ to non-perturbative domain AND

"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m p_T}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

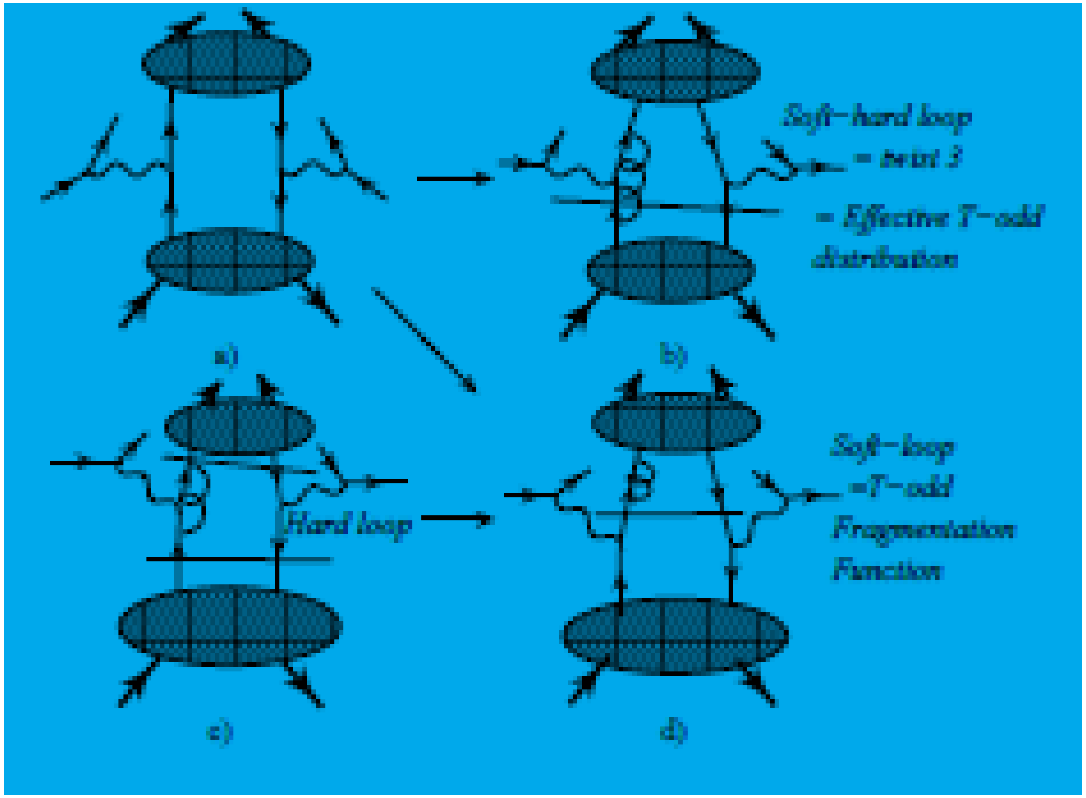
Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.

# Phases in QCD-Large distances in distributions

- Distributions: Sivers, Boer and Mulders – no positive kinematic variable producing phase
- QCD: Emerge only due to (initial of final state) interaction between hard and soft parts of the process
- Brodsky -Hwang-Schmidt model:the same SH interactions as twist 3 but non-suppressed by Q: Sivers function – leading (twist 2).
- Related in various complementary ways

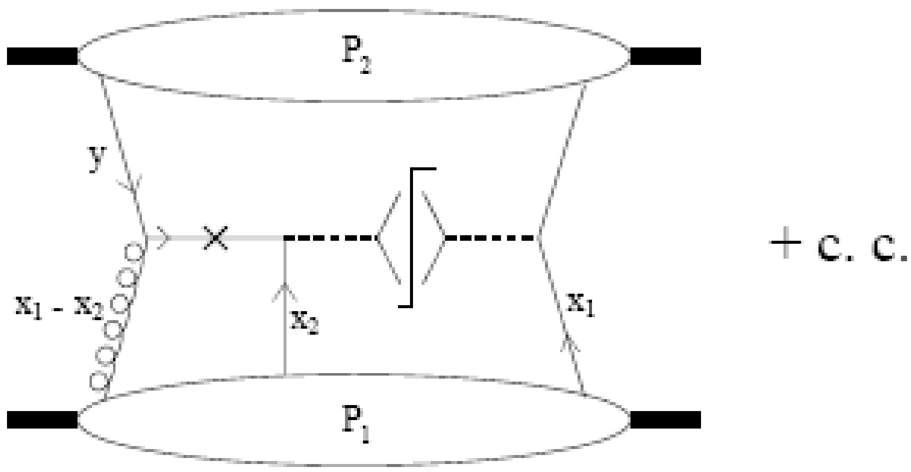
# SSAs in SIDIS

- Various opportunities for phases generation



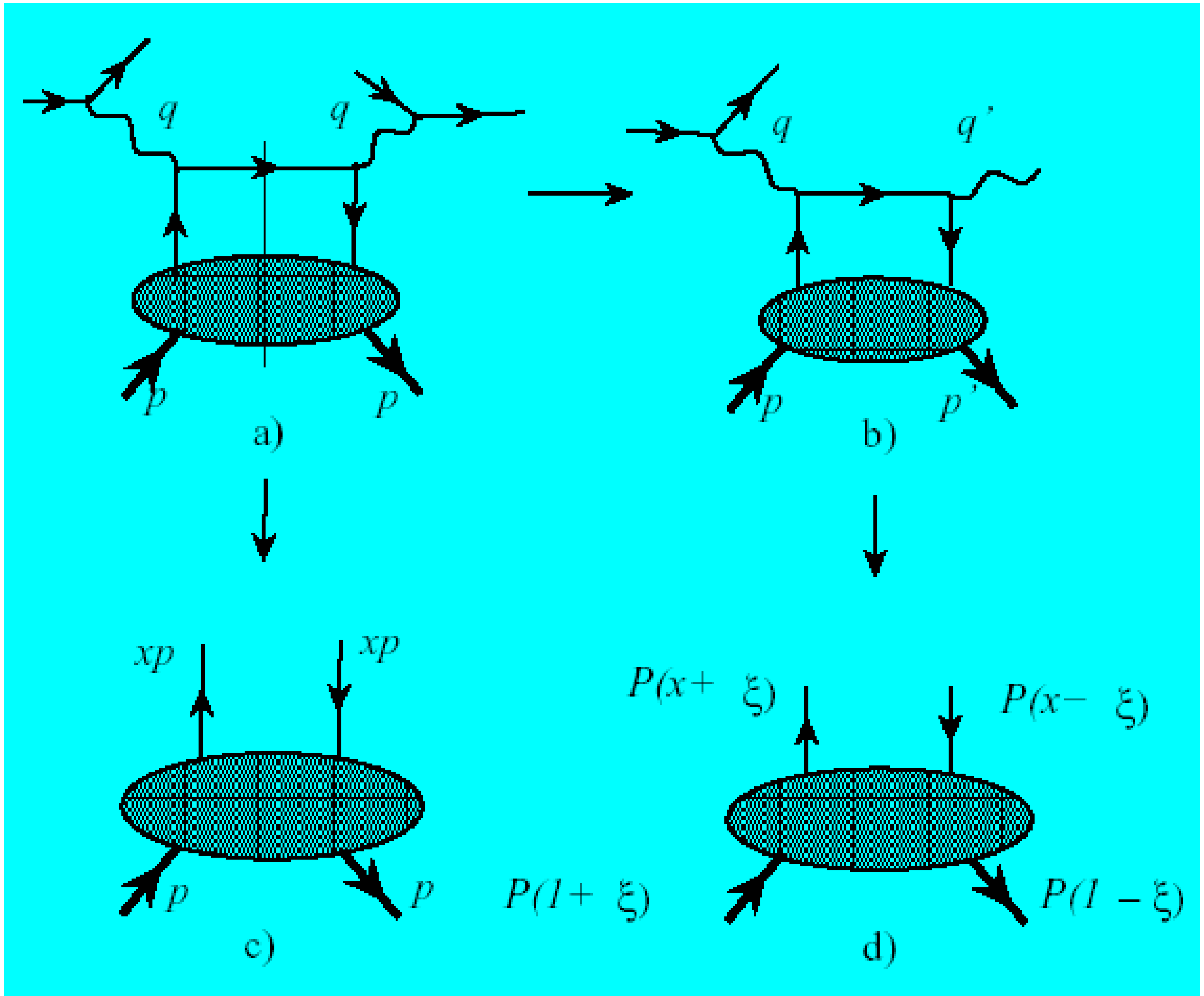
# SSA in DY

- TM integrated DY with one transverse polarized beam– unique SSA – glu onic pole (Anikin,OT –factor 2)
- Important for lower M (SPD)

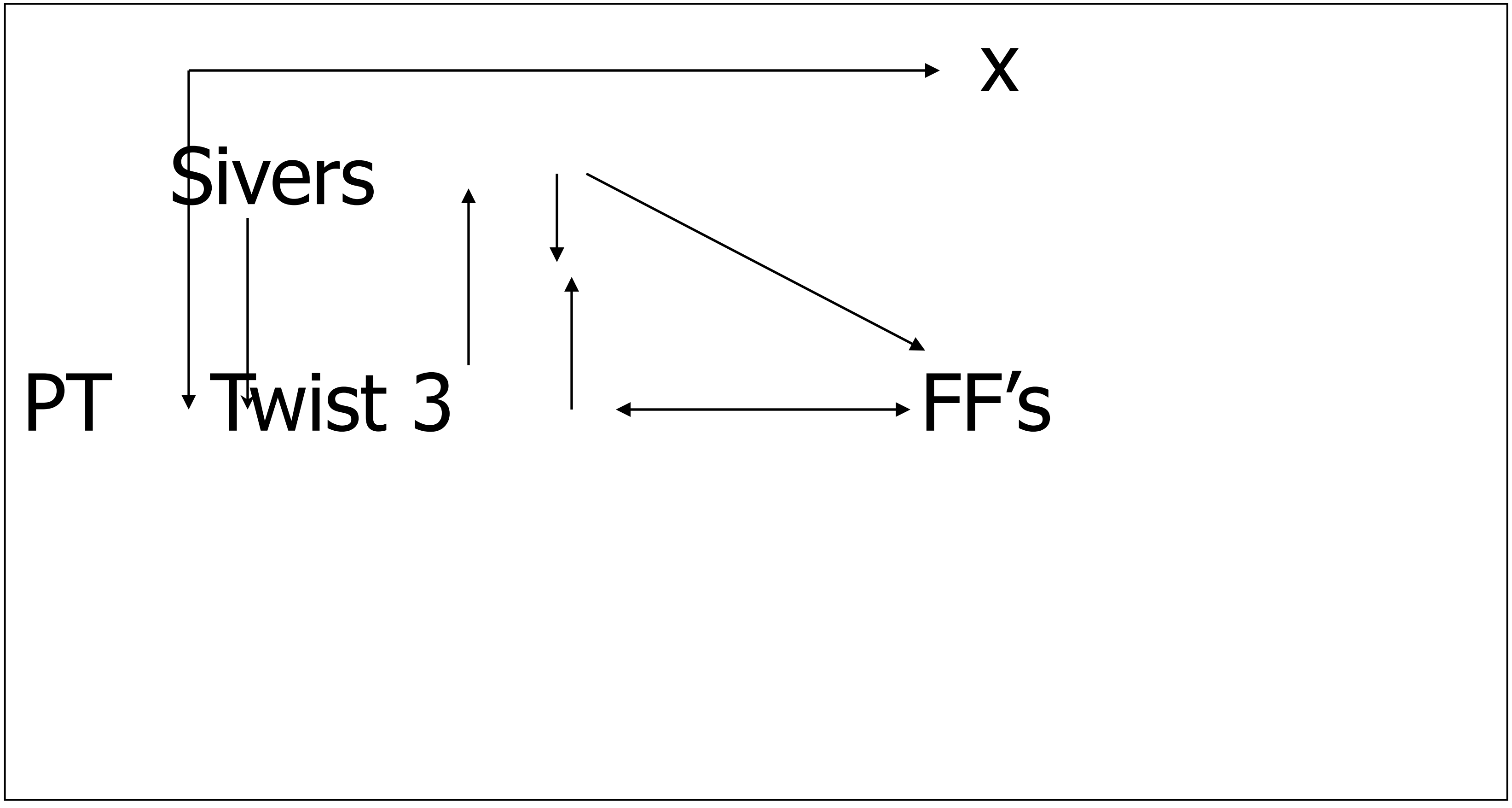



$$A = g\,\frac{\sin 2\theta\ \cos\phi\left[T(x,x) - x\frac{dT(x,x)}{dx}\right]}{M\left[1+\cos^2\theta\right]q(x)}$$

# GPDs – another source of T-odd effects



# Kinematical domains for SSA's



# Λ-polarisation

- Self-analyzing in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: ’87): Randomization – smearing – no direction normal to the scattering plane

# Global polarization

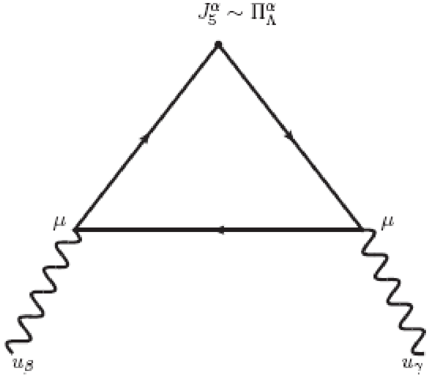
- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- How to transform rotation to spin?

# Anomalous mechanism – polarization similar to CM(V)E

- 4-Velocity is also a GAUGE FIELD (V.I. Zakharov)

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT '10)
- Analogous to anomalous gluon contribution to nucleon spin (Efremov,OT'88)
- 4-velocity instead of gluon field!



# Energy dependence

- Coupling -> chemical potential

$$Q_5^s = \frac{N_c}{2\pi^2}\int d^3x\, \mu_s^2(x)\gamma^2\epsilon^{ijk}v_i\partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- Large  chemical potential: appropriate for NICA/FAIR energies

M. Lisa, for the STAR collaboration , QCD Chirality Workshop, UCLA, February 2016;
SQM2016, Berkeley, June 2016

# One might compare the prediction below with the right panel figures

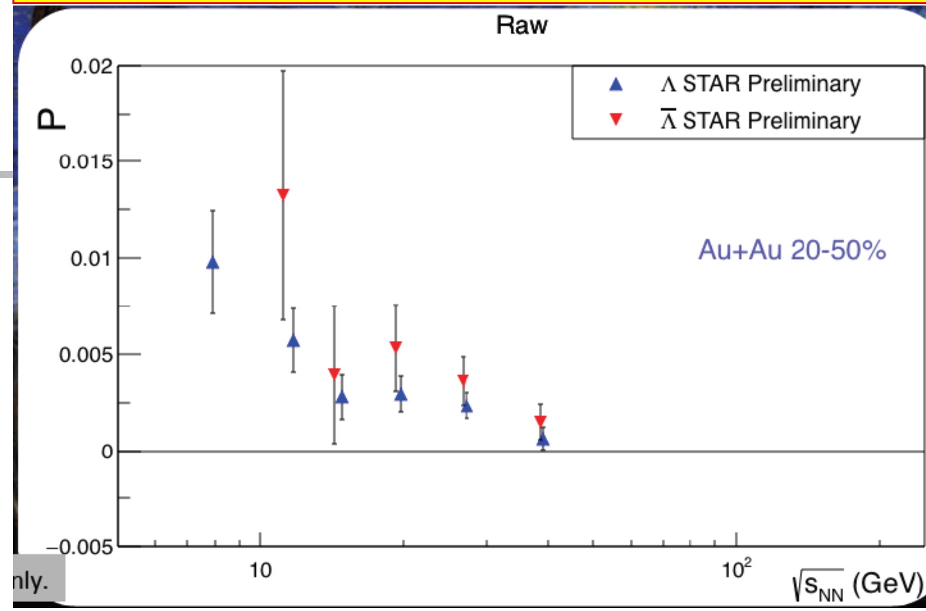
**O. Rogachevsky, A. Sorin, O. Teryaev**
**Chiral vortic effect and neutron asymmetries in heavy-ion collisions**
**PHYSICAL REVIEW C 82, 054910 (2010)**

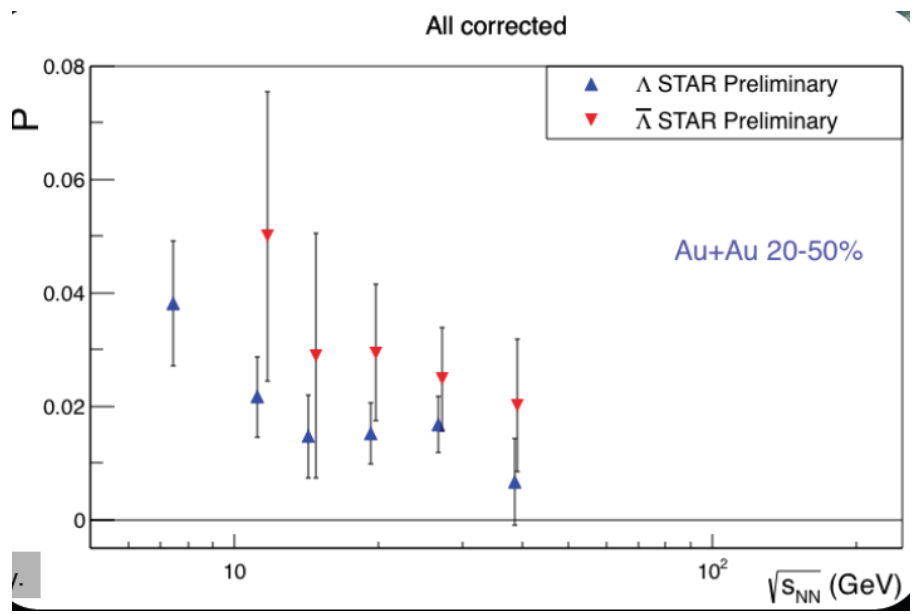
One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where $n$ and $\epsilon$ are the corresponding charge and energy densities and $P$ is the pressure. Therefore, the $\mu$ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.



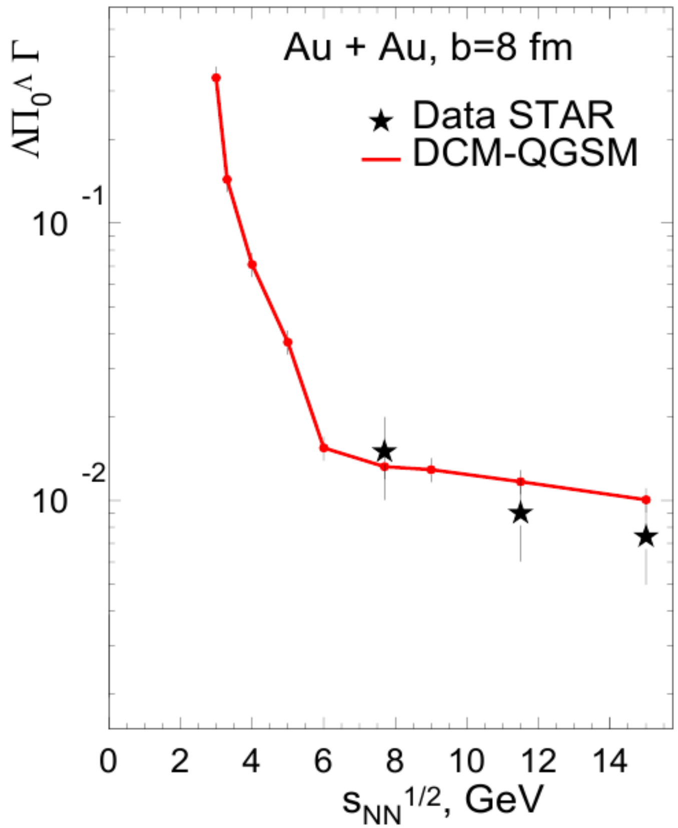



# Another NATURE article

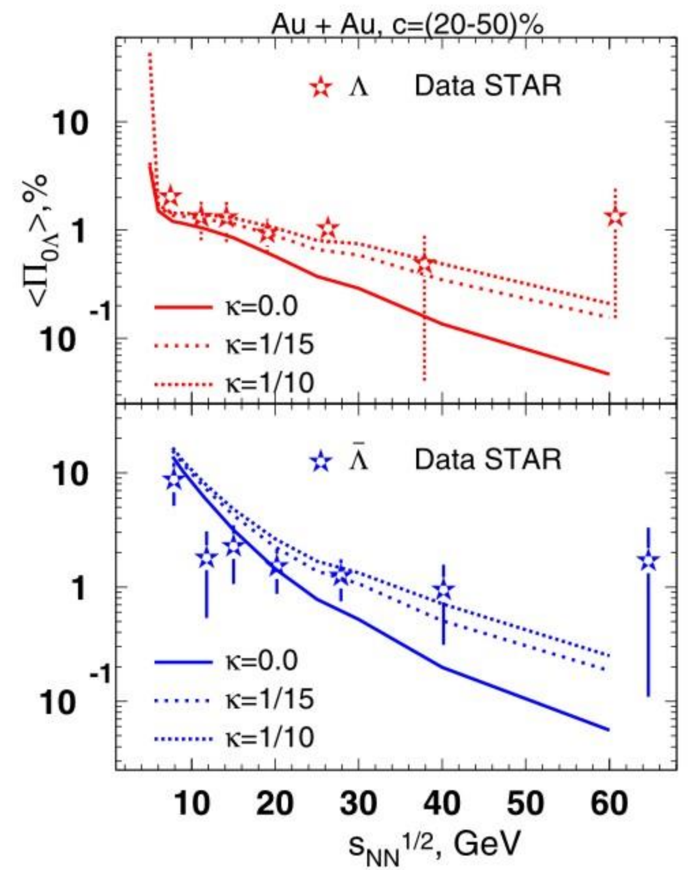
- **Global $\Lambda$ hyperon polarization in nuclear collisions**
- The STAR Collaboration
- Journal name:Nature Volume: 548, Pages:62–65 Date published: (03 August 2017

# Energy dependence (Baznat, Gudima, Sorin,OT)

- Growth at low energy
- Close to STAR data!
- Baryon-antibaryon successfully described - but a lot of work ahead



# Λ vs Anti Λ

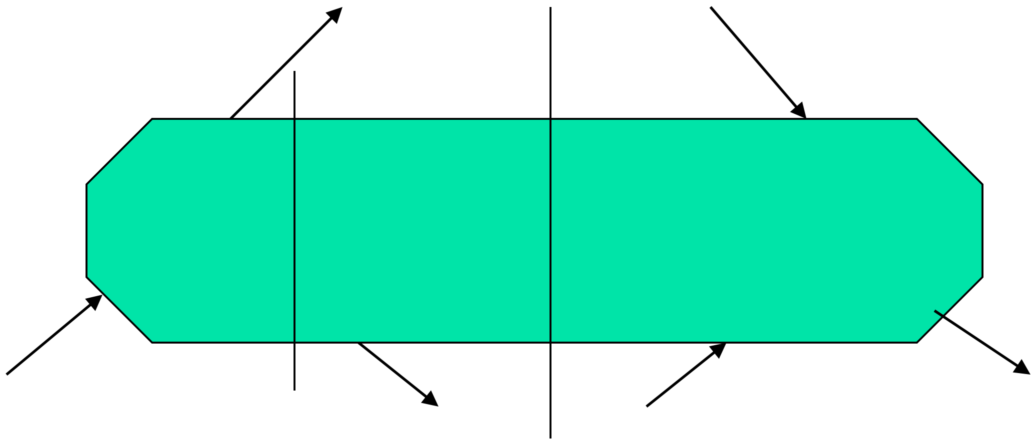


Au + Au, c=(20-50)%
Λ Data STAR
10
1
10^-1
<Π0Λ>,%
κ=0.0
κ=1/15
κ=1/10
Λ̄ Data STAR
10
1
10^-1
κ=0.0
κ=1/15
κ=1/10
10 20 30 40 50 60
sNN^1/2, GeV

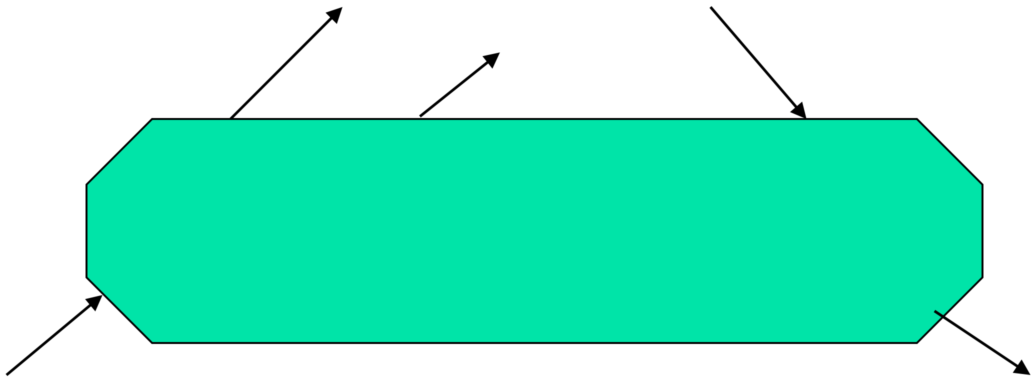
# Fracture functions

- Common NP ingredient for FRAgmentation and struCTURE
- Structure functions – parton distributions
- Fracture functions – fractural (conditional,correlational,entangling?) parton distributions
- May be T-odd (Collins'95 –polarized beam jets; OT'01-T-odd Diffractive Distributions)
- Related by crossing to dihadron fragmentation functions

# (T-odd) Fractural (conditional) parton distributions



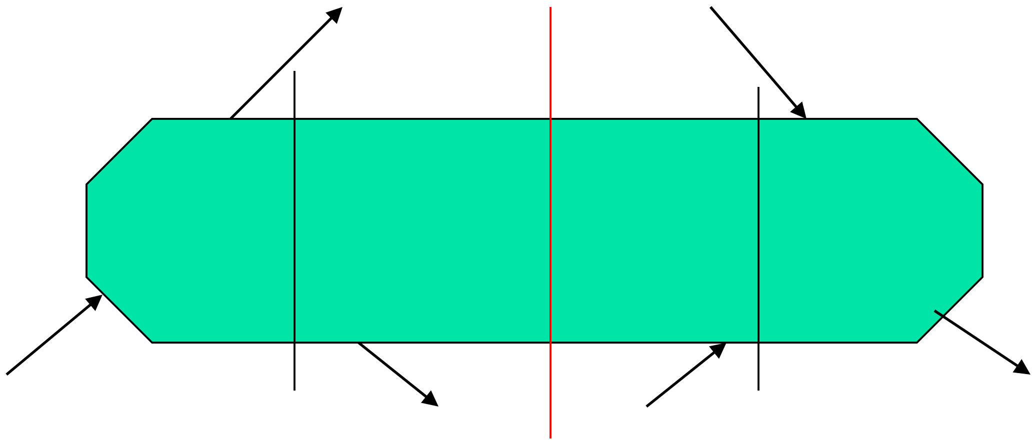
# HT parton distributions



# T-odd fracture function for hyperons polarization

- May be formally obtained from spin-dependent T-odd DIS (cf OT'99 for pions SSA- work in progress)
- Transverse spin in DIS – either transverse spin or transverse momentum of hyperon in SIDIS
- Both longitudinal and transverse polarizations appear
- SPD – extra hadrons (pions) with low TM

# GPDs in exclusive limit of fractured distributions



# Problems for NICA

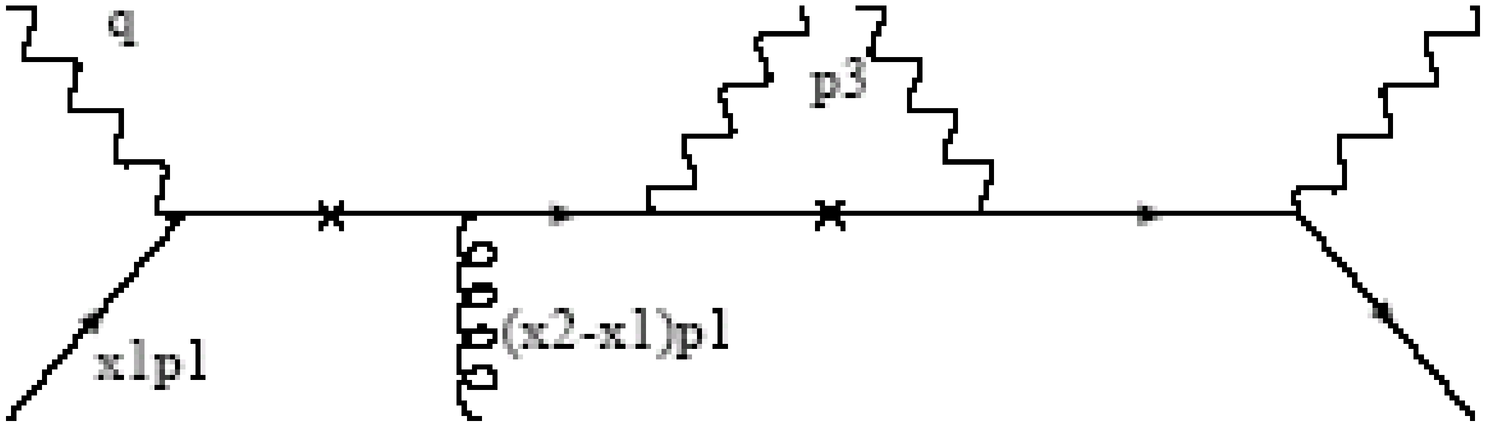
- SPD LoI: TMDs@DY
- TMDs – J/Ψ, ɣ
- GPDs: Exlusive DY-type (smaller x-section but lower background)
- GPDs from TMDs (pressure?!)
- Fracture – SSAs with extra hadrons
- Relation of HIC/hadronic spin (MPD/SPD) – polarization for hadrons, light and heavy ions

- BACKUP

# Fractural PD

- **Frac´tur`al a.1.**Pertaining to, or consequent on, a fracture.

# Twist 3 partonic subprocesses for SIDVCS



# Real and virtual photons - most clean tests of QCD

- Both initial and final – real :Efremov, O.T. (85)
- Initial – quark/gluon, final - real : Efremov, OT (86, fermionic poles); Qui, Sterman (91, GLUONIC poles)
- Initial - real, final-virtual (or quark/gluon) – Korotkiian, O.T. (94)
- Initial –virtual, final-real: O.T., Srednyak (05; smooth transition from fermionic via hard to GLUONIC poles).

# Sivers function and formfactors

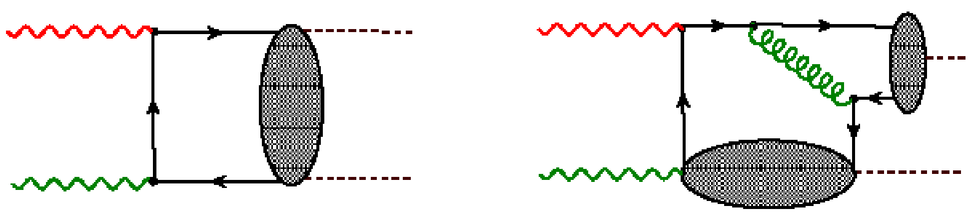
- Relation between Sivers and AMM known on the level of matrix elements (Brodsky, Schmidt, Burkardt)
- Phase?
- Duality for observables?
- Solution: SSA in DY

# SSA in exclusive limit

- Proton-antiproton – valence annihilation - cross section is described by Dirac FF squared
- The same SSA due to interference of Dirac and Pauli FF's with a phase shift
- Exclusive large energy limit; x -> 1 : (d/dx)T(x,x)/q(x) -> Im F2/F1
- No suppression of large x – large E704 SSA
- Positivity: Twist 4 correction to q(x) may be important

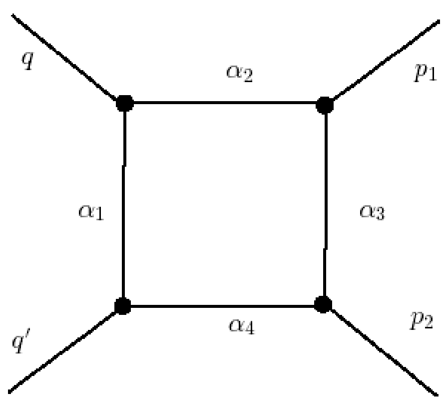
# mechanisms for exclusive amplitudes (Anikin, Cherednikov, Stefanis, OT, 08)

- 2 pion production : GDA (small s) vs TDA+DA (small t)



- Scalar model - asymptotics(Efremov,

Ginzburg, Radyushkin…)



# Duality and helicity amplitudes

- Holds if different mechanisms contribute to SAME helicity amplitudes
- Scalar- only one; QCD – L and T photons
- Other option : Different mechanisms – different helicity amplitudes (“unmatching”)
- Example -> transition from perturbative phase to twist 3 (m -> M)

# Twist 3 factorization (Efremov, OT '84, Ratcliffe,Qiu,Sterman)

- Convolution of soft (S) and hard (T) parts

$$d\sigma_s = \int dx_1 dx_2 \frac{1}{4} Sp[S_\mu(x_1,x_2) T_\mu(x_1,x_2)]$$

- Vector and axial correlators: define hard process for both double ($g_2$) and single asymmetries

$$T_\mu(x_1,x_2) = \frac{M}{2\pi}(\hat{p}_1 \gamma^5 s_\mu b_A(x_1,x_2) - i\gamma_\rho \epsilon^{\rho\mu s p_1} b_V(x_1,x_2))$$

# Twist 3 factorization -II

- Non-local operators for quark-gluon correlators

$$b_A(x_1,x_2) = \frac{1}{M}\int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \langle p_1, s|\bar{\psi}(0)\hat{n}\gamma^5(D(\lambda_1)s)\psi(\lambda_2)|p_1, s\rangle,$$

$$b_V(x_1,x_2) = \frac{i}{M}\int \frac{d\lambda_1 d\lambda_2}{2\pi} e^{i\lambda_1(x_1-x_2)+i\lambda_2 x_2} \epsilon^{\mu s p_1 n} \langle p_1, s|\bar{\psi}(0)\hat{n}D_\mu(\lambda_1)\psi(\lambda_2)|p_1, s\rangle$$

- Symmetry properties (from T-invariance)

$$b_A(x_1,x_2) = b_A(x_2,x_1),\ \ b_V(x_1,x_2) = -b_V(x_2,x_1)$$

# Twist-3 factorization -III

- Singularities

$$b_A(x_1,x_2) = \varphi_A(x_1)\delta(x_1 - x_2) + b^r_A(x_2,x_1).$$

$$b_V(x_1,x_2) = \frac{\varphi_V(x_1)}{x_1 - x_2} + b^r_V(x_1,x_2)$$

- Very different: for axial – Wandzura-Wilczek term due to intrinsic transverse momentum
- For vector-GLUONIC POLE (Qiu, Sterman ’91) – large distance background

# Sum rules

- EOM + n-independence (GI+rotational invariance) –relation to (genuine twist 3) DIS structure functions

$$\int_0^1 x^n \bar{g}_2(x)dx = \int_0^1 x^n (\frac{n}{n+1} g_1(x) + g_2(x))dx =$$

$$-\frac{1}{\pi(n+1)} \int_{|x_1,x_2,x_1-x_2|\le 1} dx_1 dx_2 \sum_f e_f^2 [\frac{n}{2} b_V(x_1,x_2)(x_1^{n-1} - x_2^{n-1}) +$$

$$b_A^r(x_1,x_2)\phi_n(x_1,x_2)], \quad \phi_n(x,y) = \frac{x^n - y^n}{x-y} - \frac{n}{2}(x^{n-1} - y^{n-1}), \quad n = 0, 2...$$

# Sum rules -II

- To simplify – low moments

$$\int_0^1 x^2 \hat{g}_2(x)dx =$$
$$-\frac{1}{3\pi}\int_{|x_1,x_2,x_1-x_2|\le 1} dx_1 dx_2 \sum_f e_f^2 b_V(x_1,x_2)(x_1-x_2)$$

- Especially simple – if only gluonic pole kept:

$$\int_0^1 x^2 \bar{g}_2(x)dx = -\frac{1}{3\pi}\int_{|x_1,x_2,x_1-x_2|\le 1} dx_1 dx_2 \sum_f e_f^2 \varphi_V(x_1)$$
$$= -\frac{1}{3\pi}\int_{-1}^1 dx_1 \sum_f e_f^2 \varphi_V(x_1)(2-|x_1|)$$

# Gluonic poles and Sivers function

- Gluonic poles – effective Sivers functions-Hard and Soft parts talk, but SOFTLY
- Implies the sum rule for effective Sivers function (soft=gluonic pole dominance assumed in the whole allowed x's region of quark-gluon correlator)

$$x f_T(x) = \frac{1}{2M} T(x,x) = \frac{1}{4}\phi_v(x)$$

$$\int_0^1 dx x^2 \bar{g}_2(x) = \frac{4}{3\pi}\int_0^1 dx x\, f_T(x)(2-x)$$

# Compatibility of SSA and DIS

- Extractions of and modeling of Sivers function: – “mirror” u and d
- Second moment at % level
- Twist -3 $g_2$ - similar for neutron and proton and of the same sign – no mirror picture seen –but supported by colour ordering!
- Scale of Sivers function reasonable, but flavor dependence differs qualitatively.
- Inclusion of pp data, global analysis including gluonic (=Sivers) and fermionic poles
- HERMES, RHIC, E704 –like phonons and rotons in liquid helium; small moment and large E704 SSA imply oscillations
- JLAB –measure SF and g2 in the same run

# Outlook (high energies)

- TMD vs UGPD
- T-odd UGPD?
- T-odd (P/O) diffractive distribiutions

(analogs - also at small energies)

- Quark-hadron duality: description of gluon coupling to “exotic” objects in diffractive production via their decay widths

# Relation of Sivers function to GPDs

- Qualitatively similar to Anomalous Magnetic Moment (Brodsky et al)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (**hep-ph/0612205** ): $x f_T(x) \propto xE(x)$

- Burkardt SR for Sivers functions is now related to Ji SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_T(x) = \sum_{q,G} \int dx x E(x) = 0$$

# How gravity is coupled to nucleons?

- Current or constituent quark masses ?– neither!
- Energy momentum tensor - like electromagnertic current describes the coupling to photons

# Sivers function and Extended Equivalence principle

- Second moment of E – zero SEPARATELY for quarks and gluons –only in QCD beyond PT (OT, 2001) - supported by lattice simulations etc.. ->
- Gluon Sivers function is small! (COMPASS, STAR, Brodsky&Gardner)
- BUT: gluon orbital momentum is NOT small: total – about 1/2, if small spin – large (longitudinal) orbital momentum
- Gluon Sivers function should result from twist 3 correlator of 3 gluons: remains to be proved!