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# MUON PAIR PRODUCTION IN STRONG INTERACTIONS AND THE ASYMPTOTIC SUM RULES

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STANFORD LINEAR ACCELERATOR CENTER

# MUON PAIR PRODUCTION IN STRONG INTERACTIONS AND THE ASYMPTOTIC SUM RULES

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## 1. Introduction

It has been noted in recent theoretical papers that study of highly inelastic scattering of leptons by protons is one of the most important means of investigating the structure of particles at short distances[1-3]. Recent experiments on the Stanford accelerator[4] have revealed a very interesting behavior of the inelastic amplitudes, and have been used as the basis for the development and verification of a number of theoretical ideas[5-7]. On the other hand, a number of restrictions were established in refs. 8 and 9 for the asymptotic behavior of the inelastic amplitudes for processes involving the participation of only strongly interacting particles. These restrictions were derived from some very general principles of field theory. Generalization of these results to inelastic processes involving the participation of leptons should lead to exact restrictions for the inelastic form factors.

In this paper we shall investigate muon pair production for highly inelastic collisions of two hadrons at high energies. It is shown that at high energies and high momentum transfers the form factors for this process are related to the matrix elements of equal-time electromagnetic current commutators. The asymptotic sum rules obtained by this procedure can be used as a basis for verifying the structure of the electromagnetic hadron current. In particular, the predictions of the quark current algebra and the field algebra for the muon polarization turn out to be qualitatively different. Studies of muon polarization in the above process are also essential for the analysis of experiments involving searches for the W-meson in proton-proton collisions using polarized muon detection at large angles[10]. The close connection between the W-meson production and the electromagnetic production of muon pairs has already been discussed in the literature.

In Part II we shall discuss the kinematics of the process and will introduce the necessary definitions and notation.

In Part III it will be shown how the cross section for the physical process is related to the electromagnetic current commutator.

In Part IV we shall derive the sum rules relating the limiting values of the form factors on the one hand and the equal-time commutators for the spatial components of the electromagnetic current and the time derivatives, on the other.

In Appendix I we shall consider the isolation of the contributions due to uncoupled diagrams, and in Appendix II we shall analyse some kinematic questions.

## II. Kinematics, Notation, and Definitions

Consider the inelastic collision between a hadron (a) and a proton, which results in the creation of a muon pair and a system of hadrons N. In the lowest-order electromagnetic interaction the process proceeds through the emission and decay of a virtual photon $\gamma^*$ :

$$a + p \rightarrow \underset{\mu^+ \ \mu^-}{\gamma^*} + N . \qquad (2.1)$$

The incident hadron can be a $\pi^{\pm}$ meson, a proton, or an antiproton:

$a = \pi^{\pm}, p, \bar{p}$ The notation which we shall employ for the moment is defined in Figure 1 :

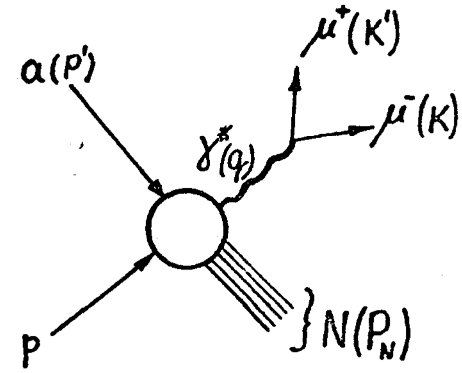


Figure 1

The conservation laws for the 4-momentum are of the form

$$p' + p = q + p_N , \qquad (2.2)$$

$$q = k' + k . \qquad (2.3)$$

The corresponding element of the T matrix is defined by the following expression

$$T_{if} = \frac{4\pi\alpha}{q^2} \epsilon^{\mu} < N\text{out} \mid J_{\mu}(0) \mid p p' \text{ in} >, \qquad (2.4)$$

where $\epsilon^{\mu} = \bar{u}(k)\gamma^{\mu} v(k')$ is the electromagnetic current associated with the muon pair, $J_\mu(x)$ is the operator for the electromagnetic current of hadrons, and $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ is the fine structure constant. The symbol c shows that the uncoupled parts in the matrix element of the current must be ignored.

The total cross section for the process defined by (2.1) in the case of unpolarized particles a and p, summed over the polarizations of the lepton pair, can be written in the form

$$\sigma = \frac{4\pi^2\alpha^2}{\sqrt{(pp')^2 - m^2 m'^2}} \int \frac{d^4q}{(2\pi)^4} \frac{\pi(q^2)}{q^2} \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \rho_{\mu\nu}, \qquad (2.5)$$

where m and m' are the masses of the proton and the particle a, respectively. In the above expressions we have used the following notation and definitions :

$$\rho_{\mu\nu}(p, p', q) = \sum_N (2\pi)^4 \delta(p + p' - q - p_N) \langle pp' \text{in} \mid J_\mu(0) \mid N\text{out}\rangle^c \langle N\text{out} \mid J_\nu(0) \mid pp'\text{in}\rangle^c \qquad (2.6)$$

$$\sum \int \frac{d\vec{k}\, d\vec{k}'}{(2\pi)^6\, 2k_0 2k_0'} (2\pi)^4 \delta(k' + k - q) \epsilon^\mu \epsilon^\nu =$$
$$= \sqrt{\frac{q^2 - 4m_\mu^2}{q}} \int \frac{d\Omega}{8\pi^2} \left(k_\mu k_\nu' + k_\nu k_\mu' - g_{\mu\nu} \frac{q^2}{2}\right) = \qquad (2.7)$$
$$= \pi(q^2)(-q^2 g^{\mu\nu} + q^\mu q^\nu),$$

where

$$\pi(q^2) = \frac{1}{4\pi}\left(1 - \frac{q^2 - 4m_\mu^2}{3q^2}\right) \sqrt{\frac{q^2 - 4m_\mu^2}{q^2}}\Bigg|_{m_\mu \equiv 0} = \frac{1}{6\pi}. \qquad (2.8)$$

The quantity $d\Omega$ is the solid angle element around the direction of the momentum of one of the muons in the center of mass system of the muon pair $\vec{q} = 0$; and $m_\mu$ is the muon mass.

Conservation of the current demands that the tensor $\rho_{\mu\nu}(p, p' q)$ can be expanded in terms of five linearly independent, gauge-invariant structures:

$$\rho_{\mu\nu}(p,p',q) = \rho_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \rho_2 \mathcal{P}_\mu \mathcal{P}_\nu + \rho_3 \mathcal{P}'_\mu \mathcal{P}'_\nu + \\ + \rho_4(\mathcal{P}_\mu \mathcal{P}'_\nu + \mathcal{P}_\nu \mathcal{P}'_\mu) + i\rho_5(\mathcal{P}_\mu \mathcal{P}'_\nu - \mathcal{P}_\nu \mathcal{P}'_\mu), \tag{2.9}$$

where

$$\mathcal{P}_\mu = p_\mu - \frac{p \cdot q}{q^2} q_\mu \qquad \mathcal{P}'_\mu = p'_\mu - \frac{p' \cdot q}{q^2} q_\mu . \tag{2.10}$$

All the form factors $\rho_i$ are real scalar functions of four independent invariant variables constructed out of the vectors p, p' and q. For example, we can choose the variable $q^2$ (the square of the mass of the virtual photon) and the usual Mandelstam variables s, t, u:

$$\begin{aligned} s &= (p+p')^2 = m^2 + m'^2 + 2m\epsilon \\ t &= (p'-q)^2 = \Delta^2, \\ u &= (p-q) = m^2 + q^2 - 2\nu, \end{aligned} \tag{2.11}$$

where the invariant $\nu = p.q$ in the laboratory system ($\vec{p} = 0$) is proportional to the virtual photon energy and $\epsilon = \frac{1}{m}(pp')$ is the energy of the incident particle in the laboratory system. If we introduce one further invariant, namely $m_N^2 = p_N^2$ (the square of the invariant mass of the hadron system), there will be a linear relation between the five invariants :

$$s + t + u = q^2 + m_N^2 + m^2 + m'^2. \tag{2.12}$$

It will be convenient to expand the tensor $\rho_{\mu\nu}$ in terms of the structures corresponding to definite polarizations of the virtual photon. The directions of the polarization vectors for the virtual photon in the laboratory system ($\vec{p} = 0$) will be defined as shown in Figure 2 :

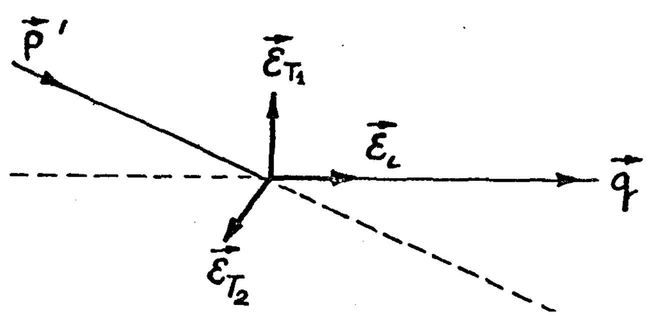


Figure 2

The corresponding relativistic polarization four-dimensional vectors are of the form

$$\epsilon_{\mu}^{(L)} = \frac{1}{\sqrt{-\mathcal{P}^{2}}} \mathcal{P}_{\mu} \, . \tag{2.13a}$$

$$\epsilon_{\mu}^{(T_1)} = \left[ \frac{\mathcal{P}^{2}}{(\mathcal{P}\mathcal{P}')^{2} - \mathcal{P}^{2}\mathcal{P}'^{2}} \right]^{1/2} \left( \mathcal{P}'_{\mu} - \frac{\mathcal{P}\mathcal{P}'}{\mathcal{P}^{2}} \mathcal{P}_{\mu} \right), \tag{2.13b}$$

$$\epsilon_{\mu}^{(T_2)} = \frac{1}{\sqrt{q^{2}}} \; \frac{1}{\sqrt{(p.p')^{2} - m^{2}m'^{2}}} \, \epsilon_{\mu\nu\lambda\rho} \; p_{\nu}\, p'_{\lambda}\, q_{\rho} \, , \tag{2.13c}$$

where

$$\mathcal{P}^{2} = \frac{1}{q^{2}} (m^{2} q^{2} - \nu^{2}),$$

$$\mathcal{P}'^{2} = \frac{1}{q^{2}} (m'^{2} q^{2} - (p'q)^{2}), \tag{2.14}$$

$$\mathcal{P}\mathcal{P}' = \frac{1}{q^{2}} (m \, \epsilon \, q^{2} - \nu (p'q)).$$

It is readily seen that the polarization vectors have the following properties :

$$\epsilon_{\mu}^{(i)} q^{\mu} = 0 \, , \quad \epsilon_{\mu}^{(i)} \epsilon^{(k)\mu} = -\delta_{ik} \qquad (i,k = T_1, T_2, L),$$

$$\sum_{i=T_1, T_2, L} \epsilon_{\mu}^{(i)} \epsilon_{\nu}^{(i)} = -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^{2}}. \tag{2.15}$$

Using (2.13) and (2.15), we can expand (2.9) so that it takes the form

$$\rho_{\mu\nu} = \rho_{T_1} \epsilon_{\mu}^{(T_1)} \epsilon_{\nu}^{(T_1)} + \rho_{T_2} \epsilon_{\mu}^{(T_2)} \epsilon_{\nu}^{(T_2)} + \rho_{L} \epsilon_{\mu}^{(L)} \epsilon_{\nu}^{(L)} +$$

$$+ \rho_{TL}^{(+)} ( \epsilon_{\mu}^{(L)} \epsilon_{\nu}^{(T_1)} + \epsilon_{\nu}^{(L)} \epsilon_{\mu}^{(T_1)} ) + i \rho_{TL}^{(-)} ( \epsilon_{\mu}^{(L)} \epsilon_{\nu}^{(T_1)} - \epsilon_{\nu}^{(L)} \epsilon_{\mu}^{(T_1)} ), \tag{2.16}$$

where

$$\rho_L = \rho_1 - \mathcal{P}^2 \rho_2 - \frac{(\mathcal{P}\cdot\mathcal{P}')^2}{\mathcal{P}^2}\rho_3 - 2\mathcal{P}\cdot\mathcal{P}' \rho_4 , \tag{2.17a}$$

$$\rho_{T_1} = \rho_1 - \frac{(\mathcal{P}\cdot\mathcal{P}')^2 - \mathcal{P}^2\mathcal{P}'^2}{\mathcal{P}^2} \rho_3 , \tag{2.17b}$$

$$\rho_{T_2} = \rho_1 . \tag{2.17c}$$

$$\rho^{(+)}_{TL} \pm i\rho^{(-)}_{TL} = \frac{\mathcal{P}\cdot\mathcal{P}'}{\mathcal{P}^2}[\mathcal{P}^2\mathcal{P}'^2 - (\mathcal{P}\cdot\mathcal{P}')^2]^{1/2}\rho_3 + [\mathcal{P}^2\mathcal{P}'^2 - (\mathcal{P}\cdot\mathcal{P}')^2]^{1/2}(\rho_4 \pm i\rho_5). \tag{2.17d}$$

Substituting (2.16) into the formula for the total cross section for the process given by (2.5), we obtain :

$$\sigma = \frac{4\pi^2\alpha^2}{\sqrt{(p\cdot p')^2 - m^2 m'^2}} \int \frac{d^4q}{(2\pi)^4} \frac{\pi(q^2)}{q^2} \rho(s, q^2, \Delta^2, \nu), \tag{2.18}$$

where

$$\rho = (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2})\rho_{\mu\nu} = \rho_{T_1} + \rho_{T_2} + \rho_L . \tag{2.19}$$

In (2.18) the integration with respect to the momentum of the virtual photon can be reduced to integration with respect to the invariant variables. For example,

$$d^4q = \frac{1}{4\sqrt{(p\cdot p')^2 - m^2 m'^2}} dq^2\, d\Delta^2\, d\nu\, d\phi, \tag{2.20}$$

where $\phi$ is the azimuthal angle.

The limits of integration can be found from the conservation laws which define the physical domain of the process (2.1) (see Appendix II).

Since we shall be concerned with high energies of the colliding hadrons for which $\epsilon \gg m$, $\epsilon \gg m'$, we have

$$\sigma(\epsilon) = \frac{a^2}{8\pi}\frac{1}{m\epsilon^2}\int_0^{2m\epsilon}\frac{dq^2}{q^2}\pi(q^2)\int_{q^2-2m\epsilon}^{0} d\Delta^2 \int_{-\frac{\Delta^2}{2m}}^{\epsilon^* + \frac{\Delta^2}{4\epsilon^*}} d\delta\, \rho(s, q^2, \Delta^2, \delta), \qquad (2.21)$$

where $\epsilon^* = \epsilon \frac{\Delta^2}{\Delta^2 - q^2}$, and $\delta = \frac{1}{m} p\Delta$ is the momentum transfer in the laboratory system.

We note that the total cross section for the process (2.1) is determined only by the form-factor sum $\rho_{T_1} + \rho_{T_2} + \rho_L$, and is independent of $\rho_{TL}^{(\pm)}$. The various form factors can be measured separately by investigating the angular distribution of the muon momentum directions in the center of mass system of the pair in which $\vec{q} = 0$.

If we take the direction of the momentum $\vec{p}$ in the rest system of the pair to lie along the z-axis, and the direction toward the production plane to be the y-direction, we obtain

$$\vec{k} = -\vec{k}' = |\vec{k}|\ (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta). \qquad (2.22)$$

As a result, the normalized angular distribution is found to be[11] (see also ref.12);

$$W(\theta,\phi) = \frac{1}{4\pi\rho}\,\frac{1}{(1-\frac{v^2}{3})}\,\{\rho_{T_1}(1-v^2\sin^2\theta\cos^2\phi)+$$
$$+\rho_{T_2}(1-v^2\sin^2\theta\sin^2\phi)+\rho_L(1-v^2\cos^2\theta)-\rho^{(+)}_{TL}v^2\sin 2\theta\cos\phi\}, \quad (2.23)$$

where $\rho = \rho_L + \rho_{T_1} + \rho_{T_2}$, and $v = \frac{|\vec{k}|}{k_0} = \sqrt{\frac{q^2-4m_\mu^2}{q^2}}$ is the velocity of the muons in the $\vec{q} = 0$ system.

It was noted in ref.11 that the form factor $\rho^{(-)}_{TL}$ is proportional to the polarization of one of the muons along the normal to the production plane, i.e. along the y-axis. We note also that measurement of the form factors $\rho_T \equiv \rho_{T_1}+\rho_{T_2}$ и $\rho_L$ can be carried out by investigating the distribution only with respect to $\Theta$ :

$$W(\theta) = \int_0^{2\pi} d\phi \; W(\theta,\phi) =$$
$$= \frac{1}{2\rho(1-\frac{v^2}{3})}\{\rho_L(1-v^2\cos^2\theta)+\rho_T(1-\frac{v^2}{2}\sin^2\theta)\}. \quad (2.24)$$

III. Current commutators

Let us relate the quantity $\rho_{\mu\nu}(p,p',q)$ in the defintion of the cross section for the process (2.1) by the Fourier transform of the diagonal matrix element of the electromagentic current commutators :

$$A_{\mu\nu}(p,p',q) = \int dx\, e^{-iqx} \langle pp' in | [J_\mu(x), J_\nu(0)] | pp' in \rangle^c =$$

$$= a_{\mu\nu}(p,p',q) - a_{\nu\mu}(p,p'-q). \qquad (3.1)$$

It is understood that the particles in brackets are unpolarized. The symbol c shows that we are taking that part of the matrix element of the two-current commutator which is coupled as a whole. It is obvious that $A_{\mu\nu}$ and $a_{\mu\nu}$ can be expanded in terms of five independent, gauge invariant structures by analogy with the expansion given by (2.9), where

$$\underset{i=1,2,3,4}{A_i}(p,p',q) = a_i(p,p',q) - a_i(p,p'-q) \qquad (3.2a)$$

and

$$A_5(p,p';q) = a_5(p,p',q) + a_5(p,p',-q). \qquad (3.2b)$$

We note that the quantities $A_i$ have no definite symmetry properties with respect to $\nu$ for fixed values of the variables s, $q^2$, and $\Delta^2$. In fact, if we substitute $q \rightarrow -q$, we have

$$\Delta^2 \rightarrow \Delta'^2 = (p'+q)^2 = 2(m'^2 + q^2) - \Delta^2. \qquad (3.3)$$

However, if instead of the variable $\Delta^2$ we fix the ratio

$$a = \frac{p'\cdot q}{p\cdot q} = \frac{m'^2 + q^2 - \Delta^2}{2\nu}, \tag{3.4}$$

then the quantities $A_{i=1,2,3,4}(s, q^2, a, \nu)$ and $A_5(s, q^2, a, \nu)$ become odd and even functions of $\nu$ respectively.

Let us now discuss in greater detail the quantity $a_{\mu\nu}$. Using the completeness of the "out" - state vectors, we obtain

$$a_{\mu\nu}(p, p', q) = \sum_N^c (2\pi)^4\, \delta(p+p'-q-p_N)\, \langle pp'\text{in}\,|\, J_\mu(0)\,|\, N\,\text{out}\rangle\langle N\,\text{out}\,|\, J_\nu(0)\,|\, pp'\,\text{in}\rangle, \tag{3.5}$$

where the symbol c above the summation sign indicates that we are selecting only the matrix elements of the two-current product that are coupled as a whole.

Let us divide the matrix element $\langle N\,\text{out}\,|\, J_\mu(0)\,|\, pp'\,\text{in}\rangle$ into uncoupled and coupled parts in accordance with Figure 3 (see Appendix I).

As a result the quantity $a_{\mu\nu}$ can be written in the form

$$a_{\mu\nu}(p, p, q) = \rho_{\mu\nu}(p, p'; q) + \tilde{\tilde{\rho}}_{\mu\nu}(p, p'; q), \tag{3.6}$$

where the quantity $\rho_{\mu\nu}$ is the fully coupled part of the matrix element for the two-current product, defined by (2.6), and $\tilde{\tilde{\rho}}_{\mu\nu}$ represents the contribution of the fifteen weakly coupled generalized z-diagrams shown symbolically in Figure 4.

From the law of conservation of momentum and the spectral character condition it follows that for $q^2 > 0$:

1. $\rho_{\mu\nu}$ is nonzero for

$$\nu > 0 \ ; \ m_N^2 \le (\sqrt{s} - \sqrt{q^2}\,)^2; \tag{3.7a}$$

and

2. $\tilde{\tilde{\rho}}_{\mu\nu}$ is nonzero for

$$\nu < 0 \ ; \ m_N^2 \ge (\sqrt{s} + \sqrt{q^2}\,)^2. \tag{3.7b}$$

It is clear from (3.7b) that in the limit as $s \to \infty$ and $q^2 > 0$ the quantity $\tilde{\tilde{\rho}}_{\mu\nu}$, which corresponds to the contribution of the z-diagrams is determined by the intermediate states of the hadrons N with infinitely heavy effective masses $m_N = \sqrt{\rho_N^2}$. Hence it follows that when $q^2 > 0$

$$\lim_{\substack{s \to \infty \\ \frac{m_N^2}{s} \to 0}} \tilde{\tilde{\rho}}_{\mu\nu} = 0. \tag{3.8}$$

## IV. Dynamics in the case of infinite momentum and asymptotic sum rules

We shall show below that the problem of the behavior of the form factors for the muon-pair production process (2.1) at high energies of the colliding hadrons and high energies and masses of the virtual photon, when

$$\underbrace{s,\ q^2}\ ,\ \nu \to \infty$$

$$a = \frac{p'\cdot q}{p\cdot q} = \text{ fixed} \qquad , \omega = \frac{q^2}{2\nu} = \text{ fixed} \tag{4.1}$$

can be reduced to an investigation of the equal-time commutation relations between the spatial components of the operators for the electromagnetic hadron currents and their time derivatives.

The use of equal-time commutation relations is considerably simplified if we use the center of mass system of the muon pair, where $q = (q_0, 0)$. We note that, in this system, only the spatial components of the gauge-invariant tensors $A_{\mu\nu}$ and $\rho_{\mu\nu}$ are nonzero.

Integrating with respect to $q_0$ in (3.1) we obtain the following relations

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dq_0\, A_{ik}(p,p',q_0) = B_{ik}\ (\vec{p},\vec{p}\,'), \tag{4.2a}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} q_0\, dq_0\, A_{ik}(p,p',q_0) = C_{ik}\ (\vec{p},\vec{p}\,') \tag{4.2b}$$

and so on.

In these expressions

$$B_{ik}(\vec{p},\vec{p}\,') = \int d\vec{x} < p, p' \text{ in} | [J_i(\vec{x},0) J_k(0)] | pp' \text{in} >, \quad (4.3a)$$

$$C_{ik}(\vec{p},\vec{p}\,') = \frac{1}{i}\int d\vec{x} < pp' \text{ in} | [J_i(\vec{x},0), J_k(0)] | pp' \text{ in} >, \quad (4.3b)$$

where the equal time commutators are determined by the particular current-algebra model.

We note that in the above system ($\vec{q} = 0$) the invariant variables on which the form factors depend are given by

$$\begin{aligned} s &= m^2 + m'^2 + 2(p_0 p'_0 - \vec{p}\vec{p}\,'), \\ q^2 &= q_0^2, \\ \nu &= p_0 q_0, \\ a &= \frac{p_0}{p_0}. \end{aligned} \quad (4.4)$$

Hence it follows that the integration in (4.2) is carried out along the parabola $q^2 = \frac{\nu^2}{p_0^2}$ in the $(q^2, \nu)$ plane for fixed values of the variables s, a.

Such sum rules with arbitrary but fixed momenta p and $\vec{p}$ will, in general, contain z-diagram contributions. As was noted in the preceding section, in the limit as $s \to \infty$ and $q^2 > 0$ the contributions of the z-diagrams are determined by the intermediate states of the hadrons N with infinitely heavy effective masses $m_N$.

In accordance with the generally accepted ideology of the current algebra, we shall assume that the contributions of the z-diagrams vanish in the limit as $s \to \infty$.

This assumption turns out to be valid when we can change the order of the transition to the limit $s \to \infty$ and of the integration in (4.2). In fact, for example, in the case of the sum rules given by (4.2a) the z-diagram contribution is determined by expressions of the form :

$$\int_{-\infty}^{\infty} dq_0 \, \tilde{\tilde{\rho}}_{ik}(p,p',q_0) = - \int_{s}^{\infty} \frac{d m_N^2}{2E_N} \tilde{\tilde{\rho}}_{ik} \; (p,p',q_0). \qquad (4.5)$$

If we now pass the limit $s \to \infty$ under the integral sign for fixed $m^2_N$, and use (3.8), we find that the z-diagram contribution to the sum rules vanishes in this limit.

In the center of mass system of the lepton pair, transition to the limit $s \to \infty$ is realized when the following condition is satisfied :

$$p_0, p_0' \to \infty, \qquad (4.6)$$
$$\alpha = \frac{p_0'}{p_0} = \text{fixed} \qquad \beta = \frac{p_z'}{p_0'} = \text{fixed}$$

and the directions of the momenta are chosen so that

$$\vec{p} = \{0, 0, p_z\} \qquad \vec{p}\,' = \{p_x', 0, p_z'\} \qquad (4.7)$$

We note that for a fixed value of the variable $\omega = \frac{q^2}{2\nu}$ we have in the limit (4.6)

$$s, q^2 \quad \nu \to \infty \qquad \frac{s}{2\nu} = \frac{\alpha(1-\beta)}{2\omega} = \text{fixed} \quad ; \quad \frac{q^2}{2\nu} = \omega = \text{fixed} \tag{4.8}$$

We shall now assume that the following limits exist:

$$B_{ik}(\alpha,\beta) = \lim_{\substack{p_0, p_0' \to \infty \\ \alpha,\beta = \text{fixed}}} 2p_0 B_{ik}(\vec{p},\vec{p}\,') \tag{4.9a}$$

$$C_{ik}(\alpha,\beta) = \lim_{\substack{p_0, p_0' \to \infty \\ \alpha,\beta = \text{fixed}}} C_{ik}(\vec{p},\vec{p}\,'), \tag{4.9b}$$

where the tensors on the left-hand side of (4.9) are dimensionless model-dependent quantities.

Proceeding now to integration with respect to the variable $\omega = \frac{q^2}{2\nu}$, in the sum rules (4.2), it can be shown that, in the limit defined by (4.8), the form factors $\rho_i(s, q^2, \alpha, \nu)$ have the following behavior :

$$\rho_i(s, q^2, \alpha, \nu) \to \frac{\omega^2}{q^2} F_i(\alpha,\beta,\omega). \qquad i = T_1, T_2, L, TL \tag{4.10}$$

At the same time we have the relations

$$\frac{1}{\pi}\int_0^{\omega_0} \omega \, d\omega \, F_{T_1}(\alpha,\beta,\omega) = C_{xx}(\alpha,\beta), \tag{4.11a}$$

$$\frac{1}{\pi}\int_0^{\omega_0} \omega \, d\omega \, F_{T_2}(\alpha,\beta,\omega) = C_{yy}(\alpha,\beta), \tag{4.11b}$$

$$\frac{1}{\pi}\int_0^{\omega_0} \omega\, d\omega\ F_L(\alpha,\beta,\omega) = C_{zz}(\alpha,\beta), \qquad (4.11c)$$

$$\frac{1}{\pi}\int_0^{\omega_0} \omega\, d\omega\ F_{TL}^{(+)}(\alpha,\beta,\omega) = C_{xz}(\alpha,\beta) = C_{zx}(\alpha,\beta), \qquad (4.11d)$$

$$\frac{1}{\pi}\int_0^{\omega_0} d\omega\ F_{TL}^{(-)}(\alpha,\beta,\omega) = B_{xz}(\alpha,\beta) = - B_{zx}(\alpha,\beta), \qquad (4.11e)$$

where

$$\omega_0 = \frac{p_0 + p_0'}{2p_0} = \frac{1}{2}(1+\alpha).$$

We have written down only those relations which are parity nontrivial in the variable $q_0$. The above relations are model-independent and are the main results of the present work.

In the quark model, in which the interaction proceeds through a neutral vector field, the commutators are of the form

$$[J_i(\vec{x},0),\ J_j(0)] = 2i\,\delta(\vec{x})\,\epsilon_{ijk}\ \bar{\Psi}\gamma_0\Sigma_k Q^2\Psi\ , \qquad (4.12a)$$

$$[\dot{J}_i(\vec{x},0),\ J_j(0)] = -\delta(\vec{x})\,\{ i(\gamma_i\partial_j + \gamma_j\partial_i - 2\vec{\gamma}\vec{\partial}\,\delta_{ij}) -$$

$$- 2g(\gamma_i B_j + \gamma_j B_i - 2\vec{\gamma}\vec{B}\,\delta_{ij}) + 4M\delta_{ij}\}\, Q^2\Psi\ , \qquad (4.12b)$$

where

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}, \quad Q^2 = \frac{2}{9} + \frac{1}{3}Q. \qquad (4.13)$$

In the vector field model the commutators are

$$[J_i(\vec{x},0), J_j(0)] = 0, \tag{4.14a}$$

$$[\dot{J}_i(\vec{x},0), J_j(0)] = \delta(\vec{x})\, C_{ab}\, J_i^a(0)\, J_j^b(0) + c \text{ numbers} \tag{4.14b}$$

Using (4.14a), we find from the sum rule (4.11a) that the field-algebra model leads to

$$\int_0^{\omega_0} d\omega\, F_{TL}^{(-)}(\alpha,\beta,\omega) = 0. \tag{4.15}$$

Therefore, it may be concluded that a nonzero left-hand side in the sum rule (4.11e) is an unambiguous indication that the quark-field algebra is valid. It is important to carry out a more detailed study of the structure of the quantities $B_{ij}$ and $C_{ij}$.

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## Appendix I

### Calculation of the contribution of weakly coupled diagrams

If the state $\langle N\ \mathrm{out}|$ contains the particle $p$ or $p'$, the matrix element $\langle N\ \mathrm{out}|\, J_\mu(0)\, |p p'\ \mathrm{in}\rangle$ will contain uncoupled parts corresponding to free propagation of these particles. In particular,

$$\langle N\ \mathrm{out}|\, J_\mu(0)\, |pp'\mathrm{in}\rangle = \langle N\ \mathrm{out}|\, J_\mu(0)\, |pp'\ \mathrm{in}\rangle^c + \langle p|p\rangle\langle N\ \mathrm{out}|\, J_\mu(0)\, |p'\rangle^c + \langle p'|p'\rangle\langle N\ \mathrm{out}|\, J_\mu(0)\, |p\rangle^c + \langle pp'\mathrm{out}|pp'\mathrm{in}\rangle\langle N\ \mathrm{out}|\, J_\mu(0)\, |0\rangle ,$$

which can be shown graphically in the form

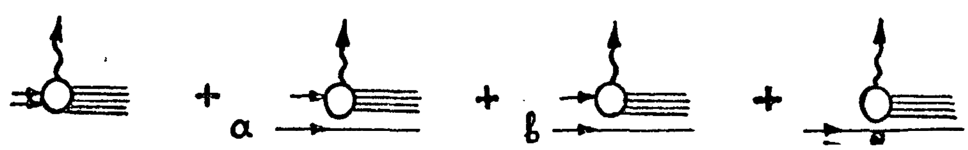


Figure 3

The first term respresents the fully coupled part of the matrix element and enters into the definition of the cross section [see (2.4)]. The remaining three terms represent the uncoupled parts and lead to the appearance of the so-called generalized z-diagrams. If we substitute this value of the matrix element into (3.3) for $a_{\mu\nu}(p,p',q)$, we obtain 16 different contributions to $a_{\mu\nu}$ :

$$a_{\mu\nu}(p,p',q) = \sum_N (2\pi)^4 \{ <pp'\text{in}|J_\mu|N\text{ out}>^c <N\text{ out}|J_\nu|pp'\text{in}>^c \delta(p+p'-q-p_N) +$$

$$+ <pp'\text{in}|J_\mu|N,p\text{ out}>^c <N\text{ out}|J_\nu|p'>^c \delta(p'-q-p_N) +$$

$$+ <pp'\text{in}|J_\mu|Np'\text{out}>^c <N\text{ out}|J_\nu|p>^c \delta(p-q-p_N) +$$

$$+ <pp'\text{in}|J_\mu|Npp'\text{out}>^c <N\text{ out}|J_\nu|0>\delta(p_N+q) +$$

$$+ <p|J_\mu|N\text{ out}>^c <Np'|J_\nu|pp'\text{in}>^c \delta(p-q-p_N) +$$

$$+ <p|J_\mu|N,p\text{ out}>^c <Np'|J_\nu|p'\text{in}>^c \delta(q+p_N) +$$

$$+ <p|J_\mu|Np'\text{out}>^c <Np'|J_\nu|p\text{ in}>^c \delta(p-p'-q-p_N) +$$

$$+ <p|J_\mu|Npp'\text{out}>^c <Np'|J_\nu|0>\delta(p'+q+p_N) +$$

$$<p'|J_\mu|N\text{ out}>^c <Np|J_\nu|pp'\text{in}>^c \delta(p'-q-p_N) +$$

$$<p'|J_\mu|N,p,\text{out}>^c <Np|J_\nu|p'>^c \delta(p'-p-q-p_N) +$$

$$<p'|J_\mu|Np'\text{out}>^c <Np|J_\nu|p>^c \delta(q+p_N)$$

$$<p'|J_\mu|N,pp',\text{out}>^c <Np|J_\nu|0> \delta(p+q+p_N)$$

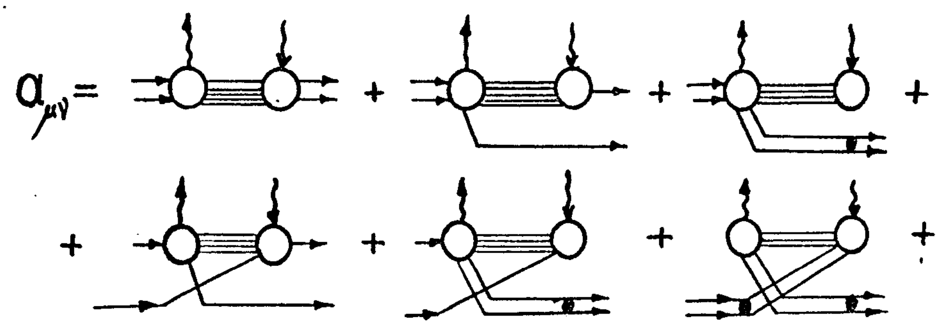
$$<0|J_\mu|N\text{ out}><Npp'|J_\nu|pp'\text{in}>^c \delta(q+p_N)$$

$$<0|J_\mu|Np\text{ out}><Npp'|J_\nu|p'>^c \delta(p+q+p_N)$$

$$<0|J_\mu|Np'\text{out}><Npp'|J_\nu|p>^c \delta(p'+q+p_N)$$

$$<0|J_\mu|Npp'\text{out}><Npp'|J_\nu|0>\delta(p+p'+q+p_N)\}$$

This expansion is shown symbolically in Figure 4 :



+ diagrams obtained by the symmetrization of the initial and final states

Figure 4

We thus have

$$a_{\mu\nu} = \rho_{\mu\nu} + \tilde{\rho}_{\mu\nu} ,$$

where $\tilde{\tilde{\rho}}_{\mu\nu}$ represents the sum of 15 weakly coupled z-diagrams. The properties of these diagrams were investigated above.

## A p p e n d i x  I I

Determination of the boundaries of the physical region*

The conservation of the 4-momentum is of the form

$$p' + p = q + p_N \,. \tag{A.1}$$

Substituting $\Delta = p' - q$, we have

$$p + \Delta = p_N \tag{A.2}$$

Hence

$$\Delta^2 = m_N^2 - m^2 - 2m\delta\,,$$

where $$\delta = \frac{1}{m}\, p\cdot\Delta = (\epsilon - q_0)$$

The case $m_N \equiv m$ corresponds to elastic scattering. $\Delta^2$ and $\delta$ are then uniquely related, i.e. they are not independent variables:

$$\delta = -\frac{\Delta^2}{2m}\,.$$

This gives the minimum value $\delta_{min}$, since $q_0$ is then a maximum. Consider the case where the virtual photon travels in the backward direction in the laboratory system. It is then clear that, for fixed invariants, it receives

---

*There is an interesting kinematic analogy between the reaction which we are investigating and inelastic neutron creation. Thus, if we replace the square of the lepton mass by our $q^2$ in the Appendix to ref.2, and $q^2$ in ref.2 by our $\Delta^2$, we reduce one problem to the other.

the minimum energy $(q_0)_{min}$, i.e.

$$\delta_{max} = \epsilon - (q_0)_{min} . \qquad (A.3)$$

Let us find this minimum energy $(q_0)_{min}$ from the relation

$$\Delta^2 = m'^2 + q^2 - 2\epsilon (q_0)_{min} - 2\sqrt{\epsilon^2 - m'^2}\,\sqrt{(q_0)^2_{min} - q^2} . \qquad (A.4)$$

Substituting $m' \equiv 0$, and solving this equation, we obtain

$$(q_0)_{min} = \frac{q^2 - \Delta^2}{4\epsilon} + \frac{\epsilon q^2}{q^2 - \Delta^2} \qquad (A.5)$$

and

$$\delta_{max} = \epsilon - (q_0)_{min} = \epsilon\left(1 - \frac{q^2}{q^2 - \Delta^2} - \frac{q^2 - \Delta^2}{4\epsilon}\right) = \qquad (A.6)$$

$$= \epsilon^* + \frac{\Delta^2}{4\epsilon^*},$$

where

$$\epsilon^* = \epsilon \frac{\Delta^2}{\Delta^2 - q^2} .$$

Therefore, in the physical region

$$-\frac{\Delta^2}{2m} \leq \delta \leq \epsilon^* + \frac{\Delta^2}{4\epsilon^*} . \qquad (A.7)$$

Let us now find the physical region of $\Delta^2$ for fixed $s$ and $q^2$. This is defined by the condition

$$\delta_{min} = \delta_{max} , \qquad (A.8)$$

and hence

$$\Delta^{2(-)} \leq \Delta^2 \leq \Delta^{2(+)}, \qquad (A.9)$$

where

$$\Delta^{2(\pm)} = \frac{q^2\epsilon + q^2 m - 2m\epsilon^2 \pm \epsilon\sqrt{4m^2\epsilon^2 + q^4 - 4q^2\epsilon m - 4q^2 m}}{2\epsilon + m}.$$

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